Dynamical systems tools to study Neurosciences & Atmospheric Flows

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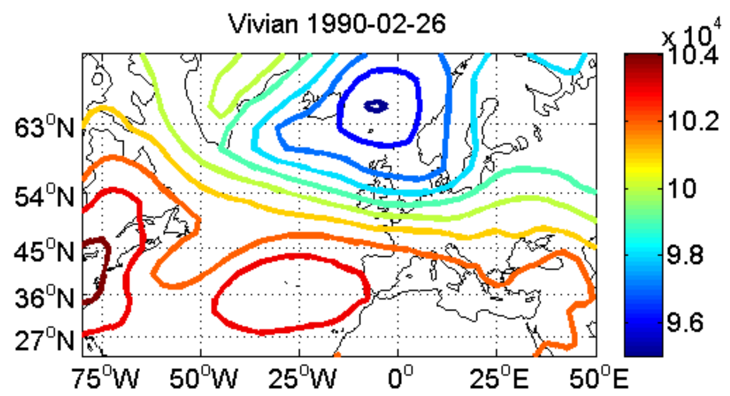
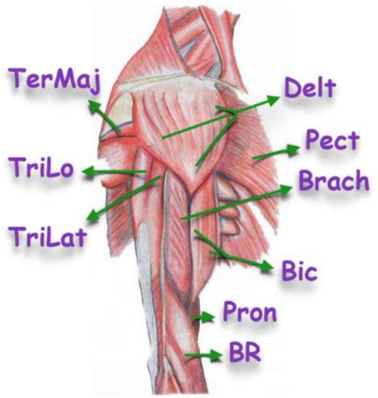
OBJECTIVES

Characterize the predictability of Configurations of systems



- *How recurrent?*
- *How rare and predictable?*
- *How persistent?*
- *Changes with parameters (age, greenhouse gases)*



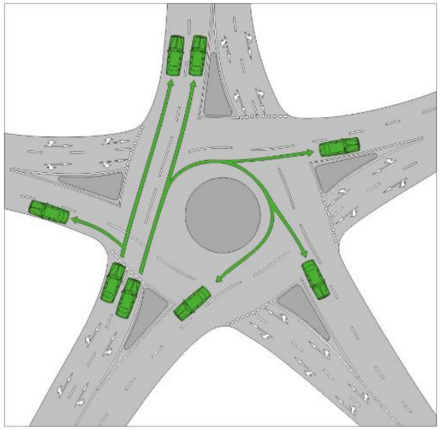
METHOD

Compute Dynamical Systems metrics to characterize atmospheric states, verifying that a long series of observations sample the underlying attractor.

Local Dimensions *d*

It is proportional to the number of possible configurations (**number of degrees of freedom**) originating and resulting from the atmospheric field analyzed.

~Number of possible exits in a roundabout



Persistence *ϴ*

Its inverse tells for **how long the atmospheric field will look like the one under examination**. For the present analysis ϴ is an inverse number of persistance days.

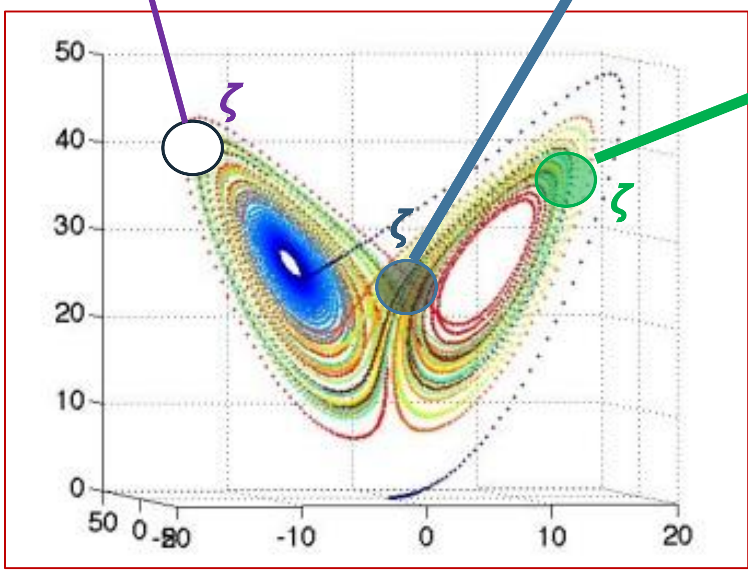
~1/Time spent choosing a direction in a roundabout

METHOD – LOCAL DIMENSIONS

Line: d(ζ)=1

Patch: d(ζ)=2

Fractal:1 < d(ζ) < 2



The local dimension depends on the point of the attractor considered

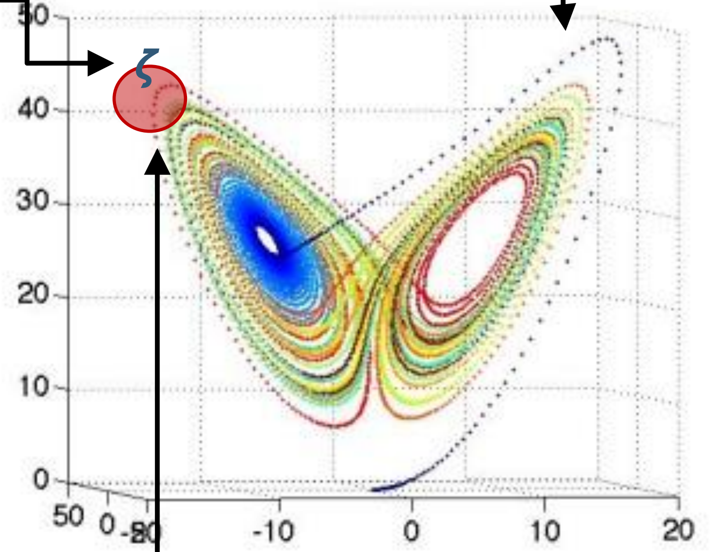
<= **Example of trajectory in the Lorenz 1963 attractor**, the colors indicate the time of the simulations, thus the chaotic behavior

METHOD – COMPUTE THE LOCAL DIMENSIONS

1) In a chaotic dynamical system, take a trajectory of the system: $f^m(x)$

2) Rare events are recurrences of a state ζ:

$$\boxed{X_m(x) = g(\mathrm{dist}(f^m(x), \zeta))}$$



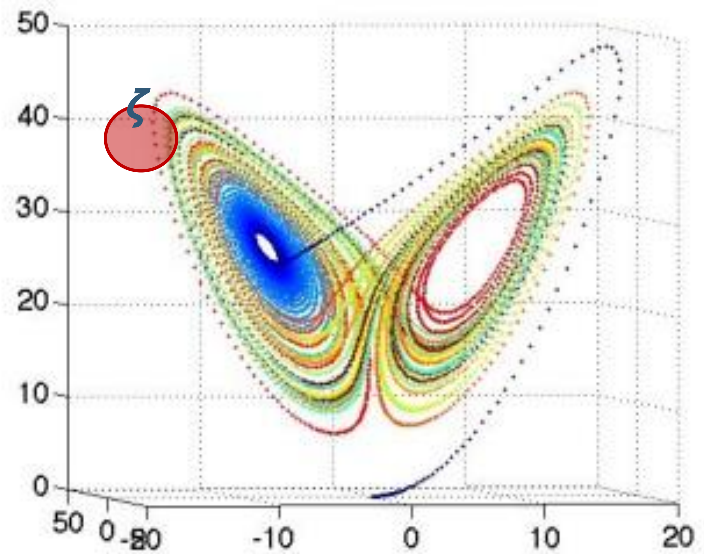
3) Then, chose observables such that the maxima of *g* correspond to minima of the distances with respect to ζ:

$$g_1^m(\zeta) = -\log(\mathrm{dist}(f^m(x), \zeta))$$

METHOD – COMPUTE THE LOCAL DIMENSIONS

For any chaotic systems entering in a ball close to ζ, is equivalent to study threeshold exceedances of:

$$P(g(x(t)) > q, \zeta)$$

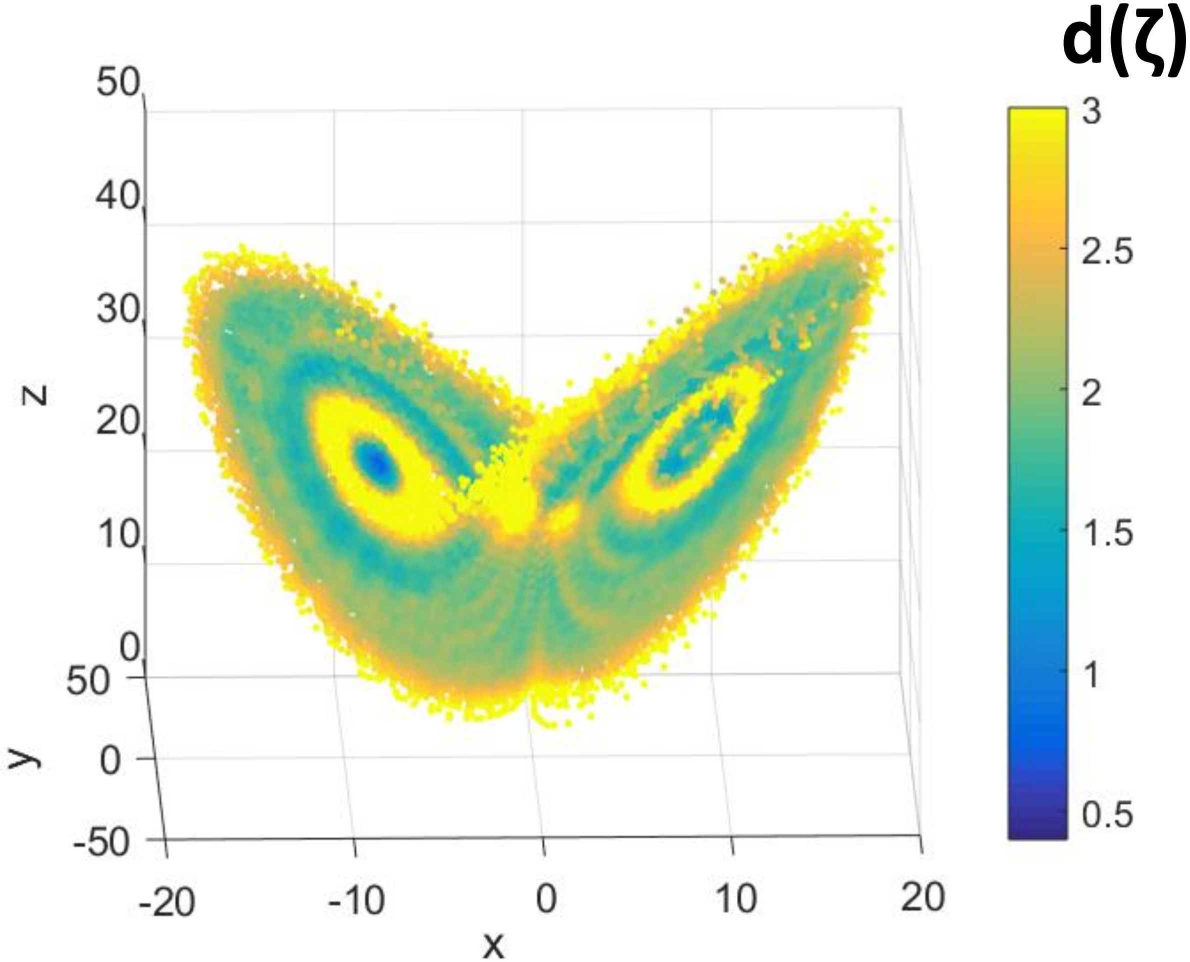


For the Freitas-Freitas-Todd theorem (2008) *P* converges asymptotically to an exponential function:

$$P(y, \zeta) = \exp(-[y-a(\zeta)]/\sigma(\zeta))$$

And **the local dimension is** $\boldsymbol{d(\zeta)=1/\sigma(\zeta)}$

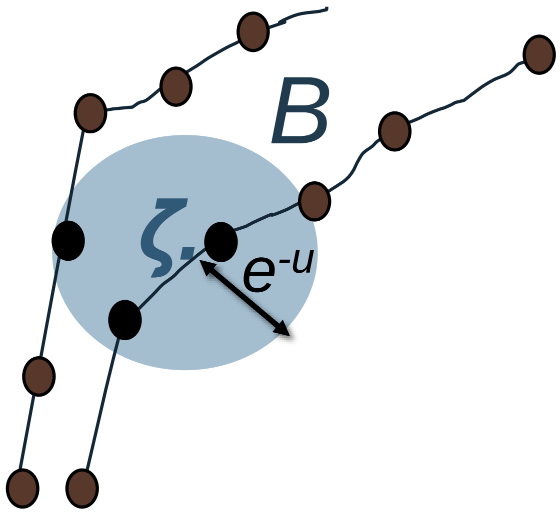
METHOD – DIMENSIONS IN LORENZ 1963

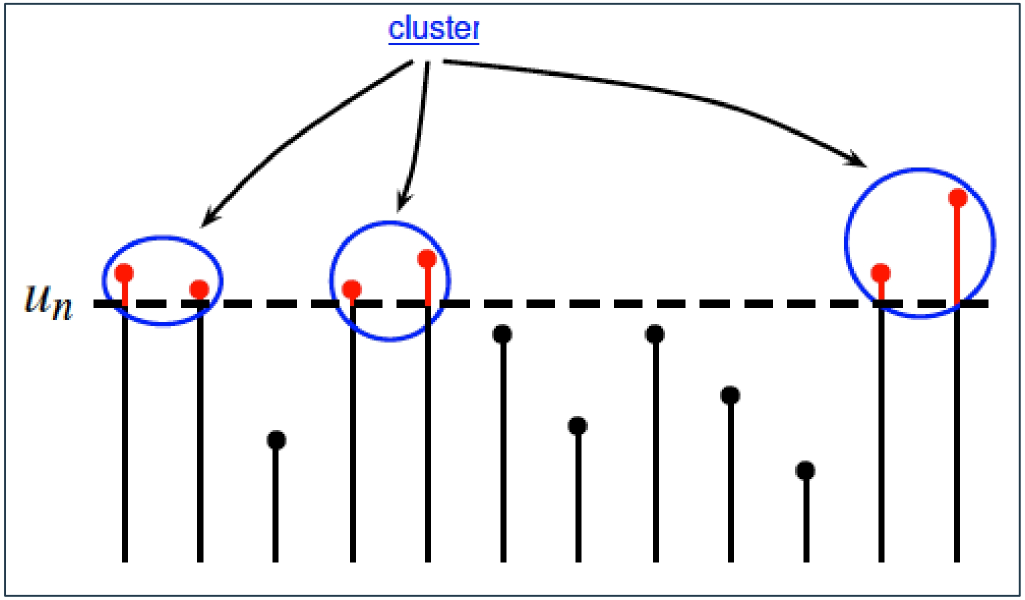


The average dimension $< d(\zeta) >_\zeta = 2.06$ is the same as classical estimates

Faranda, Messori, Yiou. Scientific Reports 2017

METHOD – LOCAL PERSISTENCE

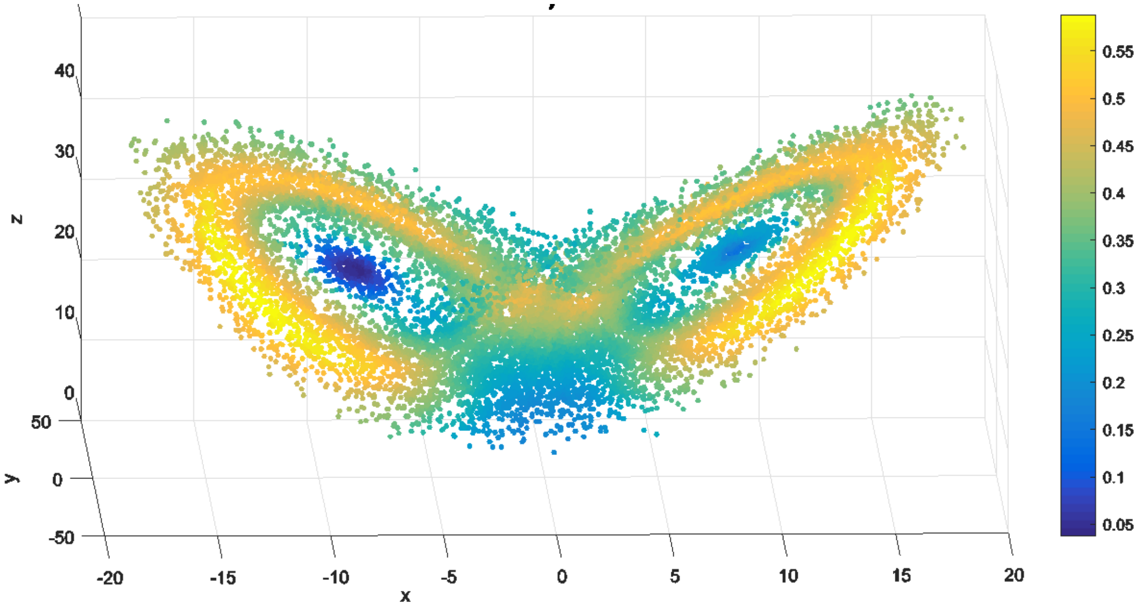




If a threshold u is applied to a series of observations x_1, x_2,...,x_s, the exceedances are those for which $x_i > u$. **The extremal index θ can then be thought of as the average inverse time spent above *u*.**

METHOD – PERSISTENCE IN LORENZ 1963

$\theta(\zeta)$



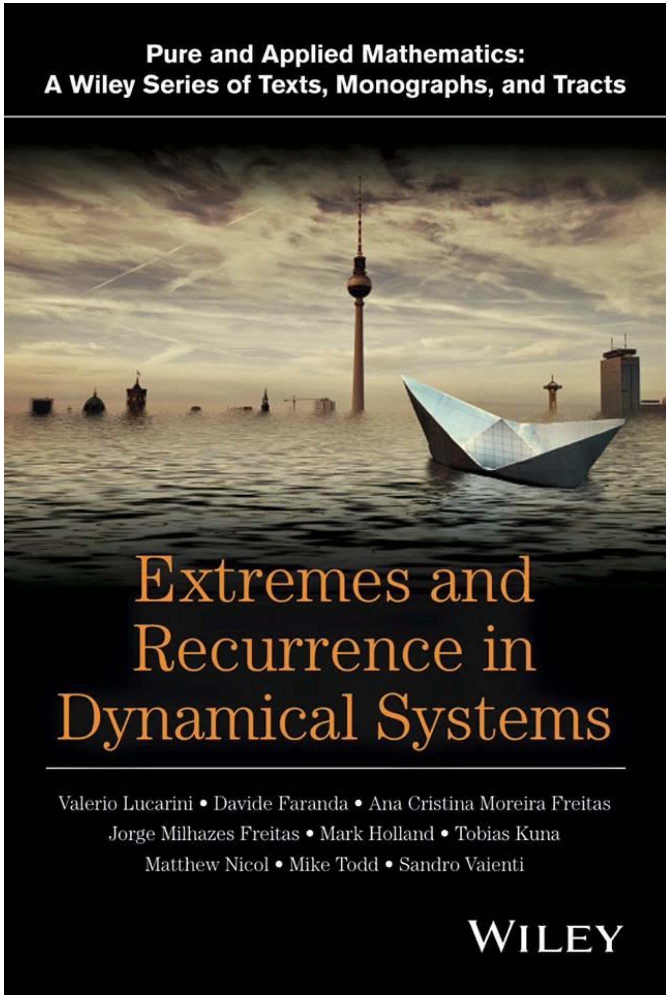
Less Persistent

More Persistent

θ is a good proxy for **unstable fixed points of the system**

Faranda, Messori, Yiou. Scientific Reports 2017

METHOD – REFERENCE



"Extremes and recurrence in Dynamical systems" contains new tools for estimating the local dimensions d(ζ)

Book: Lucarini, Faranda et al. Wiley (2016)

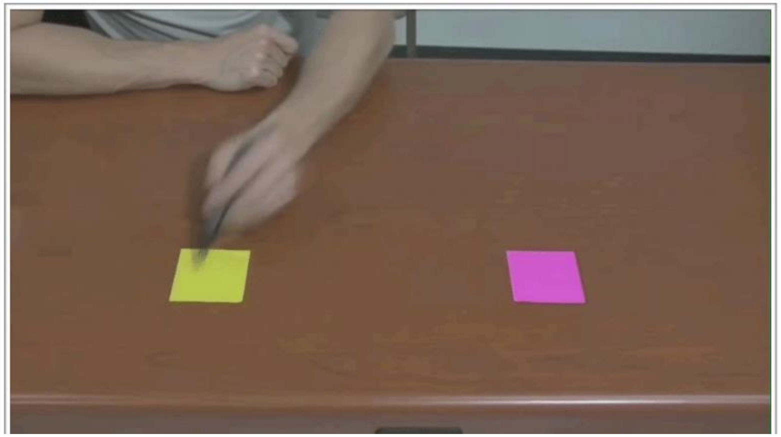
RESULTS – BONUS TRACK: NEUROSCIENCES FIT TASK

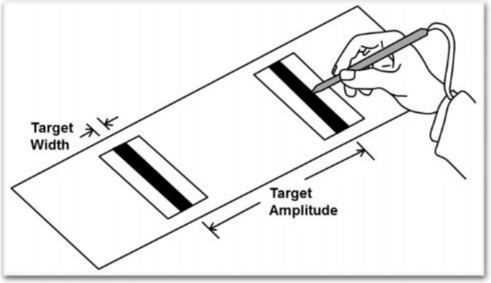
-TASK: Move the arm to target two differents small region in space as fastest as possible

-DIFFICULTY: Target size reduce

-DIFFERENT SUBJECTS OF DIFFERENT AGES

-MEASUREMENTS: physical phase space (position speed) & 12 muscles EMG activiy





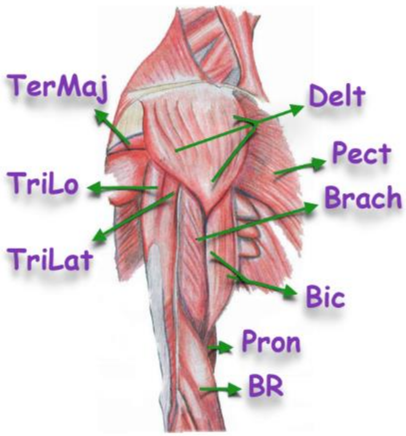
Braintime Project: with Demian Battaglia, Andrea Brovelli (La Timone, Marseille), Sandro Vaienti, Theo Caby (CPT)

RESULTS – BONUS TRACK: NEUROSCIENCES FIT TASK

1) how muscles respond to the difficulty of the task?

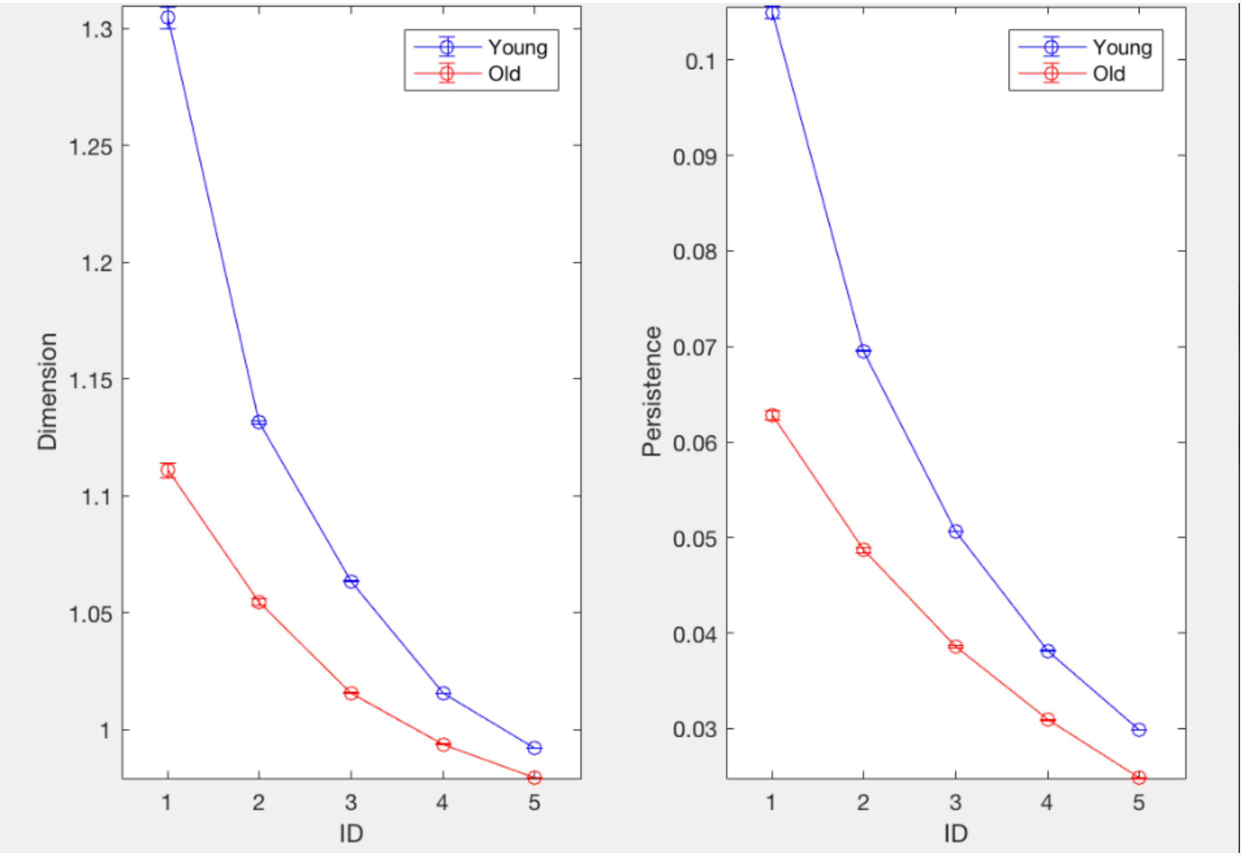
2) Differences between Old and young subjects?

3) Matching between physical phase space and muscles phase space?



RESULTS – PHYSICAL PHASE SPACE

DIMENSION **INVERSE PERSISTENCE**



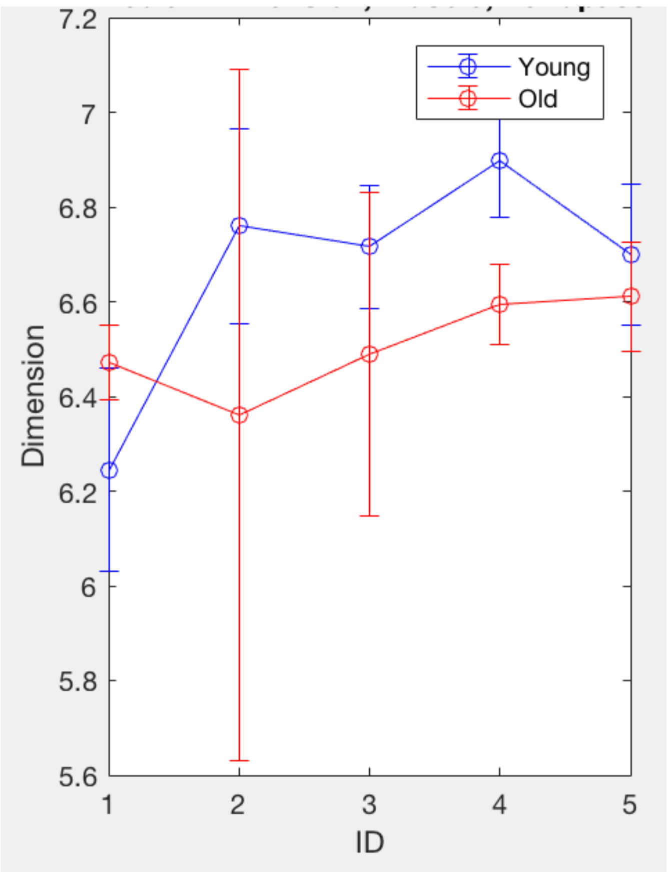
YOUNG/OLD threshold… 35 Years

- Young: more movement freedom and faster

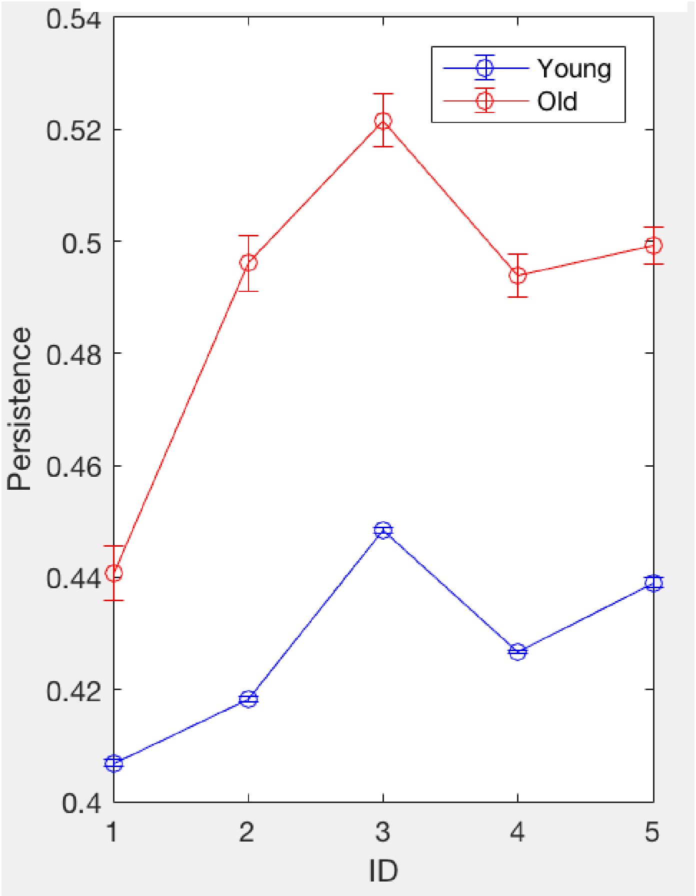
- Old: more careful but slower

RESULTS – MUSCLE PHASE SPACE

DIMENSION



INVERSE PERSISTENCE



YOUNG/OLD threshold… 35 Years

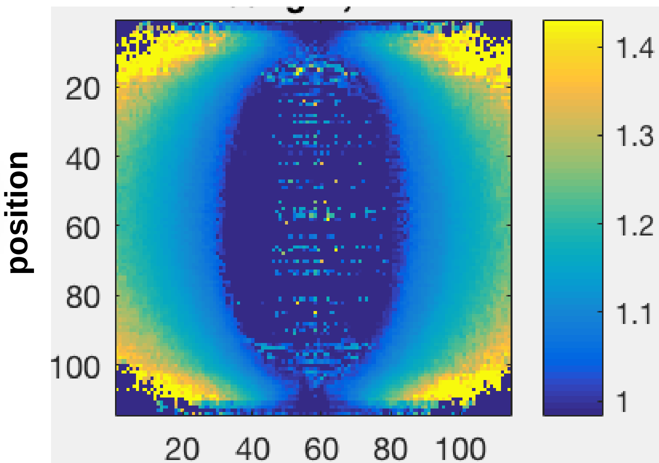
- **Young: can adapt the muscles to respond to the difficulty of the task**
- **Old: muscles less stable, « phase transitions » at diff=3**

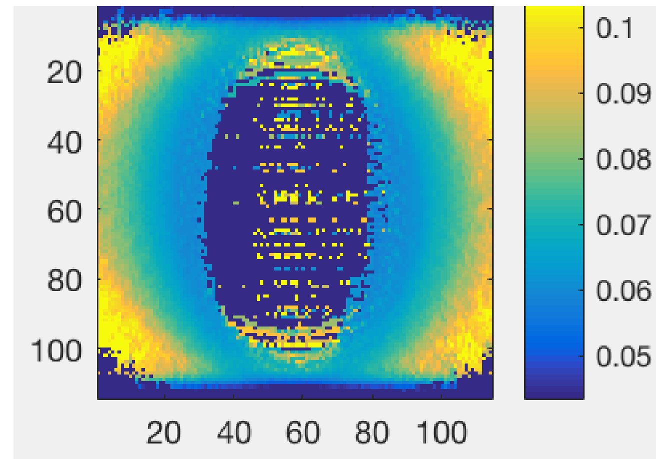
RESULTS – DIM PERSISTENCE IN TRAJECTORY (SAURON'S EYE)

DIMENSION

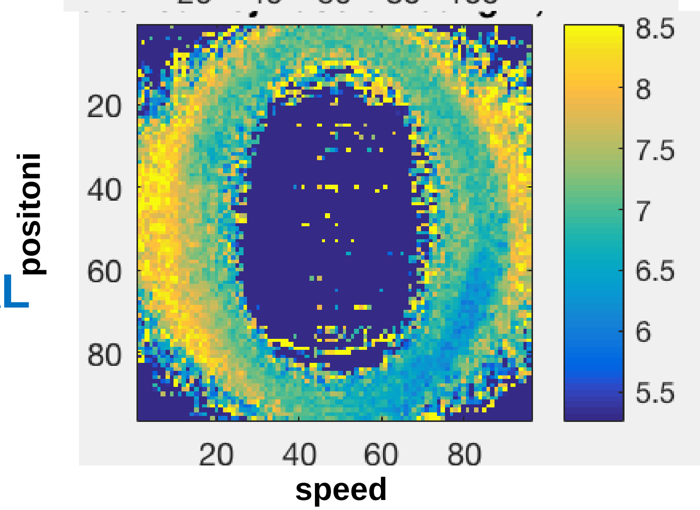
INVERSE PERSISTENCE

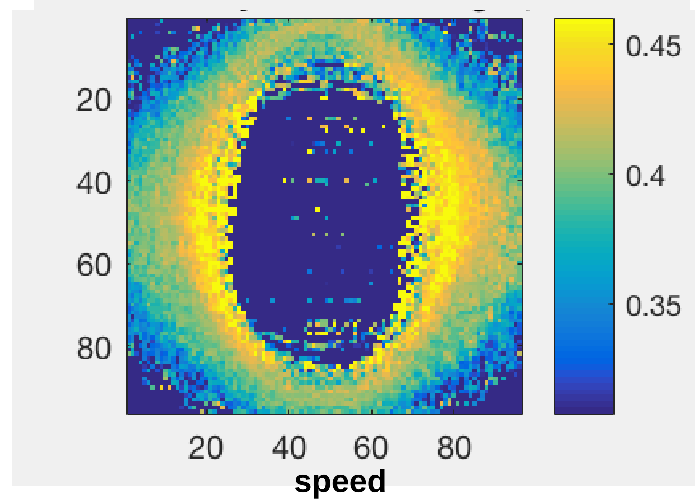
PHYSICAL SPACE



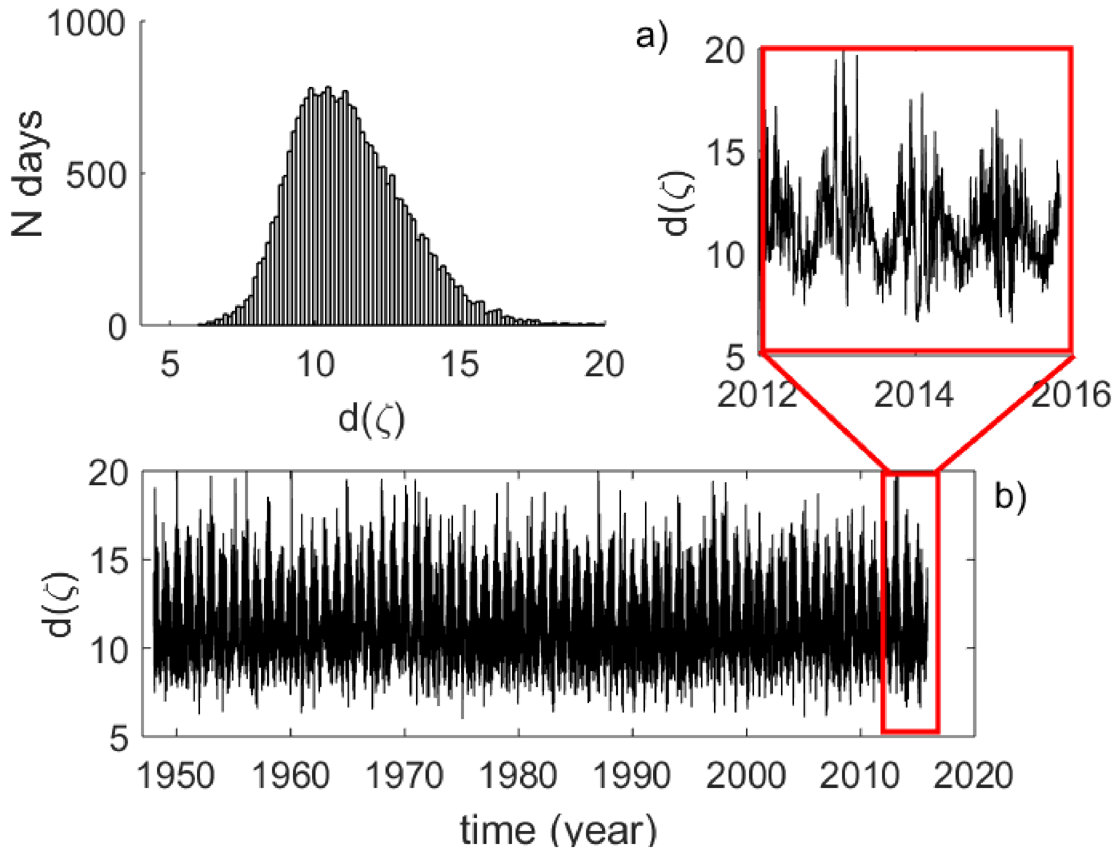


MUSCLE SPACE MAPPED TO PHYSICAL SPACE



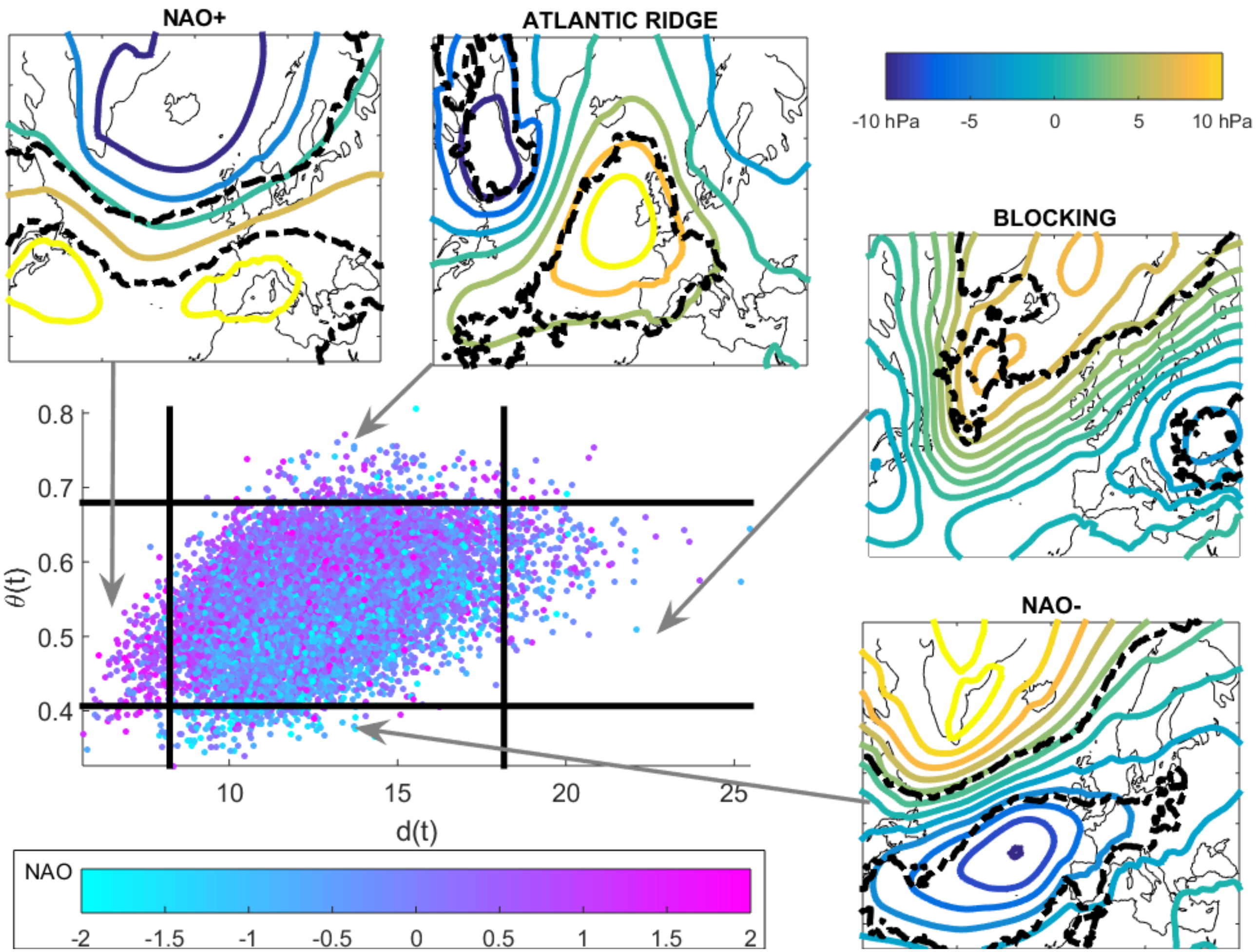


RESULTS – LOCAL DIMENSION IN NCEP 1948-2015



The instantaneous dimensions *d* (y-axis) versus the years of the database shows an interesting seasonal cycle. Extremes are found in wintertime, where sharp transitions occur between maxima and minima.

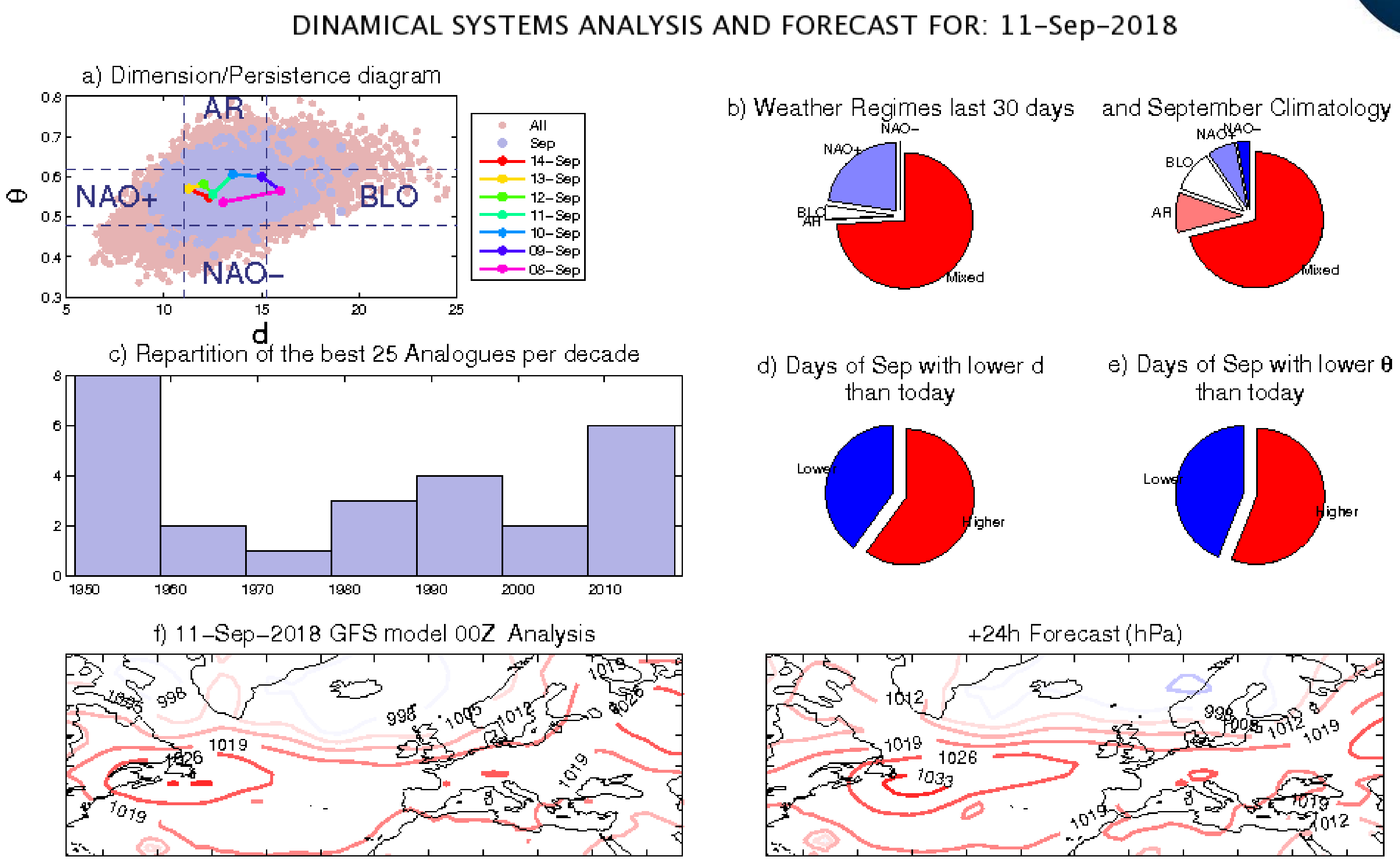
RESULTS – PERSISTENCE/DIMENSION DIAGRAM



The scatter plot displays the daily values of the instantaneous dimension *d* - the higher *d*, the more unpredictable is the atmospheric circulation - and the persistence θ - the lower θ the more stable is the atmospheric circulation - of the sea level pressure field (in hPa) extracted from the NCEP Database. The colorscale represents the North Atlantic Oscillation (NAO) index.

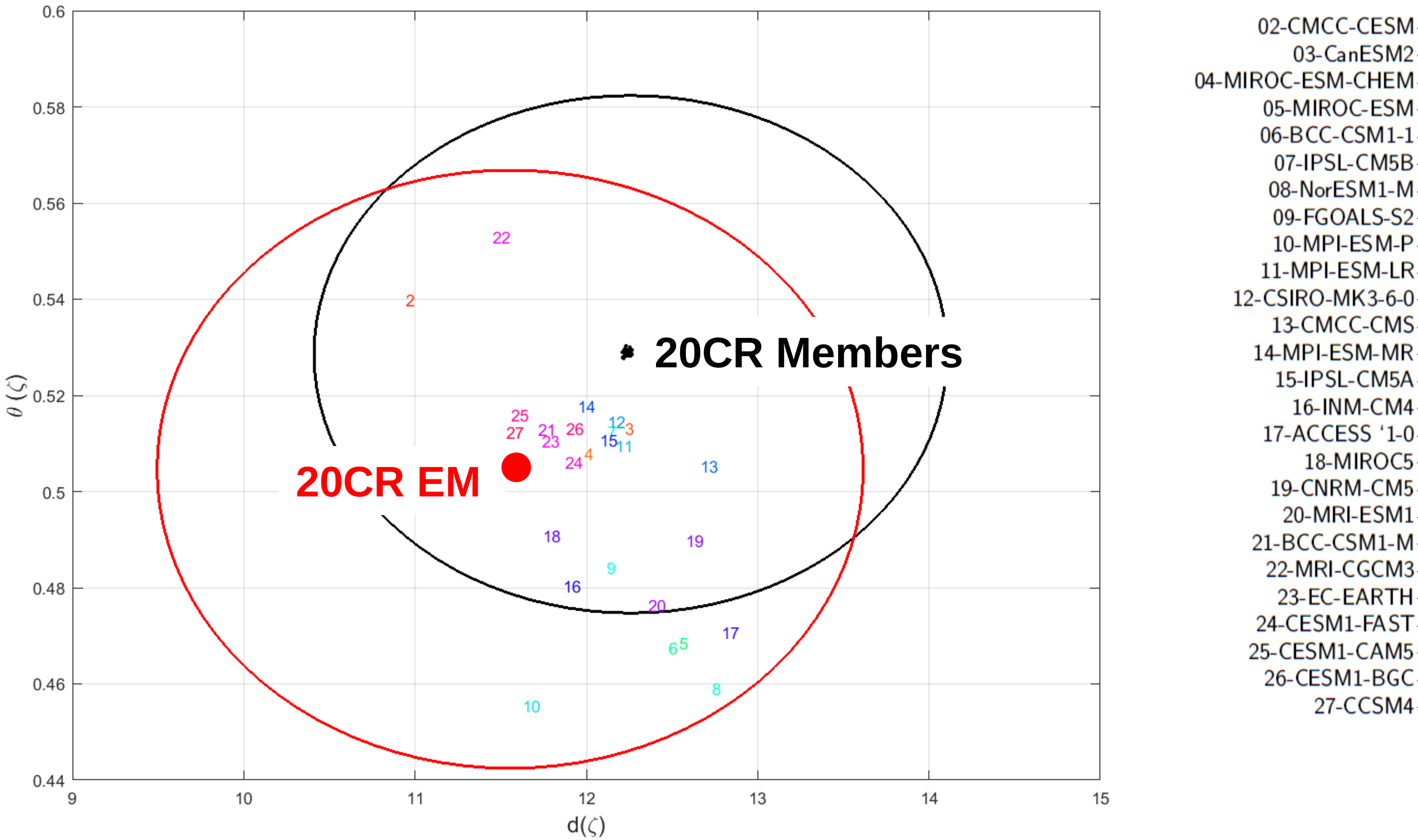
Faranda, Messori, Yiou. Scientific Reports 2017

RESULTS – PERSISTENCE/DIMENSION DIAGRAM - TODAY



https://www.lsce.ipsl.fr/Pisp/davide.faranda/

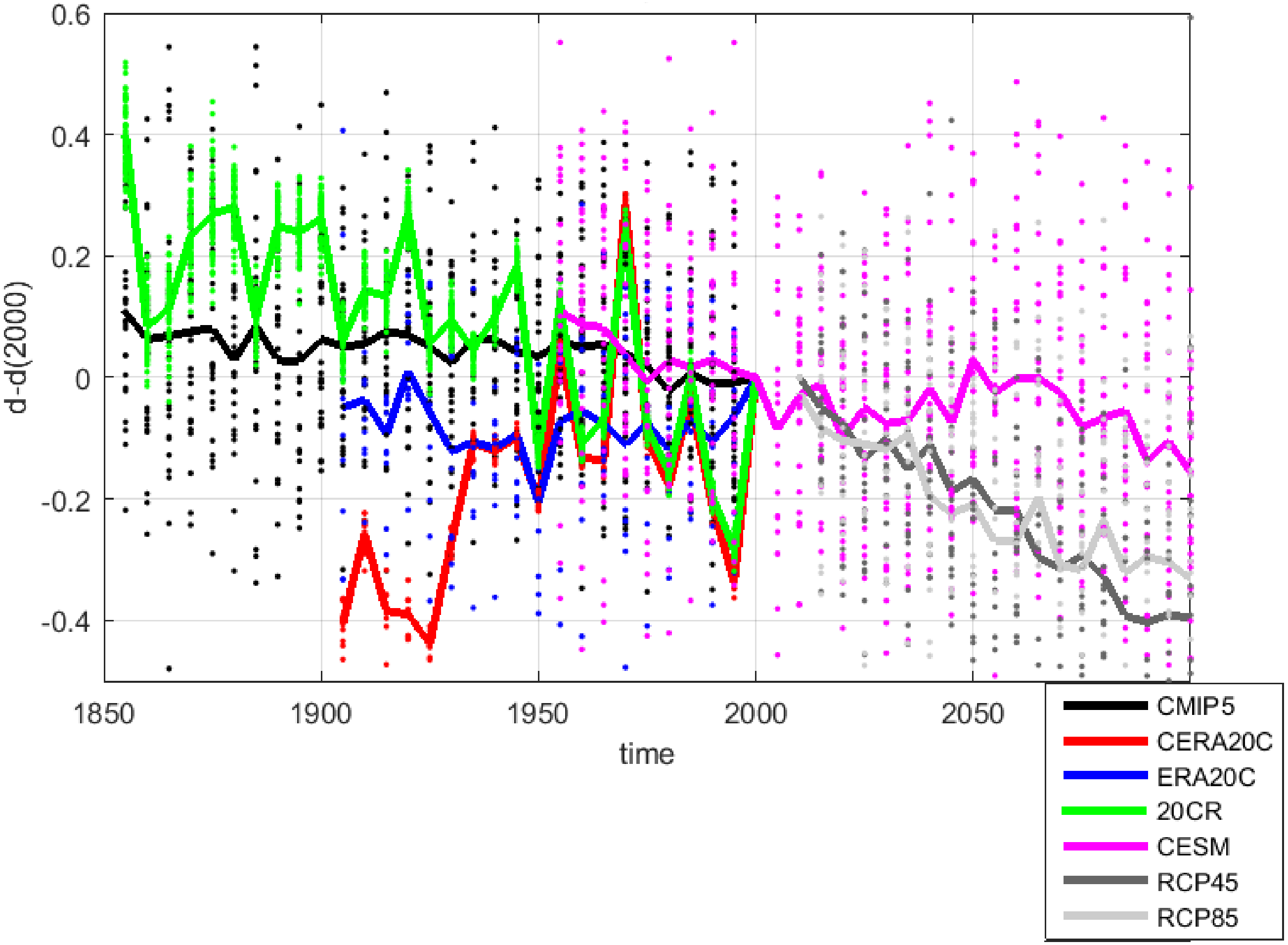
RESULTS – CMIP5 ANALYSIS – AVERAGES 1850-2100



Median values of local dimensions *d* and the persistence *θ*. Semiaxes of each ellipse represent one standard deviation of *d* and θ.

Rodrigues et al. Journal of Climate 2018

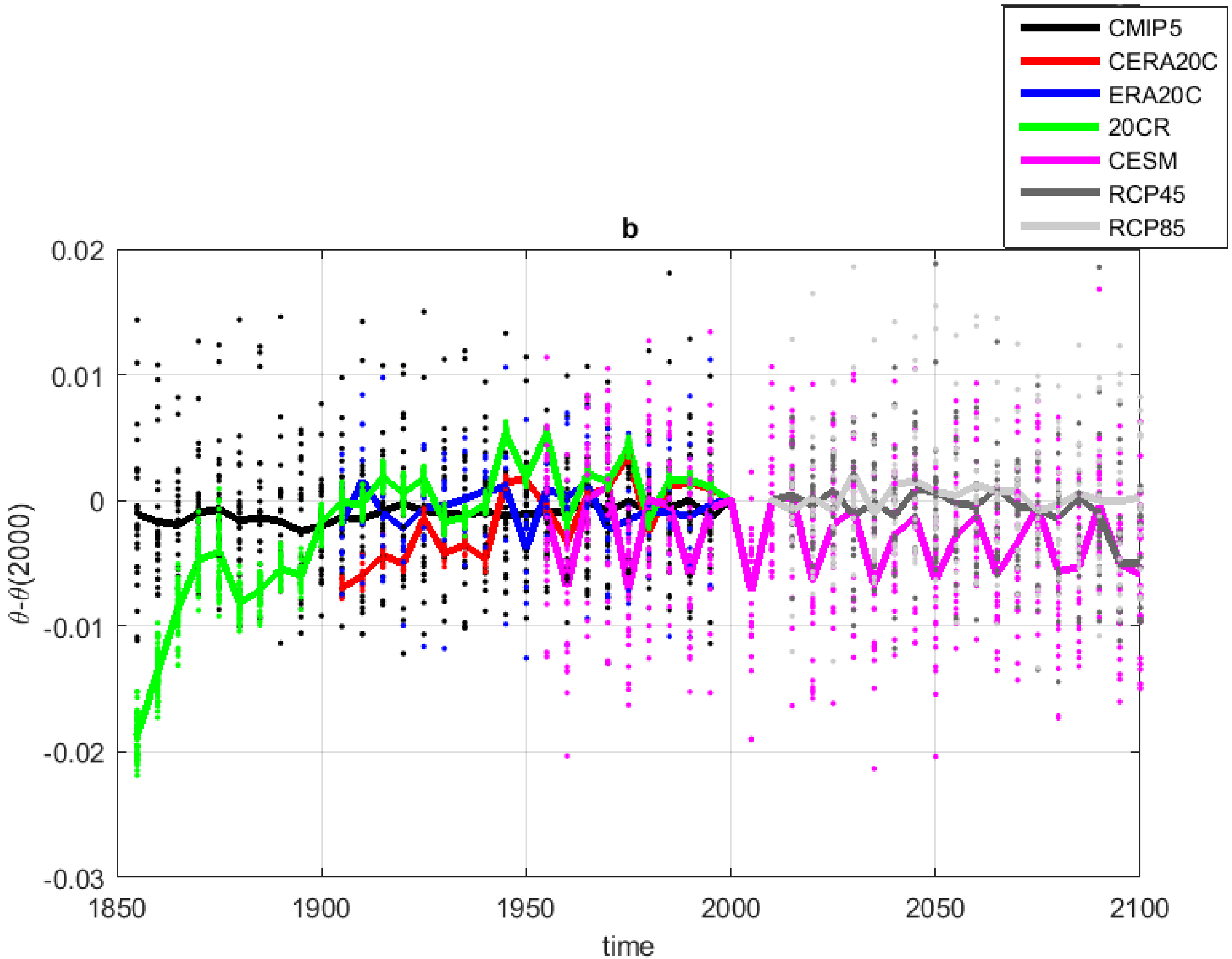
RESULTS – CHANGES OF DIMENSION IN TIME



5 years averaged values of local dimensions *d*

Faranda et aL. Submitted 2018

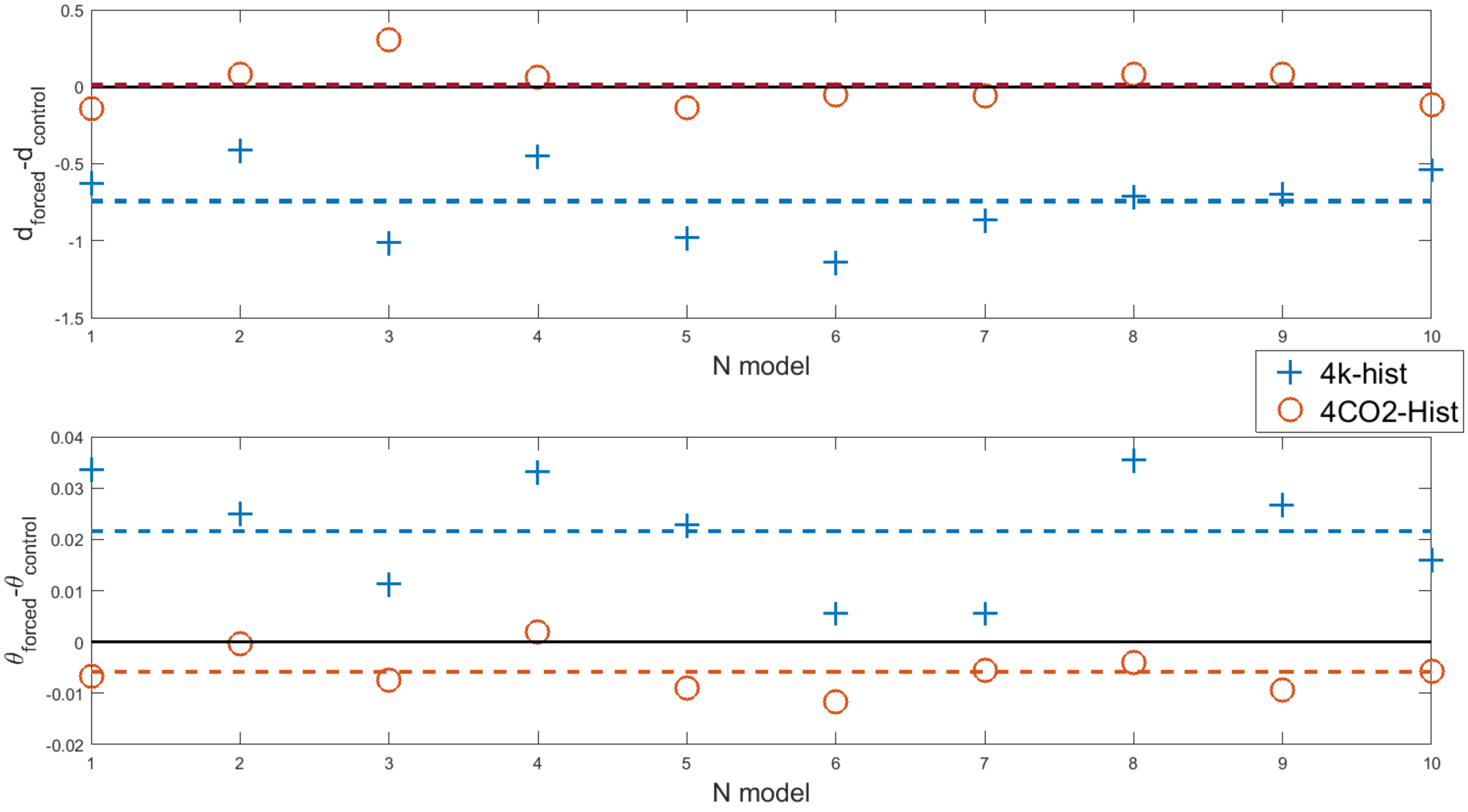
RESULTS – CHANGES OF PERSISTENCE IN TIME



5 years averaged values of the persistence θ.

Faranda et aL. Submitted 2018

RESULTS – HOW TO EXPLAIN THE DECREASE OF D



Ocean's inertia effects reduce the dimension of North Atlantic circulation when ocean temperature is increased

OUTLOOK: INDICATORS AND SWG

Sheng Cheng postdoc: provide both a theoretical framework and a numerical tool to build conditional Stochastic Weather Generators (SWG) driven by dynamical systems properties.

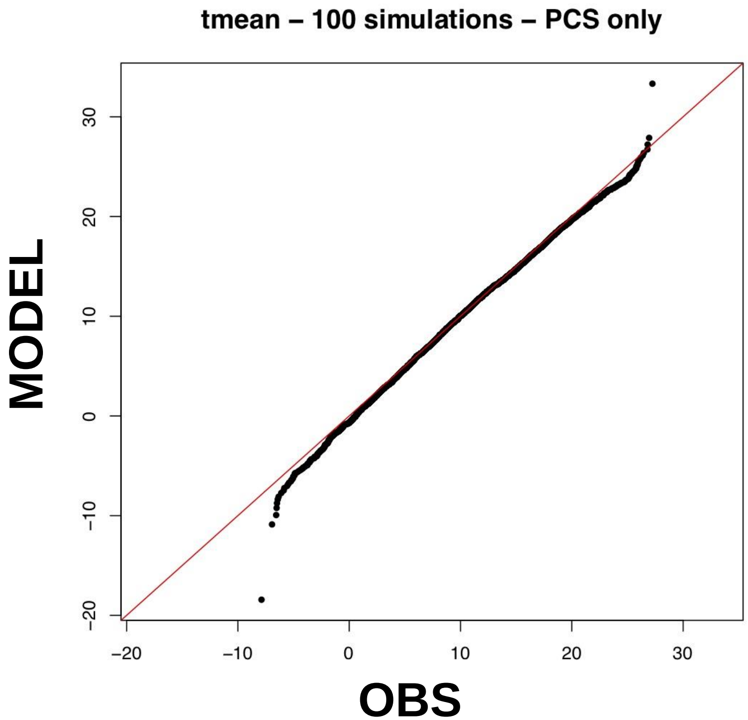
1/ Introduce **the indicators in simple autoregressive models** to mimic the dynamical properties of univariate time series X_t of climate variables and simple dynamical systems.

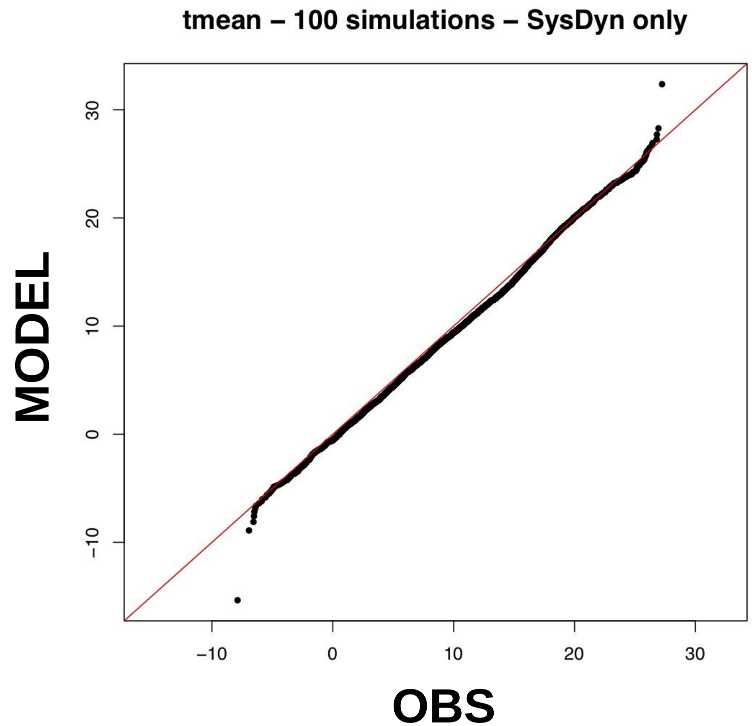
$$X_{t+1} = \phi\big(\theta(X_t)\big)X_t + \epsilon\big(d(X_t)\big)\xi(t)$$

Where ϕ is the autoregressive coefficient depending on the persistence index and ϵ is the noise amplitude ($\xi(t)$ is a random variable drawn from a normal distribution at each time)

2/ Those **state-dependent indicators introduces nonlinearities** . We use them to condition progressively more sophisticated SWGs (e.g., Vrac et al., 2007), up to spatial models (i.e., simulating field) for climate variables as wind, temperature or precipitation.

OUTLOOK: EXAMPLE QQ PLOT





Data: daily temperature mean over Ile de France 1979-2018;
Left using 20 **principal components (PCS)** for Sea Level pressure fields over the North Atlantic as predictor for SWG.
Right using system **dynamical indicators** computed for sea level pressure fields over the North Atlantic as predictor for SWG**.**

CONCLUSIONS

From an innovative application of recent results in dynamical system theory, we obtain that:

- The distribution of the local dimensions **capture the features of mid-latitude circulation dynamics**.

- Extremes of local dimensions **correspond to real-life extreme weather** (storms and blocking).

- There is a robust decrease in the dimension with greenhouse forcing, which implies a more predictable atmosphere. **This could be explained by the larger inertia feedback on atmospheric motions given by a warmer ocean.**

REFERENCES

[1] Davide Faranda, Gabriele Messori and Pascal Yiou. Dynamical proxies of North Atlantic predictability and extremes**. Scientific Reports**, 7-41278, **2017**.

[2] Valerio Lucarini, Davide Faranda, Ana Cristina Gomes Monteiro Moreira de Freitas,Jorge Miguel Milhazes de Freitas, Mark Holland, Tobias Kuna, Matthew Nicol, Mike Todd, Sandro Vaienti. Book: **Extremes and Recurrence in Dynamical Systems.** ISBN 978-1-118-63219-2, 312 pages, **Wiley**, **2016**.

[3] David Rodrigues, M Carmen Alvarez-Castro, Gabriele Messori, Pascal Yiou, Yoann Robin , Davide Faranda. Changes in the dynamical properties of the North Atlantic atmospheric circulation in the past 150 years. Accepted in ***Journal of Climate*, 2018**