Groupe de Travail Climat-Mécanique Statistique

On the nature of intermittency in a turbulent von Kármán flow

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CEA SPEC & CNRS LIMSI

A turbulent flow

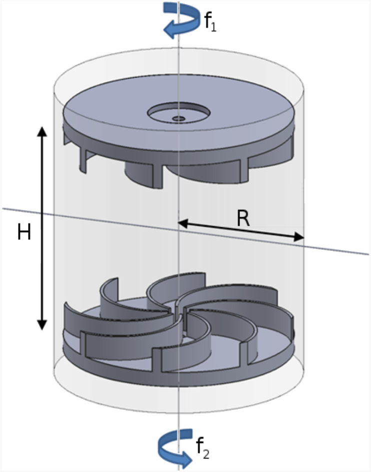

Figure 1: Von Kármán geometry
$\mathrm{Re} = \frac{2\pi f R^2}{\nu}$

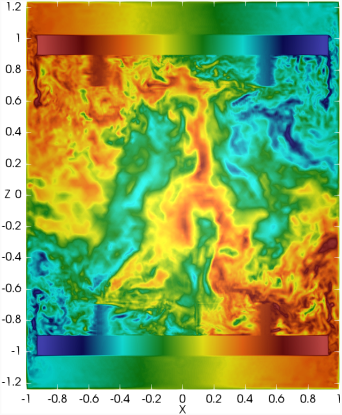

Figure 2: Out of plane velocity, at $\mathrm{Re} = 10^5$ from Cappanera et. al. [2020], submitted do Computers & Fluids

Structure functions : characterize the flow

It is natural to consider statistical properties as:

$$S_p(\ell) = \langle (\delta_\ell u)^p \rangle_{||\mathbf{r}||=\ell},$$

where δ_ℓ u is computed as $\delta_\ell u = \frac{\mathbf{r}}{\ell} \cdot \mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x})$ for 1D measurements, or from

$$G_{i,j}(\mathbf{x}, \ell) = \int \nabla_j \phi_\ell(\mathbf{r}) u_i(\mathbf{x}+\mathbf{r}) \mathrm{d}\mathbf{r},$$

where $\phi_\ell(x) = \ell^{-3}\phi(x/\ell)$ is a smooth non-negative function with unit integral. From this, we get the wavelet velocity increments as

$$\delta W_\ell(\mathbf{x}) = \ell \max_{ij} |G_{i,j}(\mathbf{x}, \ell)|.$$

In other terms $\delta W_\ell(\mathbf{x}) \sim \ell \nabla \overline{\mathbf{u}}^\ell(\mathbf{x})$

Theoretical prediction from Kolmogorov (1941) suggests that for stationary turbulent flows, in the inertial range:

$$S_3(\ell) = -\frac{4}{5}\epsilon\ell, \text{(Kármán Howarth Monin)}$$

and then if the flow is self-similar

$$S_p(\ell) = -C_p(\epsilon\ell)^{\frac{p}{3}}.$$

To test this theory, people have measured exponents $\zeta(p)$ as

$$S_p(\ell) \propto \ell^{\zeta(p)}$$

We then introduce $\tau(p) = \zeta(p) - \frac{p}{3}$.

Test of K41 theory

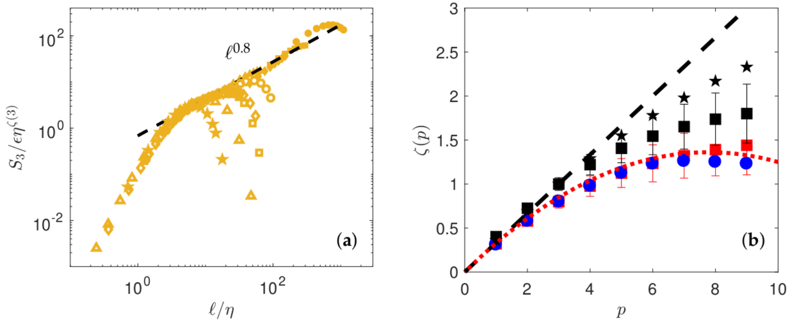

Figure 3: Determination of $\zeta(3)$ from DNS (open symbols) and experiments (filled symbols). (b) Scaling exponents $\zeta(p)$ of the wavelet structure functions of δW as a function of the order p for DNS (blue circle) and experiments (red square and black stars. Black squares correspond to $\frac{\zeta(p)}{\zeta(3)}$. From Geneste et. al. (2019)

Where does intermittency come from ?

Kolmogorov (1962) refined theory connects intermittency of the local energy dissipation with (intermittent) correction to scaling of the velocity structure functions up to the inertial scales. There should be a direct link between the 'active' regions of intense local dissipation, and the intermittent corrections to scaling.

Rotationnal parts of the flow seems therefore to play an important role, as found by Chainais Abry and Pinton (1999).
By computing conditional statistics, they remove the contribution of spots where $\omega = \|\nabla \times \mathbf{u}\|$ is large.

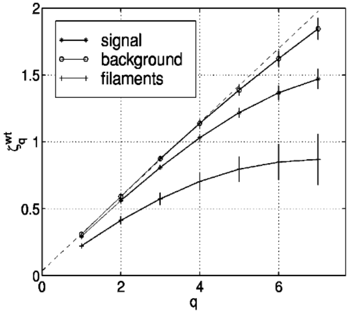

FIG. 13. Exponents of the wavelet based structure functions for the whole signal and the filament and background phases as a function of q. The dashed straight line is given as a linear reference.

Where does intermittency come from ?

The procedure was improved using wavelet filtering by Chainais et al. (1999), to conclude that the coherent structures do affect the intermittency by acting on the way the cascade develops. This suggests that the vorticity is not the only important ingredient of the intermittency, and that energy transfers should be somehow taken into account, as first argued by Kraichnan (1975).

Energy budget of filtered Navier-Stokes equation Dubrulle (19).

$$\partial_t\left(\frac{\mathbf{u}\cdot\overline{\mathbf{u}}^\ell}{2}\right) + \mathrm{div}\ \mathbf{J}^\ell$$
$$= -\frac{1}{4}\int_{\mathbf{r}} \nabla\phi_\ell \cdot \delta\mathbf{u}(\mathbf{x},\mathbf{r})^3 \mathrm{d}\mathbf{r}$$
$$-\frac{\nu}{2}\int_{\mathbf{r}} \Delta\phi_\ell \left(\delta\mathbf{u}(\mathbf{x},\mathbf{r})\right)^2 \mathrm{d}\mathbf{r}$$
$$= -\mathcal{D}_I^\ell - \mathcal{D}_\nu^\ell.$$

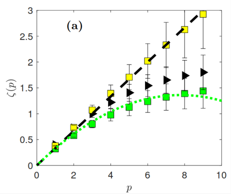

Figure 4: Scaling exponents $\zeta(p)$ of the wavelet structure functions of $\delta_\ell W$(green symbols) and $\delta_\ell W$ excluding strong $\mathcal{D}_I^\ell$ (yellow symbols). The black-triangles are the rescaled scaling exponents $\frac{\zeta(p)}{\zeta(3)}$.

Description of the data sets

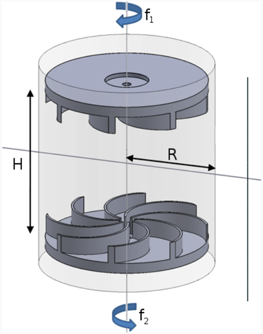

Figure 5: Von Kármán geometry $Re = \frac{2\pi f R^2}{\nu}$

Case	F (Hz)	Grid Points	Re	R_λ	η(mm)	Δx(mm)	Symbol
A	5	89 × 65	$3.1\ 10^5$	610	0.016	2.1	●
B	5	77 × 79	$3.1\ 10^5$	920	0.016	0.49	■
C	5	162 × 157	$3.1\ 10^5$	890	0.016	0.24	◆
D	1	77 × 80	$4.1\ 10^4$	300	0.073	0.49	▲
E	1.2	151 × 174	$5.8\ 10^3$	72	0.32	0.24	⋆
T-1	5	149 × 103 × 20	$3.1\ 10^5$	890	0.016	0.35	○
T-2	1	139 × 101 × 20	$6.3\ 10^4$	390	0.054	0.35	□
T-3	0.5	148 × 103 × 20	$3.1\ 10^4$	250	0.09	0.35	◇
T-4	0.1	149 × 100 × 20	$6.3\ 10^3$	80	0.3	0.35	△
DNS	$\frac{1}{2\pi}$	400 × 800 × 509	$6\ 10^3$	72	0.37	0.1-0.4	—○—

Table 1: Parameters describing the main data sets

Can we chose between ω and $\mathcal{D}_I^\ell$?

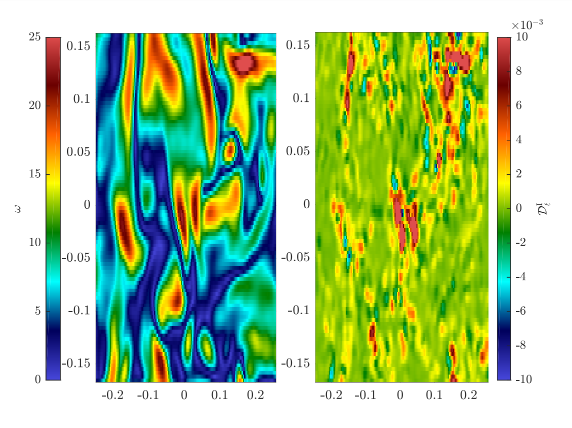

Figure 6: Visualization of amplitude of ω (left panel) and $\mathcal{D}_\ell^I$ at $\ell = 3.2\eta$ (right panel) in experimental measurements in a plane containing the cylinder's axis.

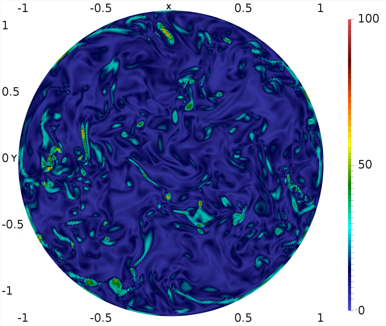

Figure 7: Visualization of the vorticity amplitude ω.

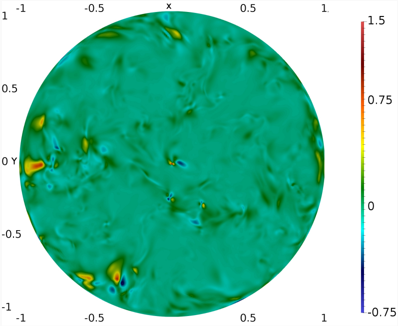

Figure 8: Visualization of $\mathcal{D}_\ell^I$ for $\ell = 8\eta$ on a plane containing the center perpendicular to the cylinder's axis (DNS).

Joint statistics

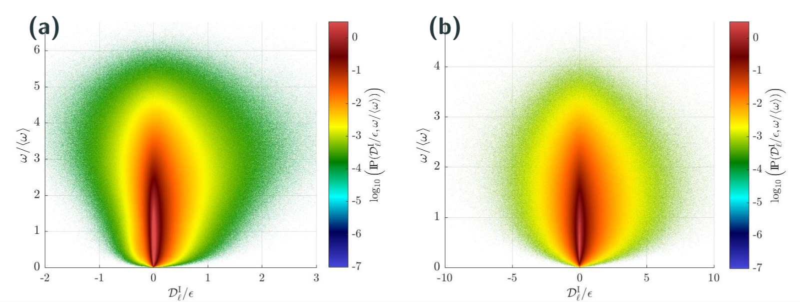

Figure 9: Joint-PDF of $\mathbb{P}\left(\frac{\mathcal{D}^{\mathrm{I}}_{\ell}}{\epsilon}, \frac{\omega}{\langle\omega\rangle}\right)$ for different scales from experimental measurements in table 1 computed over several uncorrelated snapshots. Here ω refers to the norm of the vorticity, and $\mathcal{D}^{\mathrm{I}}_{\ell}$ is the energy transfer. **a**) T-4: $\ell = 3.2\eta$ (3 10^4 snapshots) **b**) T-2: $\ell = 17.9\eta$ (1.02 10^4 snapshots).

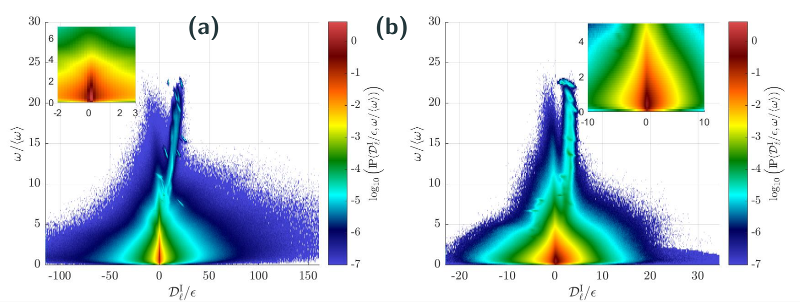

Figure 10: Joint-PDF of $\mathbb{P}(\frac{\mathcal{D}^{\mathrm{I}}_\ell}{\epsilon}, \frac{\omega}{\langle\omega\rangle})$ for different scales from the DNS in table 1 computed over 21 uncorrelated snapshots. ω refers to the norm of the vorticity, and $\mathcal{D}^{\mathrm{I}}_\ell$ is the energy transfer. A zoom on the central region is presented on each panel to compare with figure 9. **a**) $\ell = 1.06\eta$ **b**) $\ell = 26.5\eta$.

Joint statistics

Conditional average $\mathbb{E}(\mathcal{D}_\ell^I|\omega)$ for different scales ℓ, from DNS (filled circles) and experimental data (other symbols). At small scales, we find a linear relation between energy transfers $\mathcal{D}_\ell^I$ and ω^2.

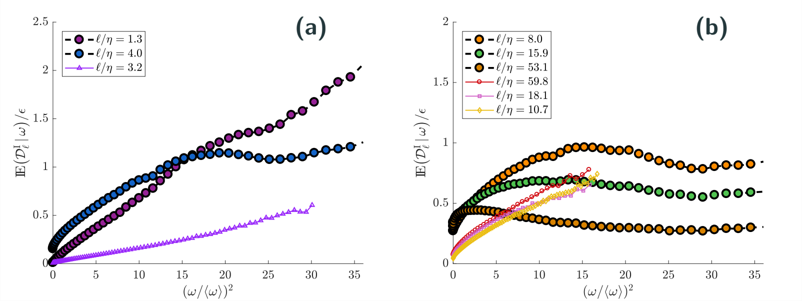

Figure 11: Conditional average $\mathbb{E}(\mathcal{D}_\ell^I|\omega)/\epsilon$ as a function of $(\omega/\langle\omega\rangle)^2$for different scales ℓ. They are computed from joint PDFs as the ones in figures 9 and 10 from datasets of the DNS and cases T-1 to T-4 in table 1. Symbols are coded according to table 1.

Maybe a common structure ?

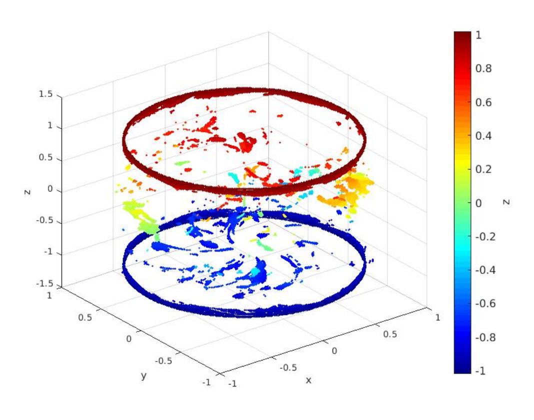

Figure 12: Positions of DNS points in the tail where $\mathcal{D}_\ell^I > \epsilon$ and $\omega > 5\langle\omega\rangle$ for $\ell = 26.5\eta$ The colors correspond to the z values of the points, indicating that most of them are close to the impellers ($0.7 < |z| < 1.02$), or at the tank mid-height.

Conditionned scaling exponents

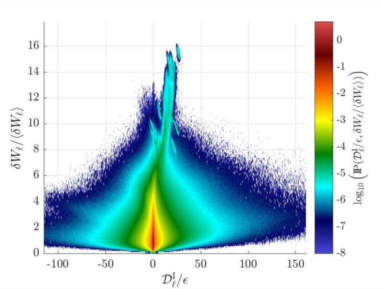

Figure 13: Joint Pdf of $\mathcal{D}^I_\ell$ and $\delta W\ell$ for $\ell = 1.06\eta$

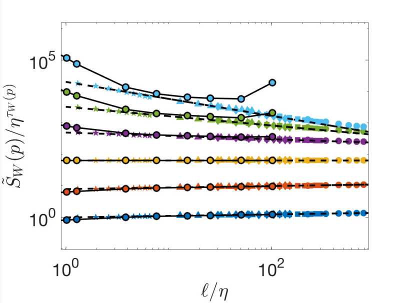

Figure 14: Scale variation of the normalized non-dimensional wavelet structure functions $\tilde{S}_W(p,\ell) = \frac{S_p(\ell)}{S_p(\ell)^{p/3}}$ for $p = 1$ to $p = 6$ for SPIV experiments A to E (filled symbol) and the DNS (filled symbols with black line).

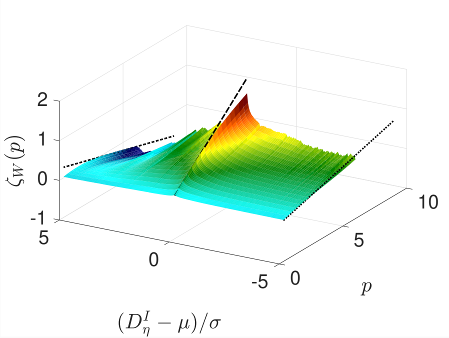

Figure 15: Scaling exponent of the non-compensated wavelet structure functions $S_W(p)$ computed using the joint PDF.

The dotted lines have a slope of -0.1 for large positive values of centered and reduced value of $\mathcal{D}_\eta^I$, $1/3$ for small values of centered and reduced value of $\mathcal{D}_\eta^I$, and 0.1 for large negative values of centered and reduced value of $\mathcal{D}_\eta^I$.

Conclusion

- Both energy transfers and vorticity are implied in intermittency
- The transfers are important, and therefore if the data is not resolved enough then the fluctuations are impacted.
- Cyclones et Tornadoes as strong events of energy transfers and vorticity ?
- The analogy angular momentum in von Kármán versus temperature in a Rayleigh-Bénard cell suggests that temperature fluctuations are also impacted by the heat fluxes.

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... and you for your attention !