A new Mathematical Framework for Atmospheric Blocking Events

Valerio Lucarini (1,2,3)

1) Department of Mathematics and Statistics, University of Reading, Reading
2) Centre for the Mathematics of Planet Earth, University of Reading, Reading
3) CEN, University of Hamburg, Hamburg

Joint work with A. Gritsun (INM-RAS, Moscow)

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A new mathematical framework for atmospheric blocking events

Valerio Lucarini[1,2,3] · Andrey Gritsun[4]

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Abstract
We use a simple yet Earth-like hemispheric atmospheric model to propose a new framework for the mathematical properties of blocking events. Using finite-time Lyapunov exponents, we show that the occurrence of blockings is associated with conditions featuring anomalously high instability. Longer-lived blockings are very rare and have typically higher instability. In the case of Atlantic blockings, predictability is especially reduced at the onset and decay of the blocking event, while a relative increase of predictability is found in the mature phase. The opposite holds for Pacific blockings, for which predictability is lowest in the mature phase. Blockings are realised when the trajectory of the system is in the neighbourhood of a specific class of unstable periodic orbits (UPOs), natural modes of variability that cover the attractor the system. UPOs corresponding to blockings have, indeed, a higher degree of instability compared to UPOs associated with zonal flow. Our results provide a rigorous justification for the classical Markov chains-based analysis of transitions between weather regimes. The analysis of UPOs elucidates that the model features a very severe violation of hyperbolicity, due to the presence of a substantial variability in the number of unstable dimensions, which explains why atmospheric states can differ a lot in term of their predictability. Additionally, such a variability explains the need for performing data assimilation in a state space that includes not only the unstable and neutral subspaces, but also some stable modes. The lack of robustness associated with the violation of hyperbolicity might be a basic cause contributing to the difficulty in representing blockings in numerical models and in predicting how their statistics will change as a result of climate change. This corresponds to fundamental issues limiting our ability to construct very accurate numerical models of the atmosphere, in term of predictability of the both the first and of the second kind in the sense of Lorenz.

Keywords Atmospheric blockings · Unstable periodic orbits · Covariant Lyapunov vectors · Lyapunov exponents · Predictability · Numerical modelling

The Physics of Climate Variability and Climate Change

Michael Ghil

Geosciences Department and Laboratoire de Météorologie Dynamique (CNRS and IPSL),
Ecole Normale Supérieure and PSL Research University,
F-75231 Paris Cedex 05,
France
Department of Atmospheric and Oceanic Sciences,
University of California,
Los Angeles, CA 90095-1565,
USA

Valerio Lucarini

Department of Mathematics and Statistics,
University of Reading,
Reading, RG66AX,
UK
Centre for the Mathematics of Planet Earth,
University of Reading,
Reading, RG66AX,
UK
CEN - Institute of Meteorology,
University of Hamburg,
Hamburg, 20144,
Germany

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Rev. Mod. Physics, submitted 2019

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The climate system is a forced, dissipative, nonlinear, complex and heterogeneous system that is out of thermodynamic equilibrium. The system exhibits natural variability on many scales of motion, in time as well as space, and it is subject to various external forcings, natural as well as anthropogenic. This paper reviews the observational evidence on climate phenomena and the governing equations of planetary-scale flow, as well as presenting the key concept of a hierarchy of models as used in the climate sciences. Recent advances in the application of dynamical systems theory, on the one hand, and of nonequilibrium statistical physics, on the other, are brought together for the first time and shown to complement each other in helping understand and predict the system's behavior. These complementary points of view permit a self-consistent handling of subgrid-scale phenomena as stochastic processes, as well as a unified handling of natural climate variability and forced climate change, along with a treatment of the crucial issues of climate sensitivity, response, and predictability.

Menu

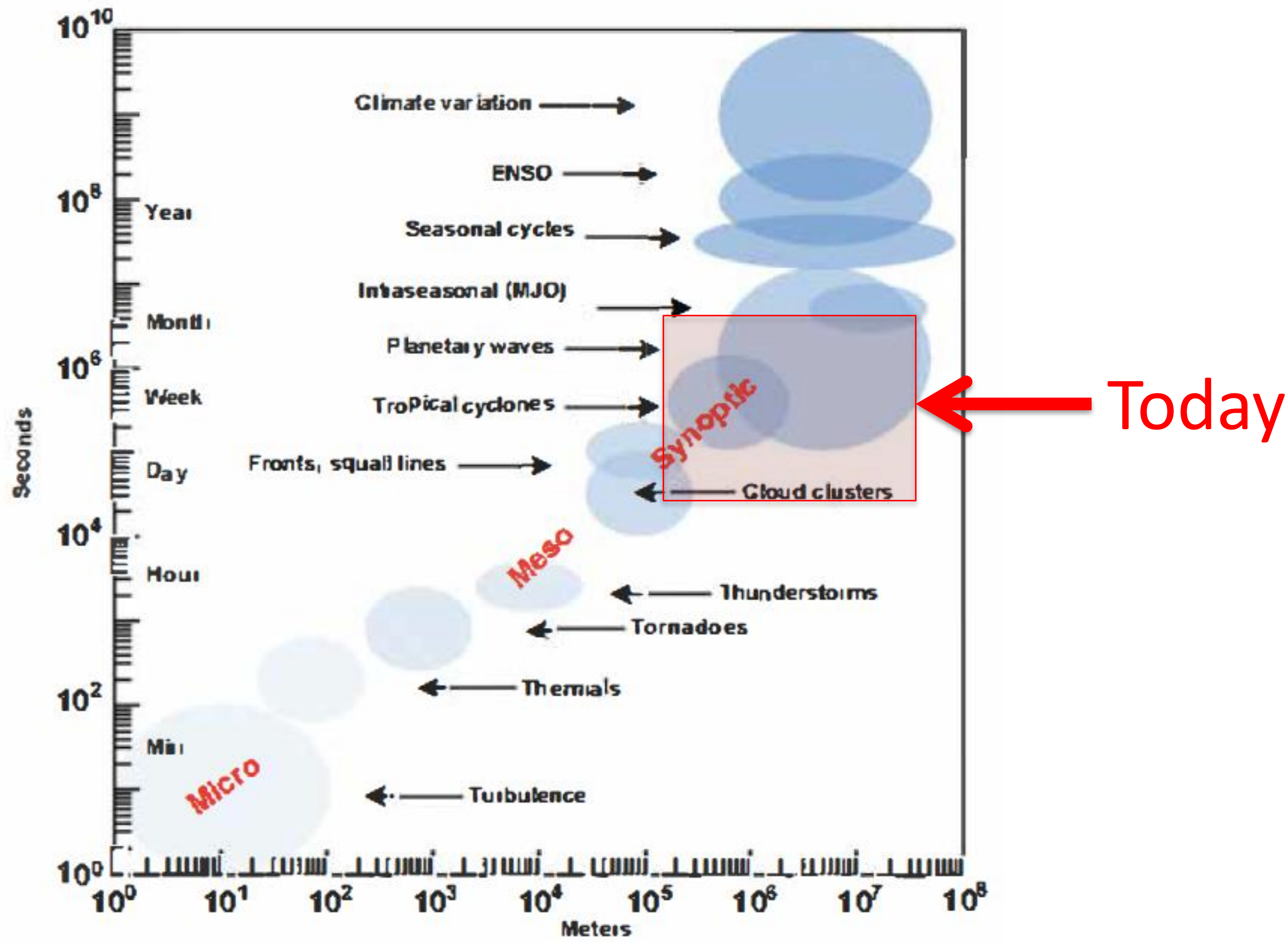
- We will talk about the low-frequency variability of the atmosphere in the mid-latitudes
- We will propose a mathematical framework to address these questions:

1) Are blockings associated to anomalously unstable conditions of the atmosphere?
2) Is there something special about the onset and decay phases?
3) Can we associated blockings withspecial modes of the atmosphere?

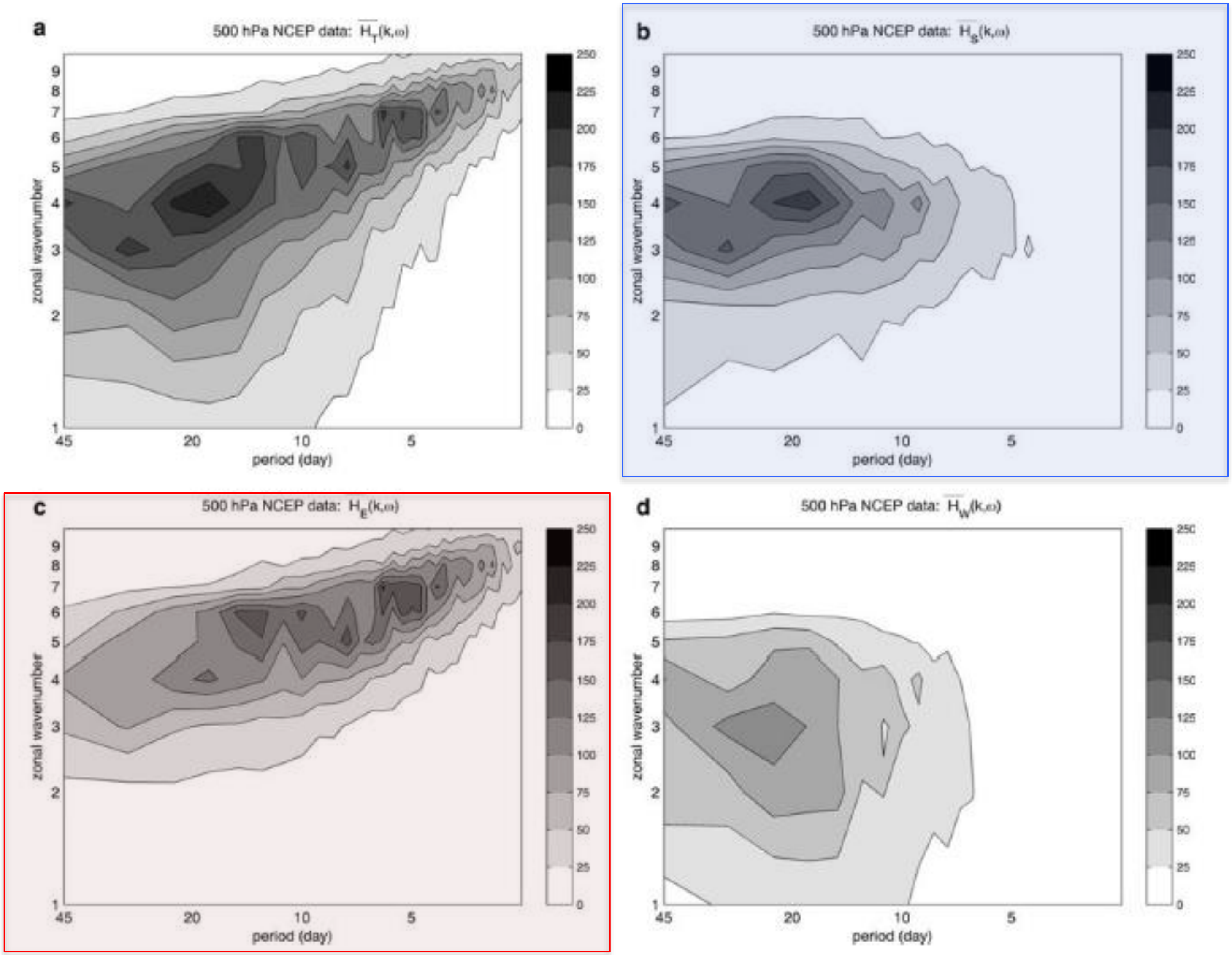
And:

Serendipitous Bonus Result

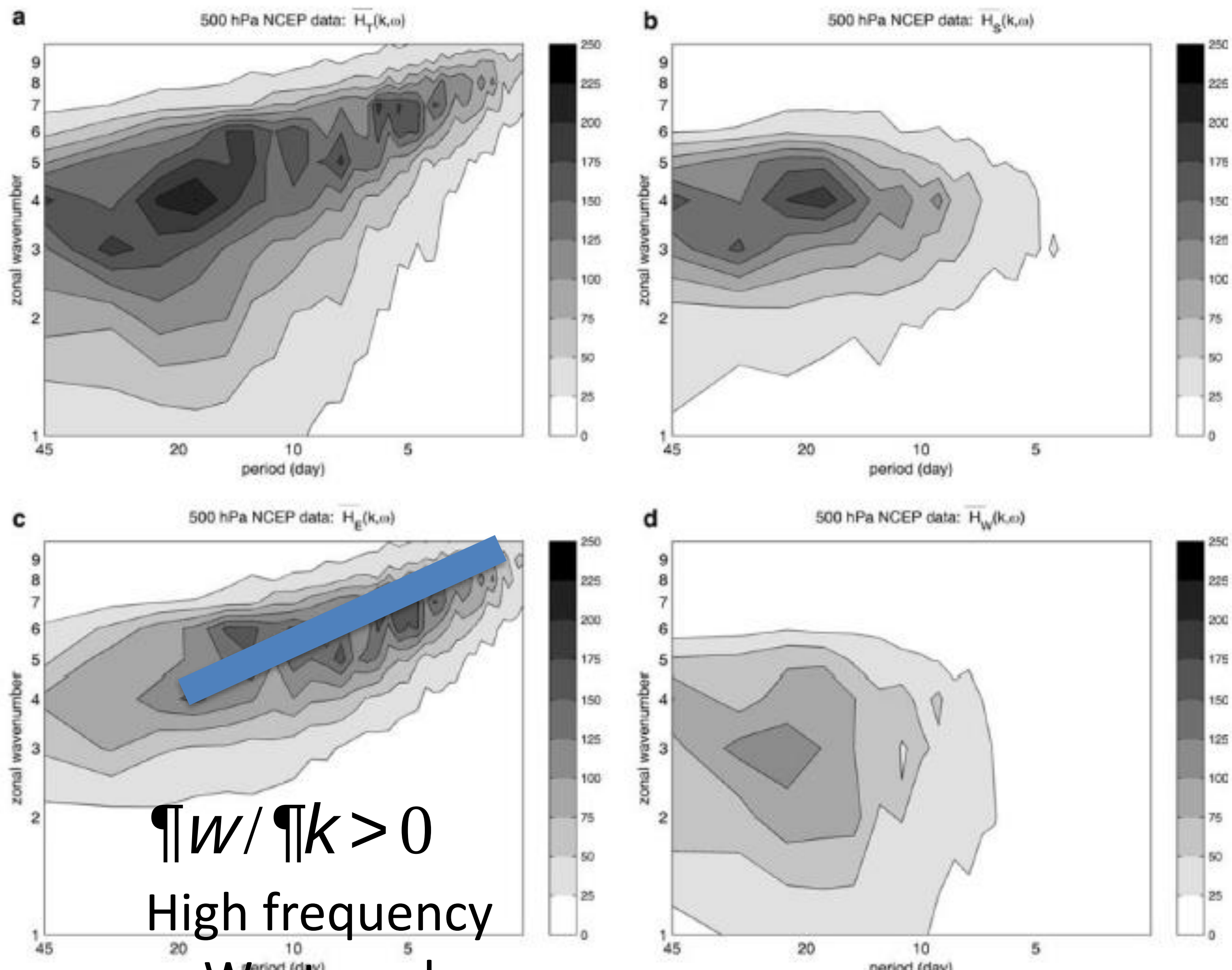
Spatial and Temporal Scales



Hayashi Spectra – Winter Mid-latitudes



Hayashi Spectra – Winter Mid-latitudes

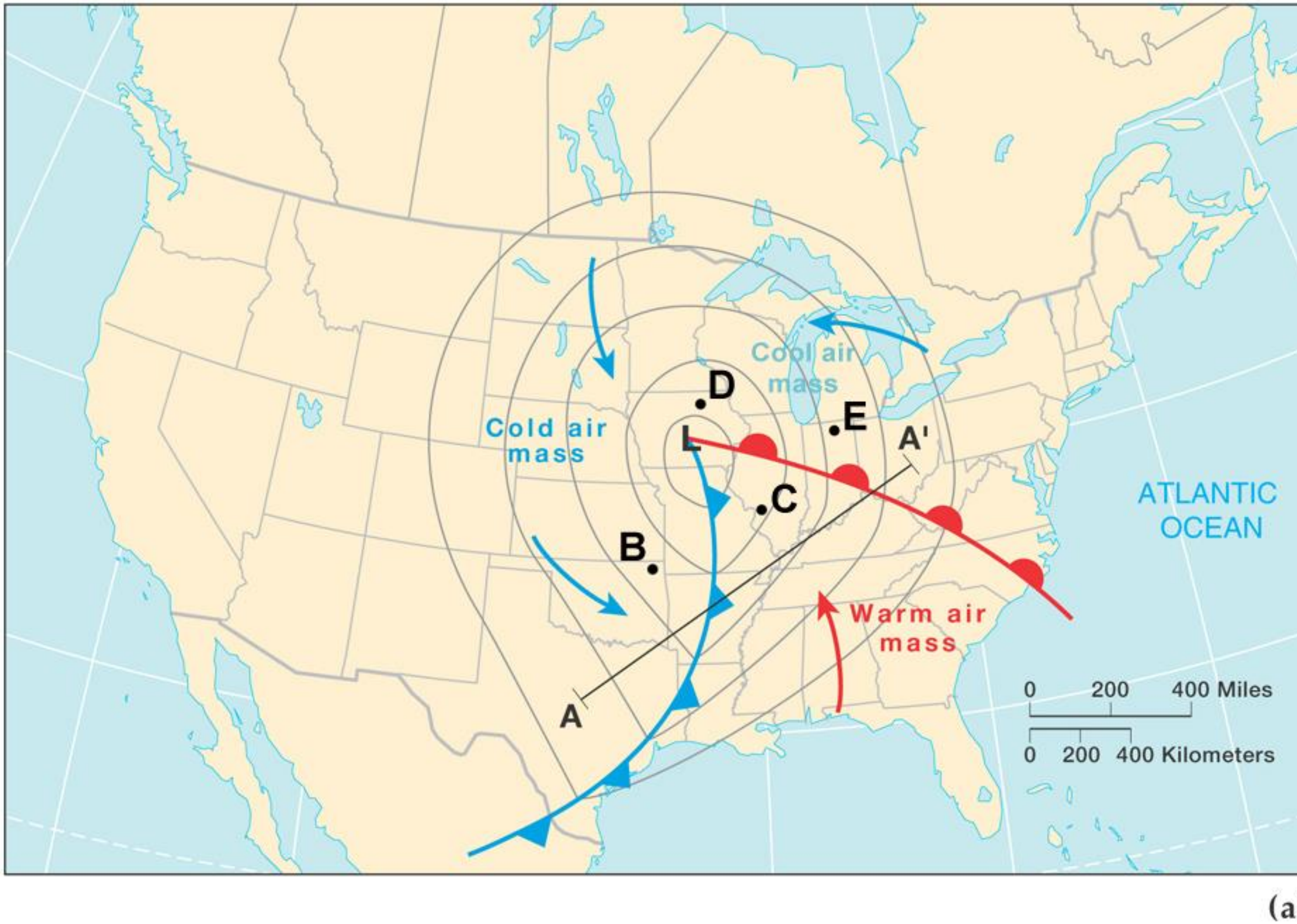


$\partial \omega / \partial k > 0$

High frequency

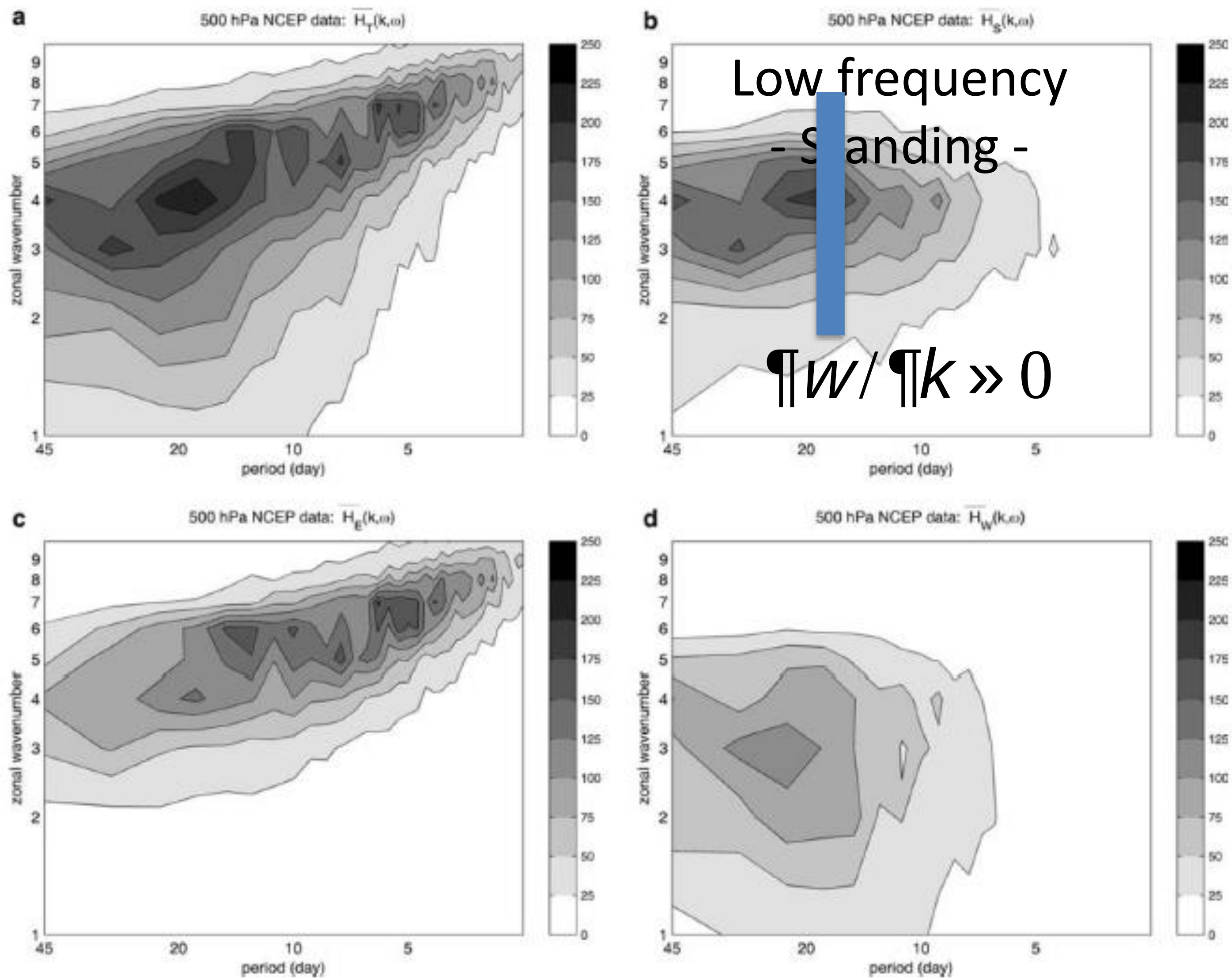
- Westward -

Mid-latitude Cyclone

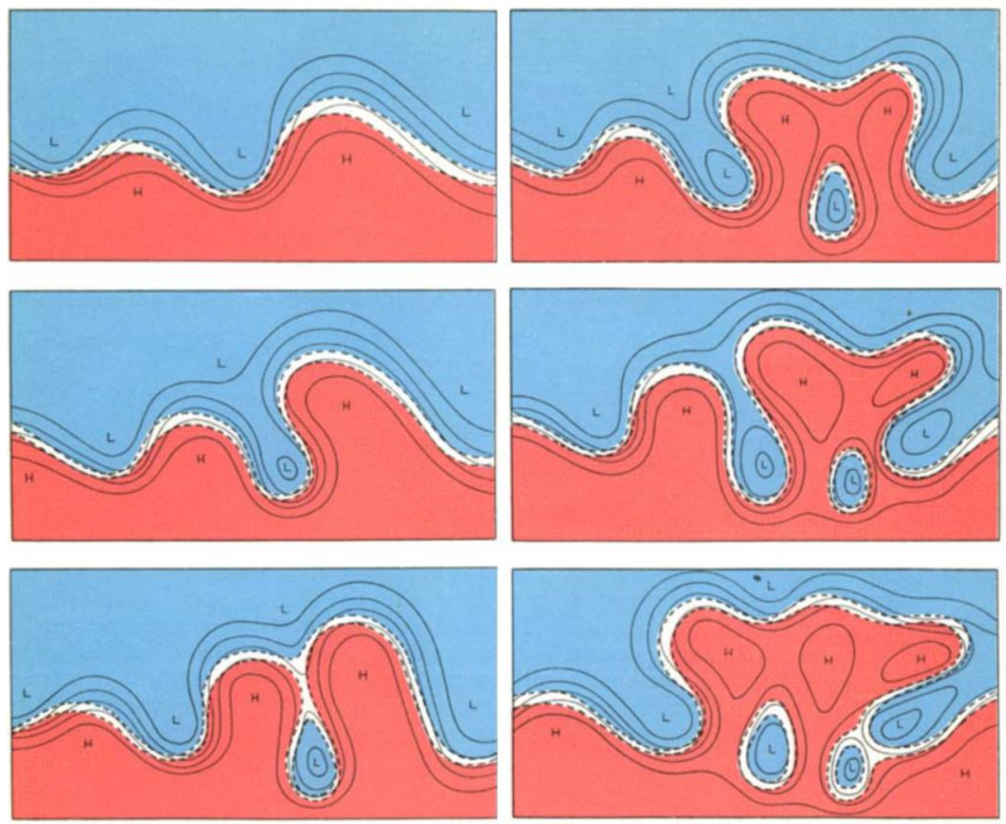


period (day) period (day)

Hayashi Spectra – Winter Mid-latitudes



Breakdown of Zonal Flow

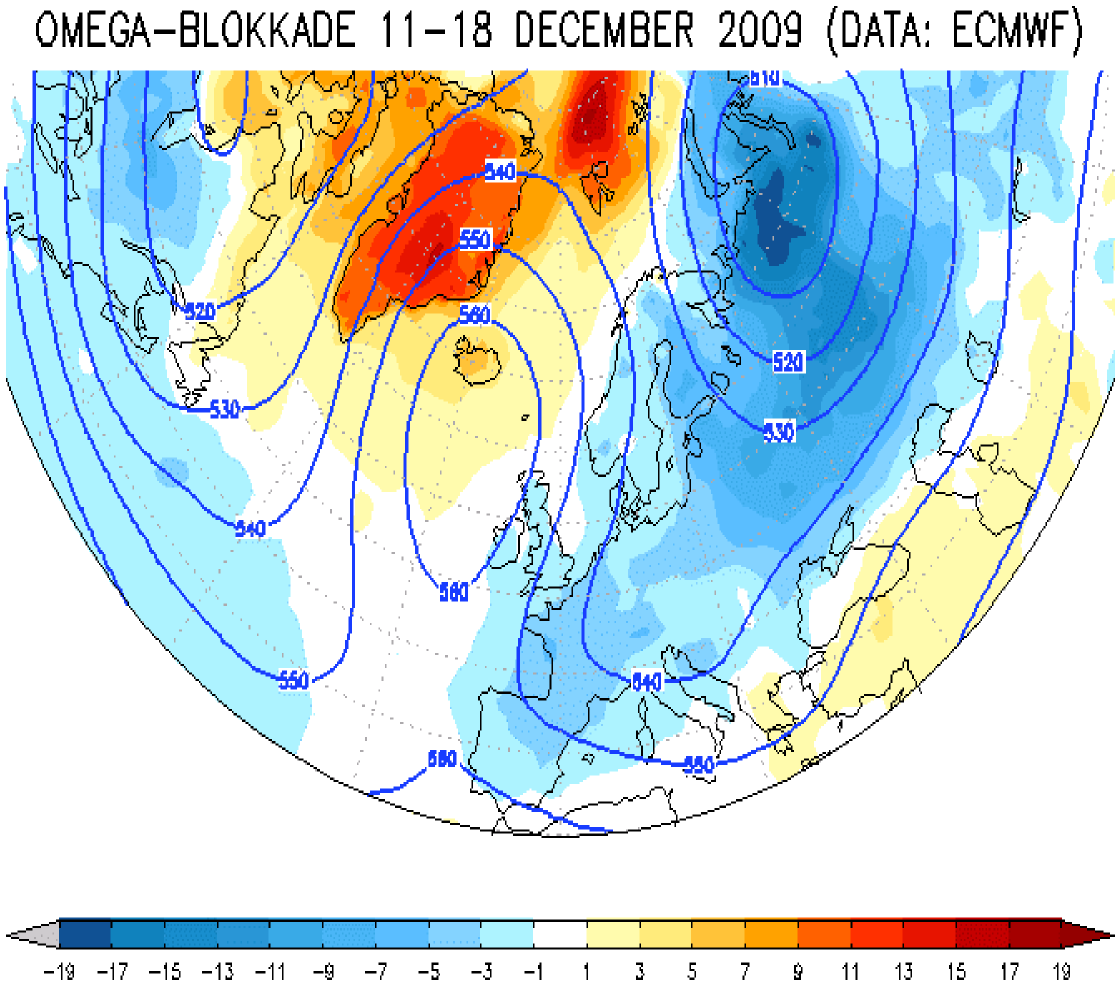


a d b e c f

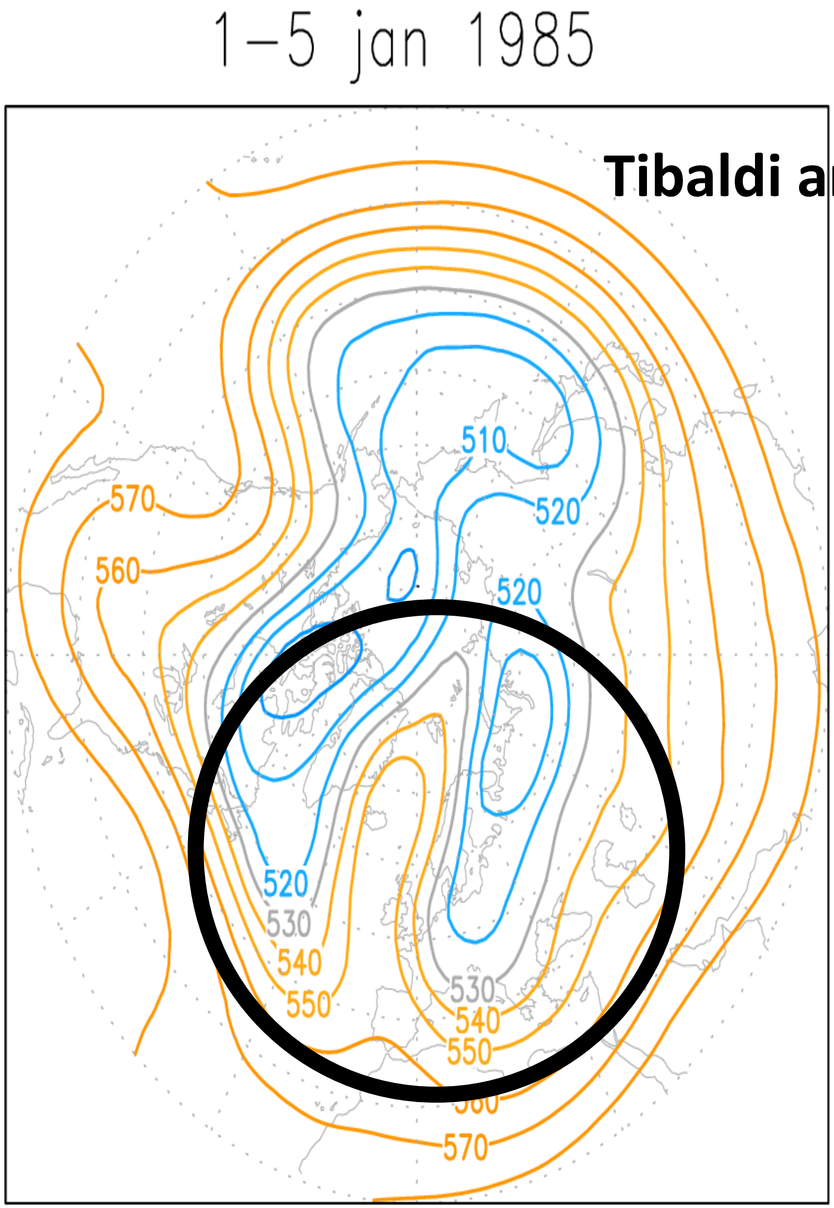
Fig. 26. Idealized sketches of the development of unstable waves at the 500 mb level, in association with the establishment of a blocking anticyclone in high latitudes. Cold air in blue colour, warm air in red. Solid lines are stream lines and broken lines the frontal boundaries.

Berggren et al. 1949

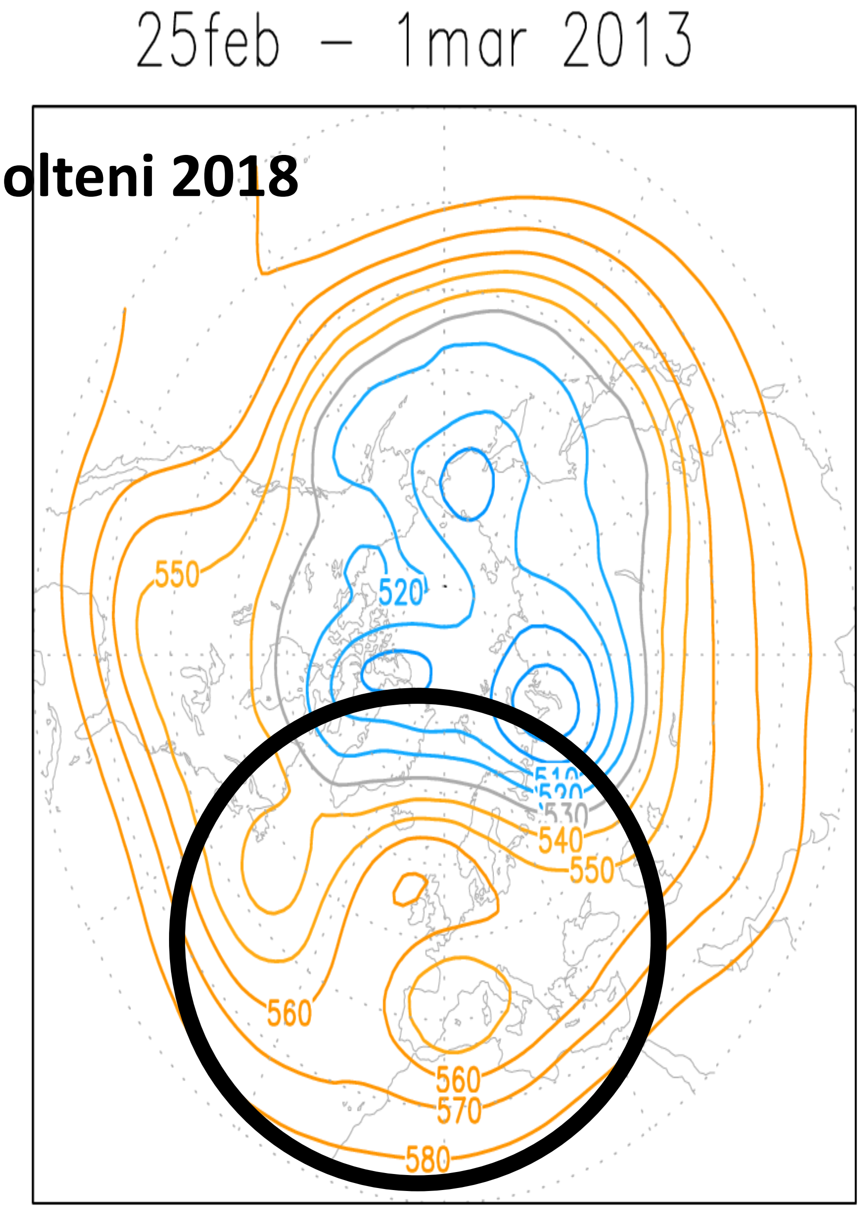
Example of Atmospheric Blocking



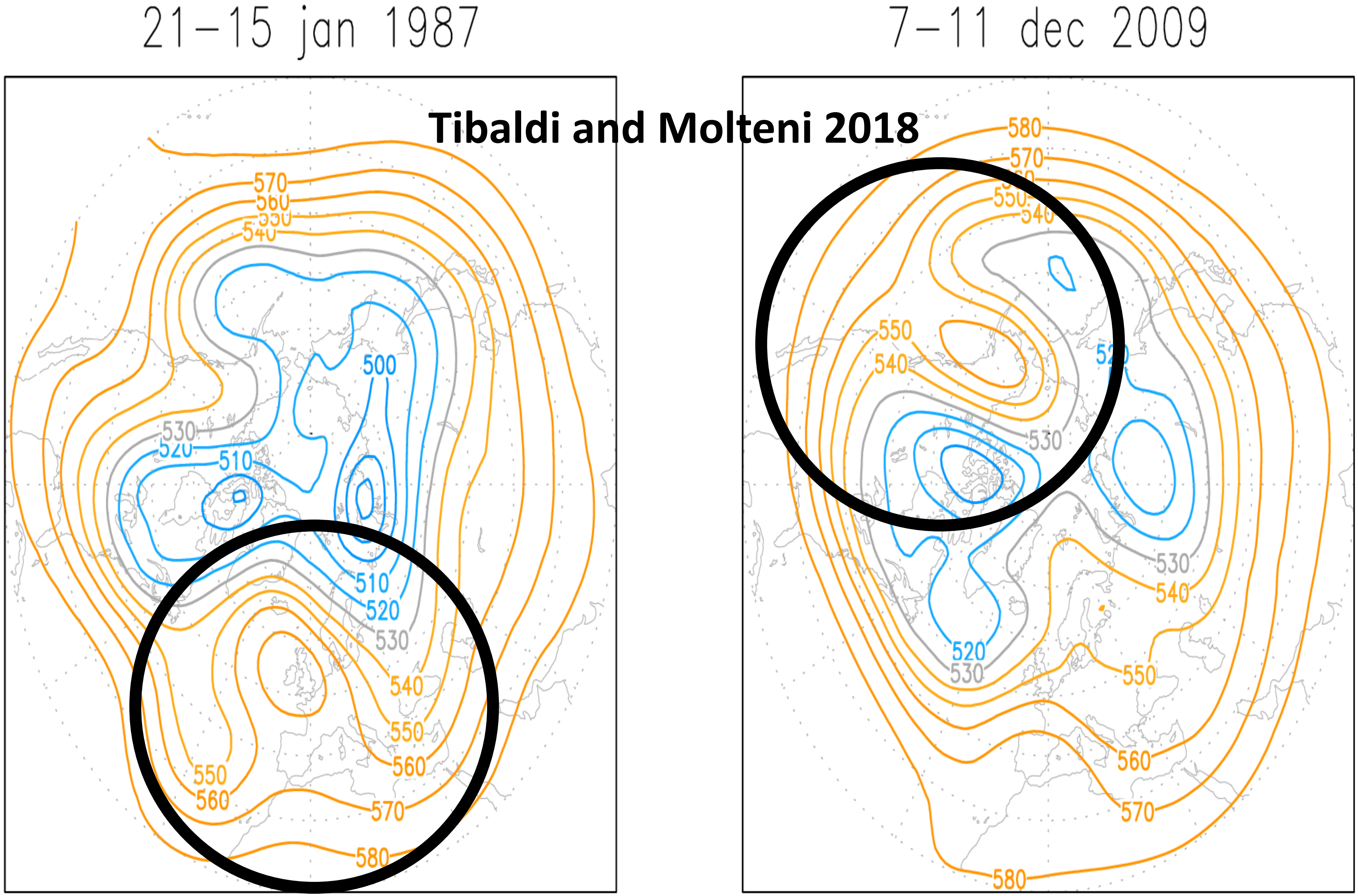
Tibaldi and Molteni 2018



Omega Block

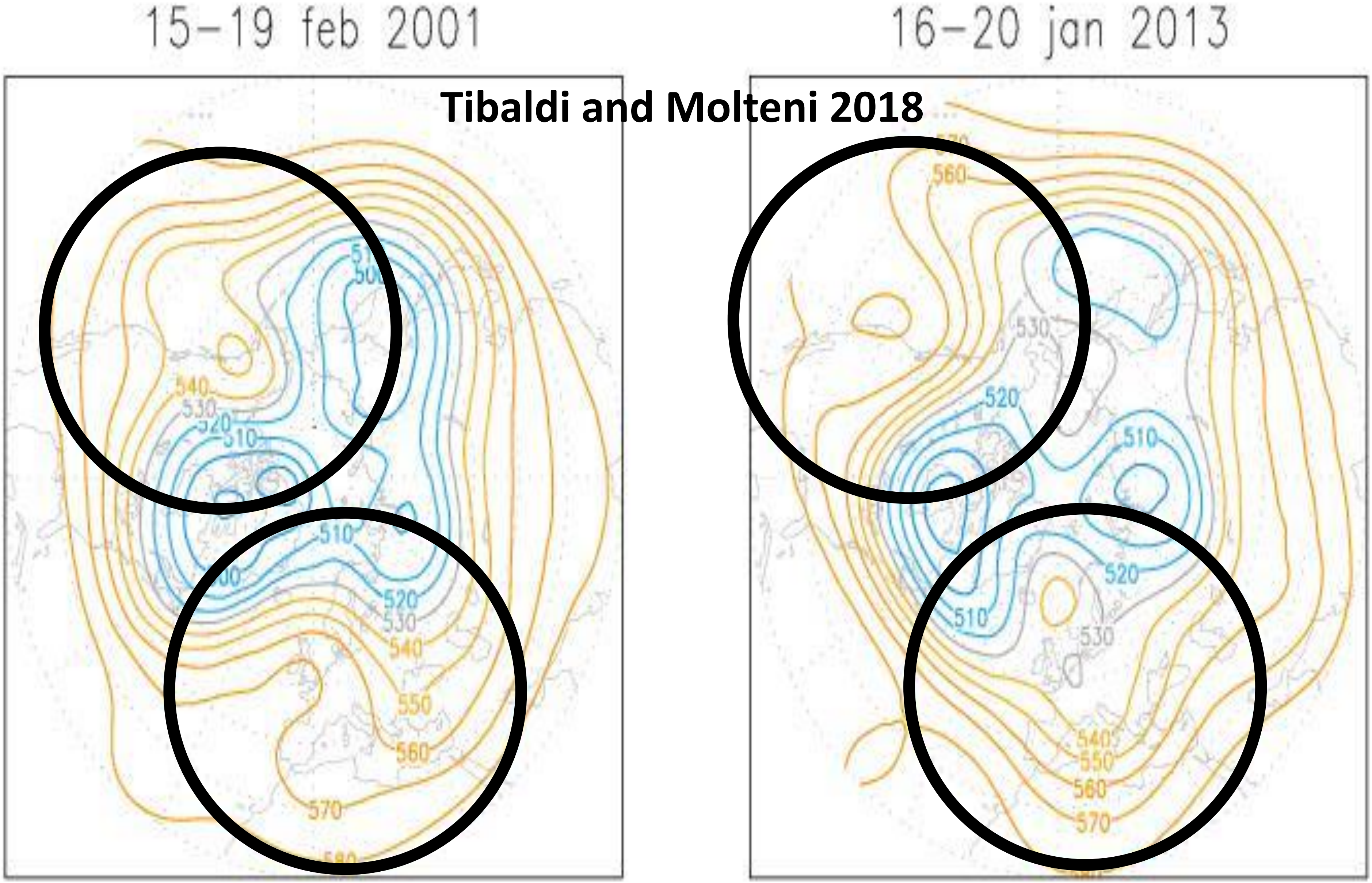


Dipole Block



Euro-Atlantic Block

Pacific Block



Two examples of "Global" Block

Is there “a” Blocking Episode?

<u>Phenomenolology is extremely complex and varied</u>

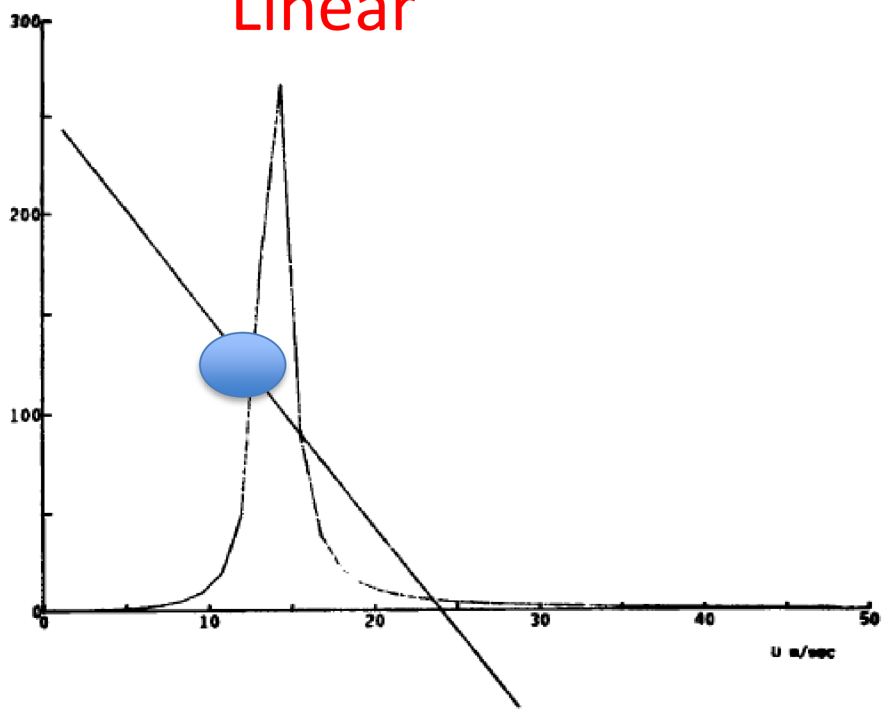
Examples:

- Pacific Blocking vs. Euro-Atlantic Blocking
- Omega block vs. dipole block
- Global block vs. sector block
- NH Blocking vs. SH Blocking
- ……….

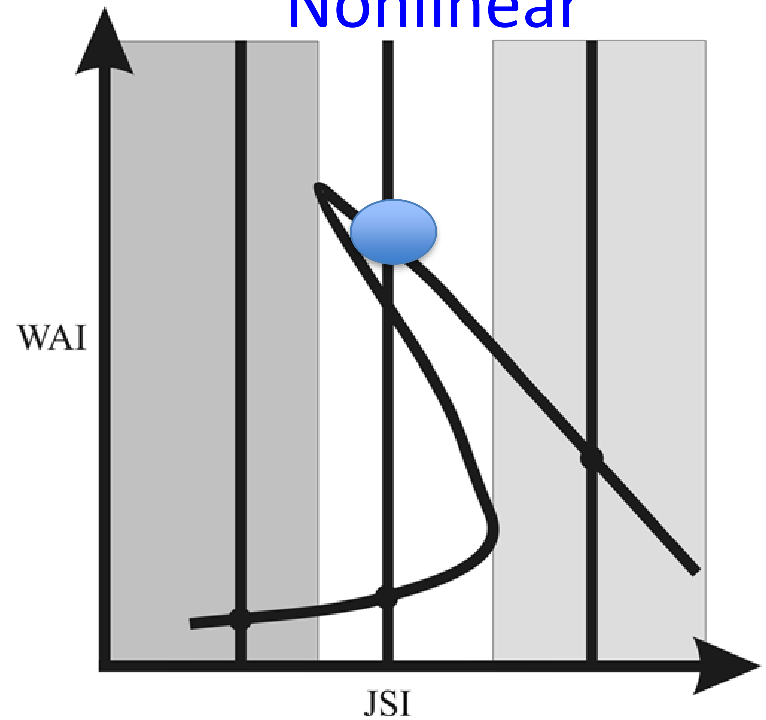
Difficulties in formulating a comprehensive theory

LFV: Blocking as Resonant Rossby Wave on Topography

Linear



Nonlinear



BTROP: Charney DeVore (1979)

BCLINIC Charney and Strauss (1981)

Benzi et al. (1986)

Ruti et al. (2006)

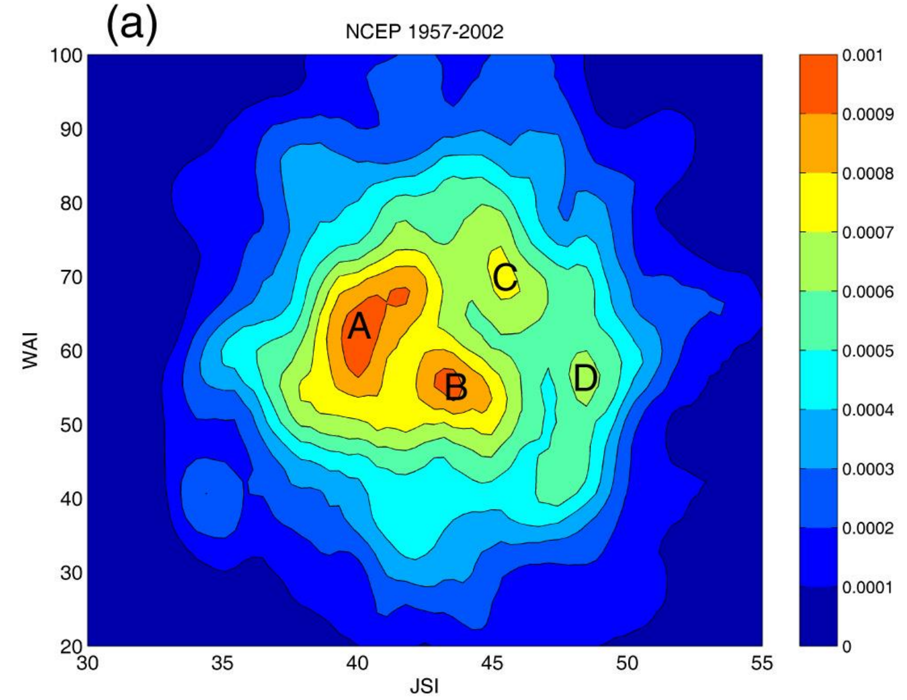
Multistability of the Atmosphere

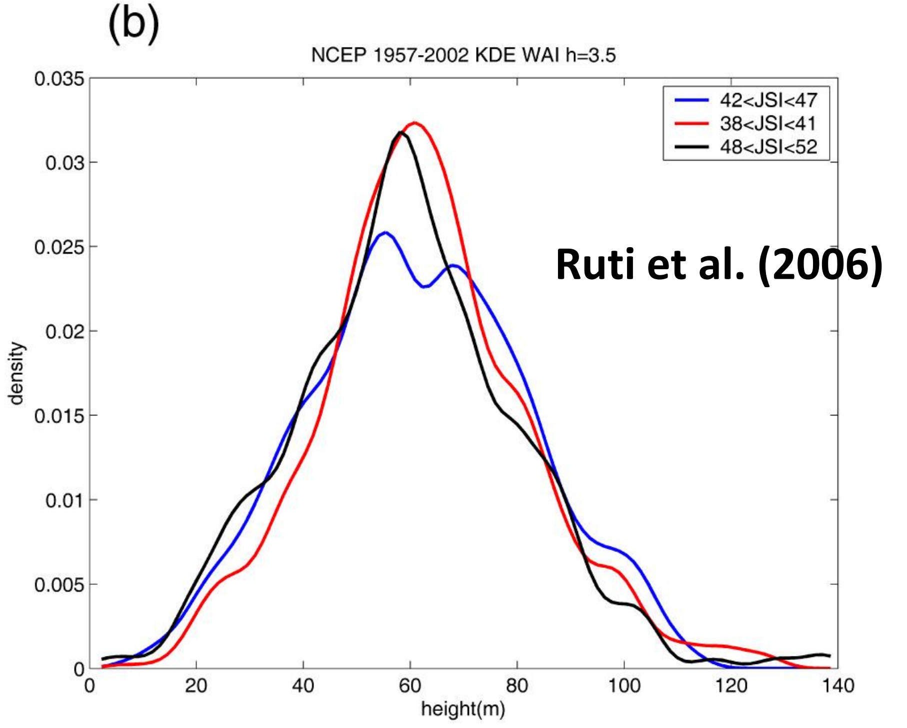
Theory and “Evidence” of Weather Regimes

Zonal vs Blocked

High-Frequency variability facilitating the transitions

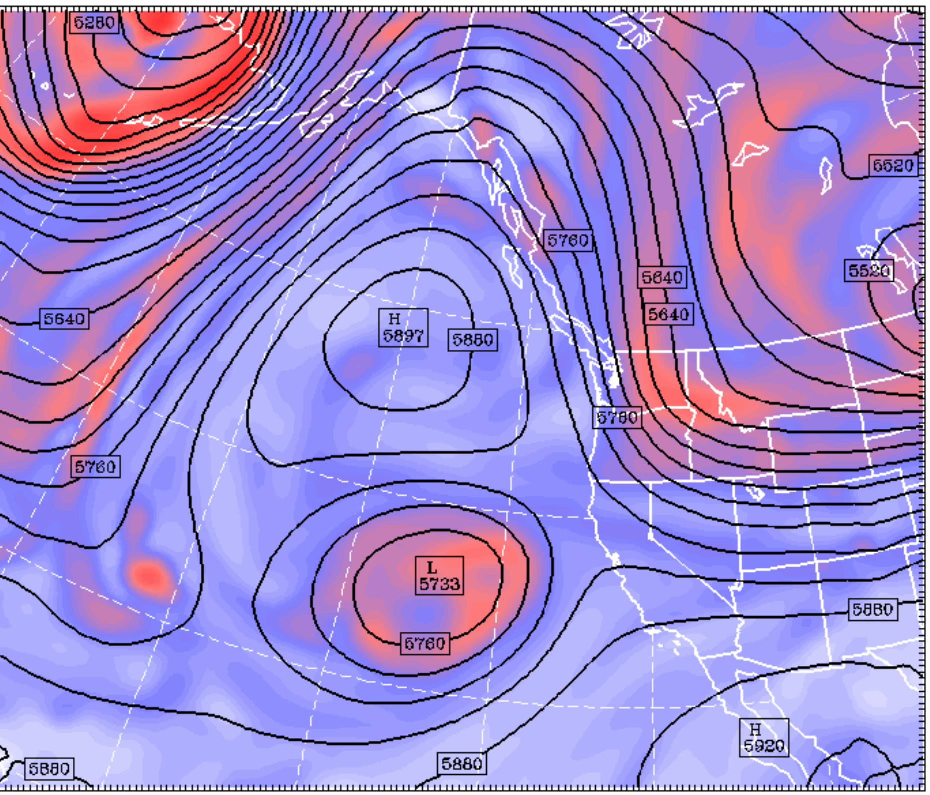
Hansen and Sutera 1985
Legras and Ghil 1985
Benzi et al. 1986
Ghil (et X) 198Y



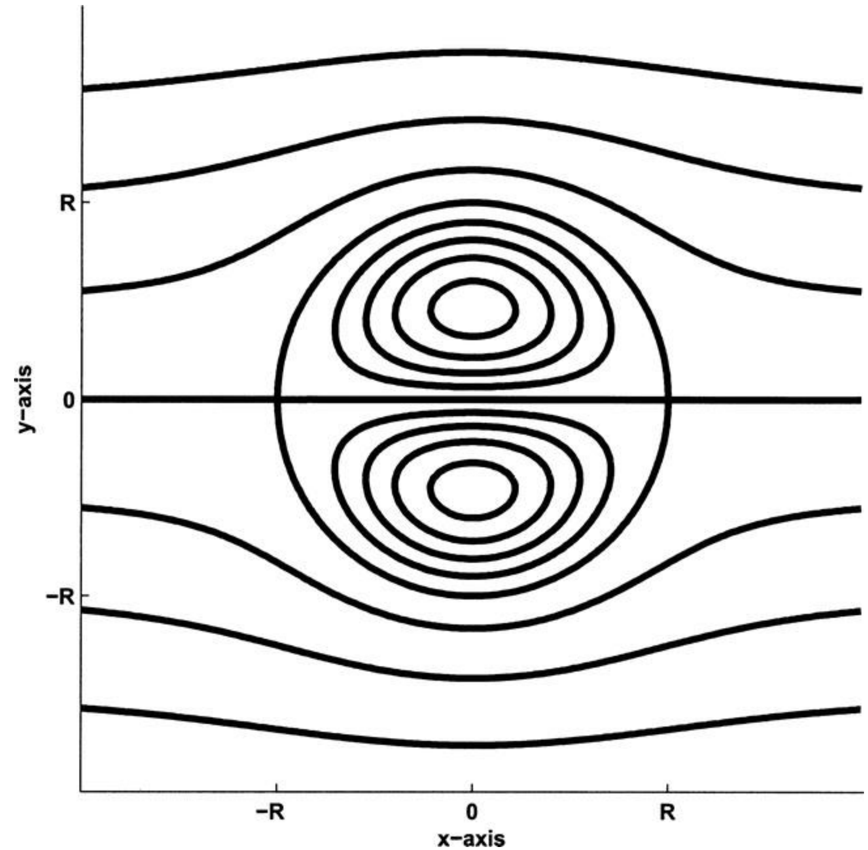


Ruti et al. (2006)

Blockings as Modons



Rex/Dipole/Modon Block

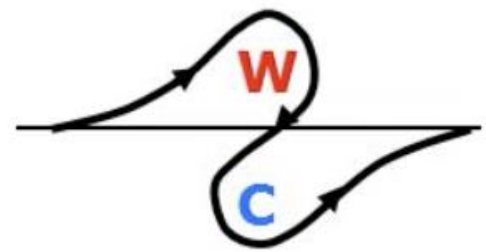


$$J\left(Q, \Psi\right) = 0$$

(Flierl, 1980; McWilliams, 1980, Haines and Marshall, 1987)

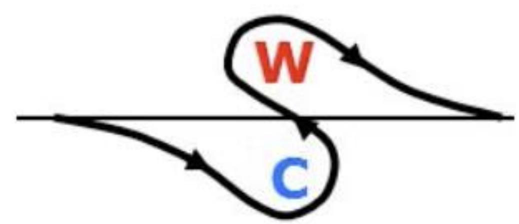
Blockings as Result of Rossby Wave Breaking

The breaking of upper-level Rossby waves is a precursor of Blocking episodes



Anticyclonic wave-breaking

Wave breaking leads to a slowdown of the zonal flow



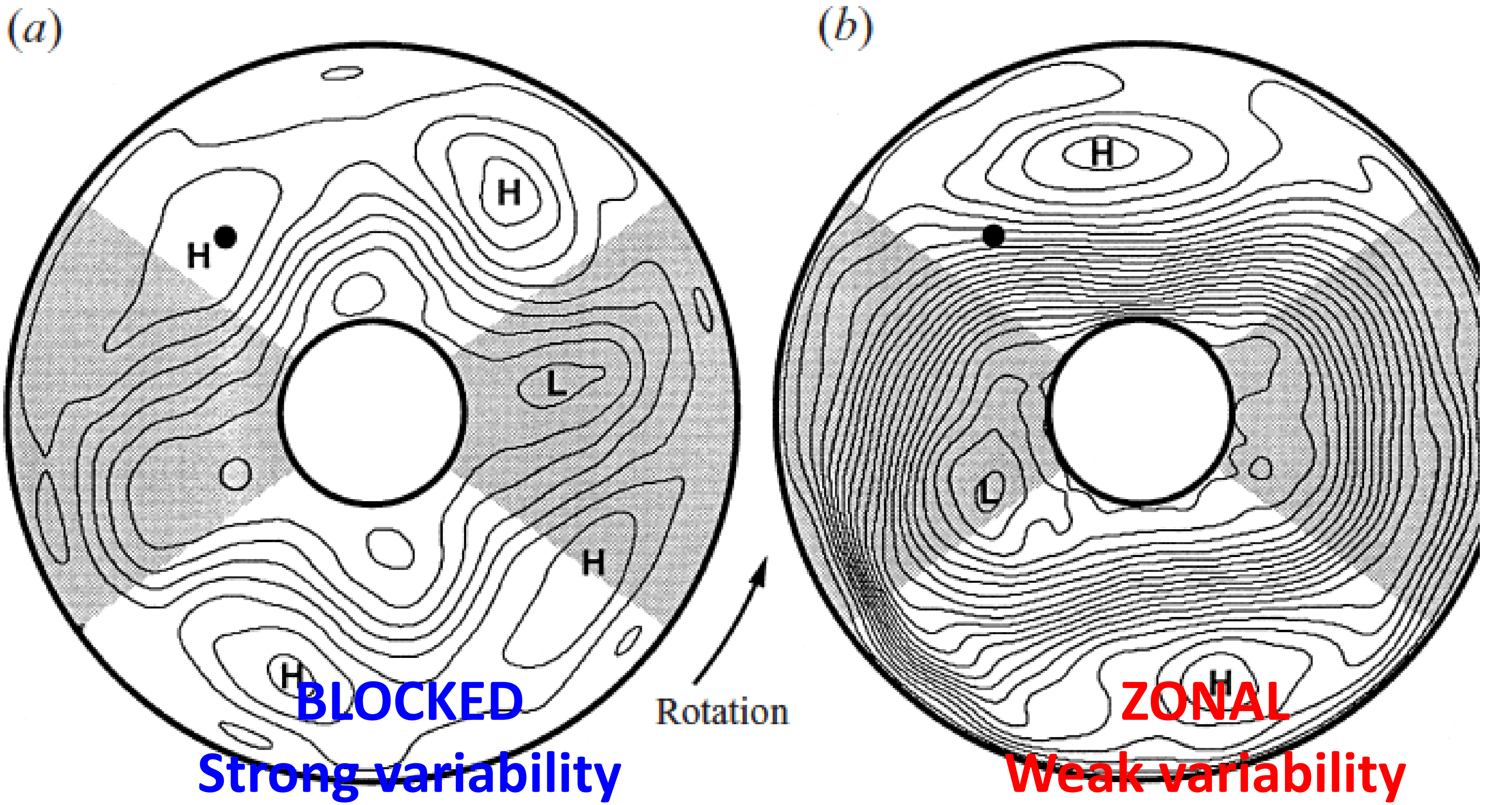
Cyclonic wave-breaking

Multiscale phenomena

(Pelly and Hoskins 2003, Weijenborg et al. 2012)

Rotating Annulus Experiment

Barotropic Dynamics
Weeks et al. 1997; Tian et al. 2001



Markov Chain Models

	Pattern 1	Pattern 2	Patern 3
Pattern 1	0.8	0.25	0.2
Pattern 2	0.15	0.7	0.1
Pattern 3	0.05	0.05	0.7

- Vautard and Ghil (1990), Kimoto and Ghil (1993), proposed to describe the transitions between weather patterns (e.g.: blocking, zonal, Atlantic ridge) using Markov chains
- Each day a pattern has a certain probability to persist or morph into another pattern
- Optimal number of patterns?

Detecting Blocking Events

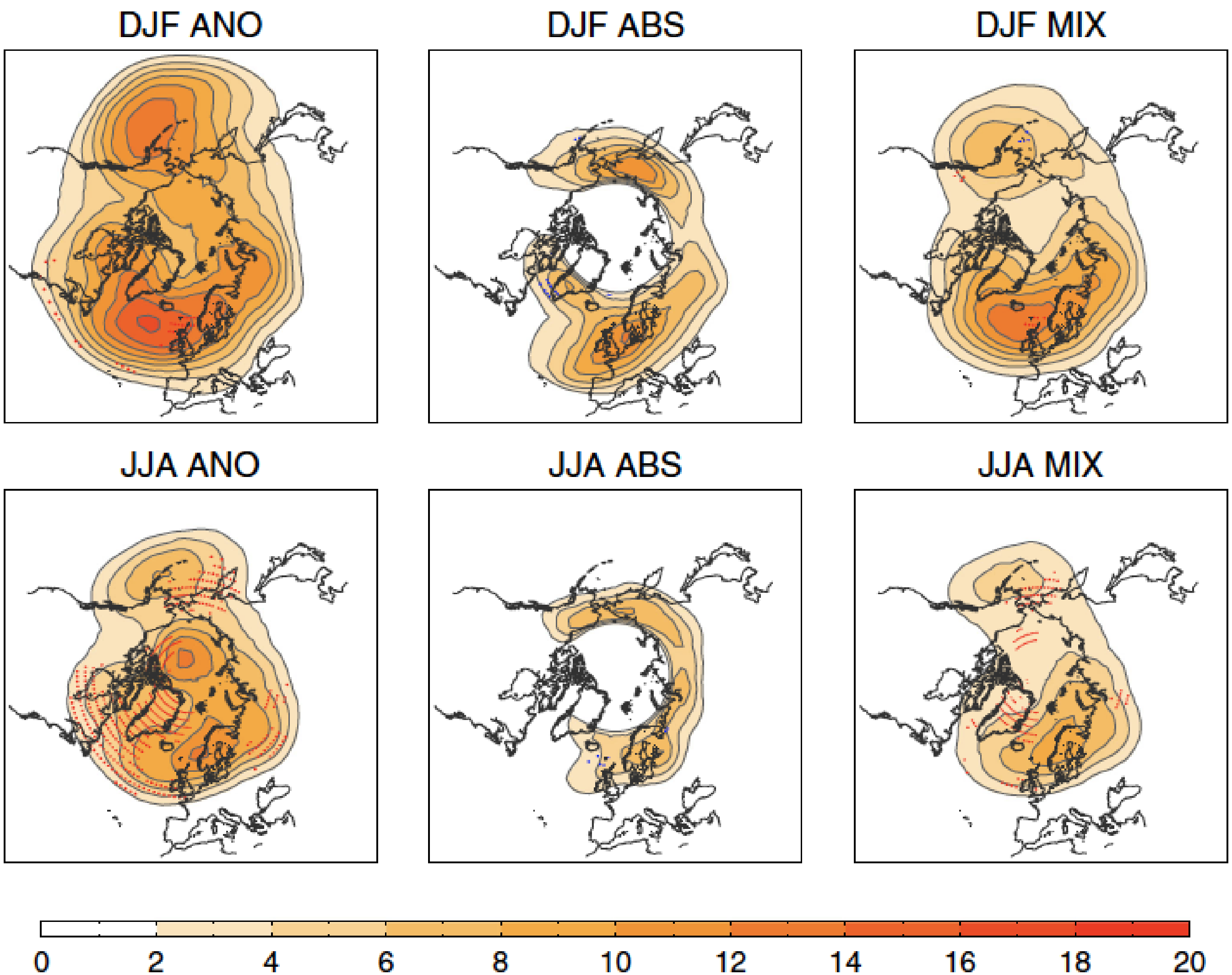
- Many ways to detect blockings
- Classical Blocking Index – Tibaldi & Molteni 1990

$$BN(l,t) = \frac{Z500(f_N, l, t) - Z500(f_0, l, t)}{f_N - f_0}$$

$$BS(l,t) = \frac{Z500(f_0, l, t) - Z500(f_S, l, t)}{f_0 - f_S}$$

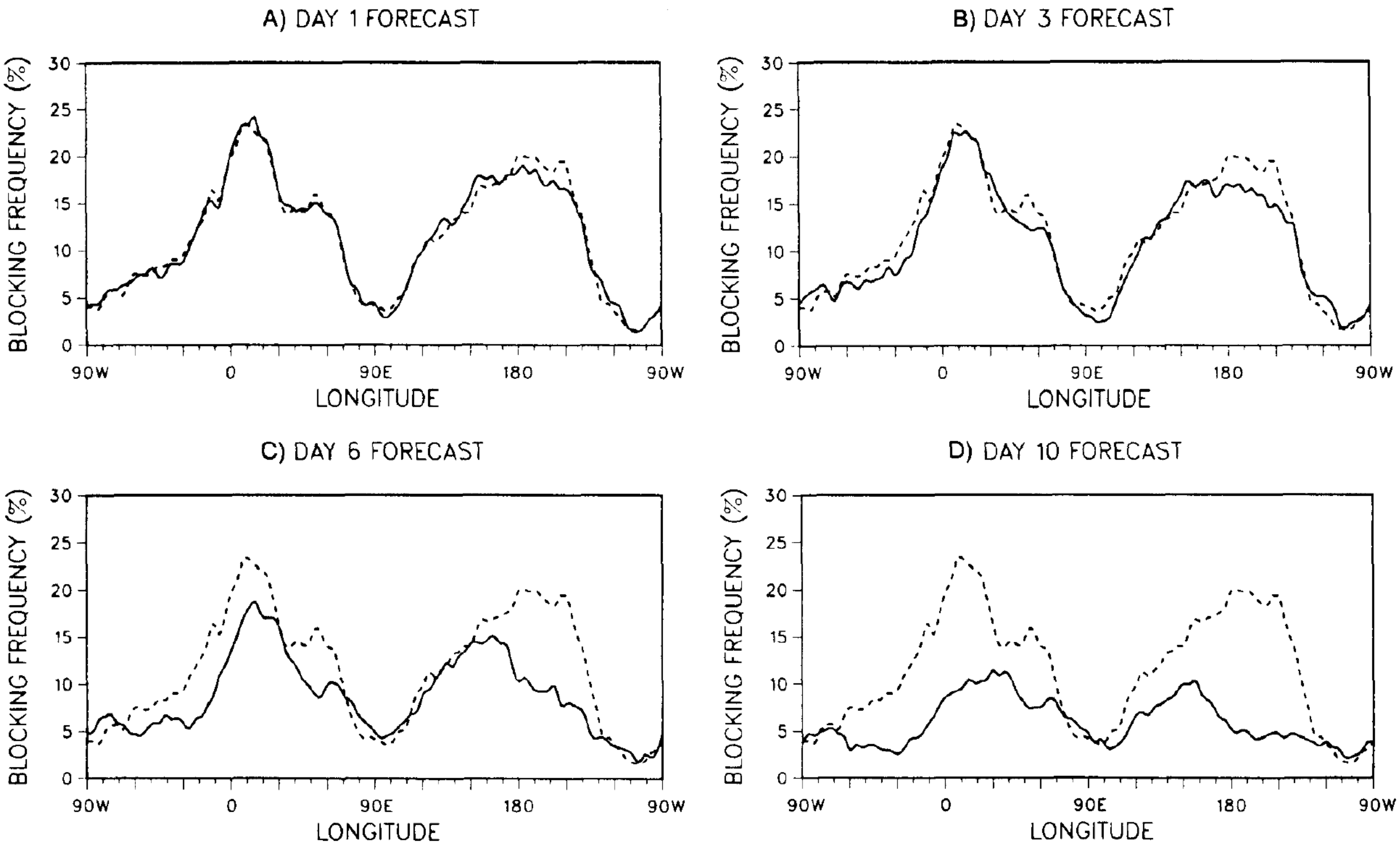
- One investigates whether flow is reversed in the mid-latitudes defined by ϕ_0
- Blocking is detected if BN<0 and BS>threshold

Occurrence of Blockings



1958-2012, various reanalyses, Woolings et al. 2018

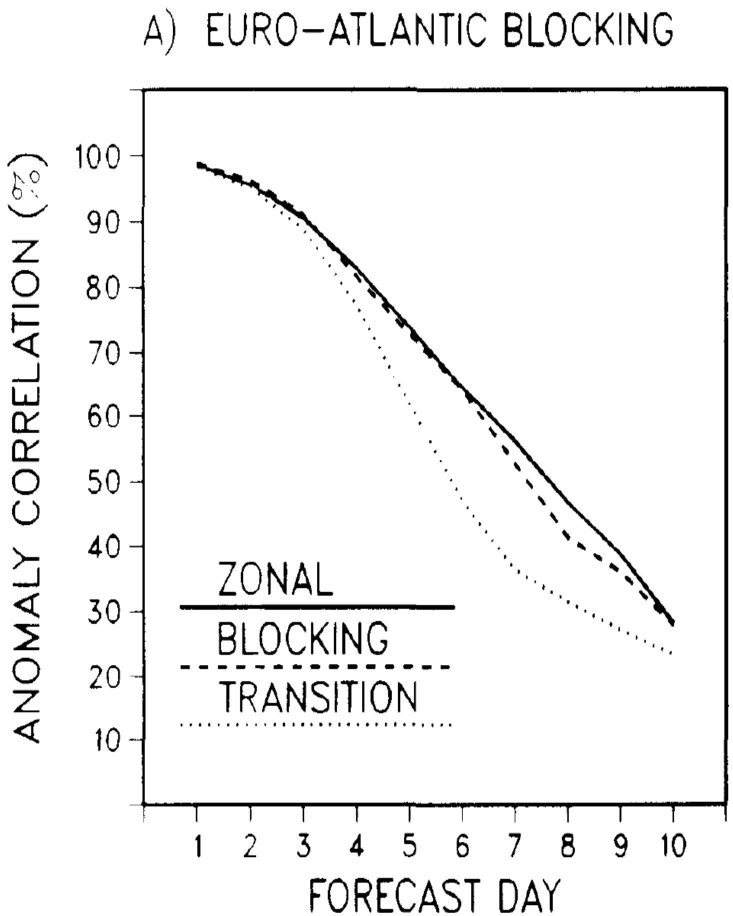
Performance of NWP Models (a)



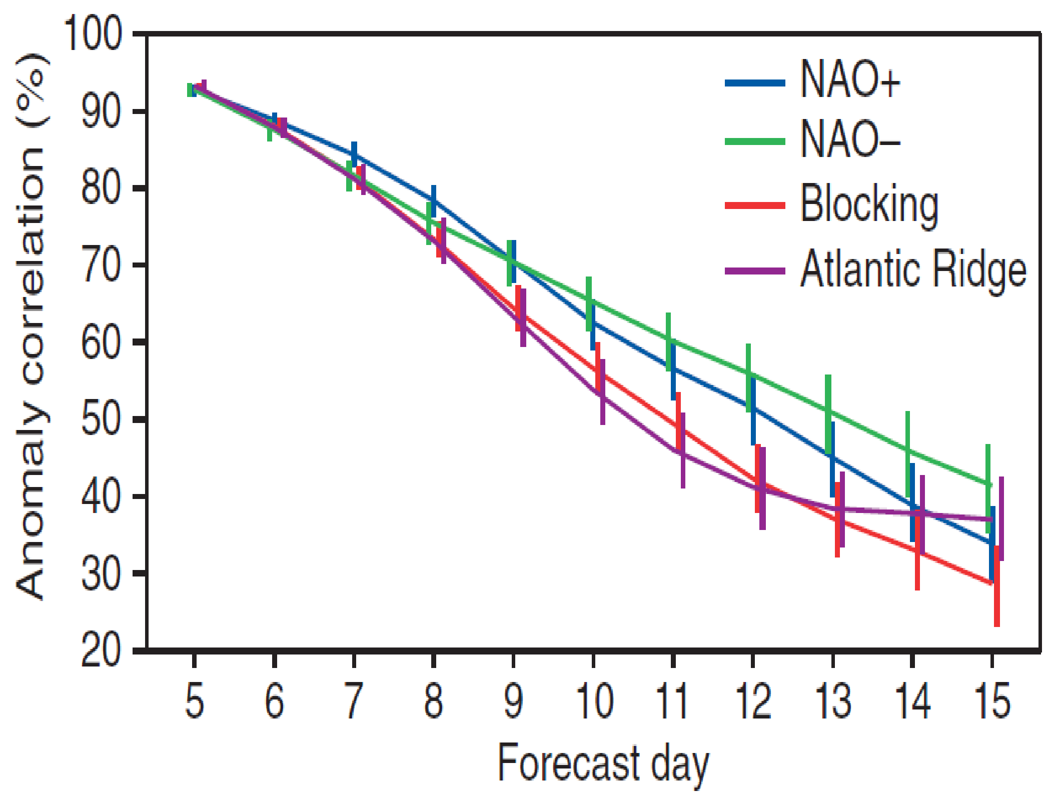
Tibaldi and Molteni 1990

Performance of NWP Models (b)

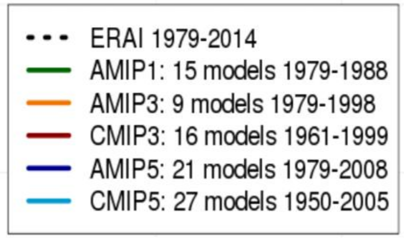
Tibaldi and Molteni 1990



Corti et al. 2014



Performance of Climate Models (a)



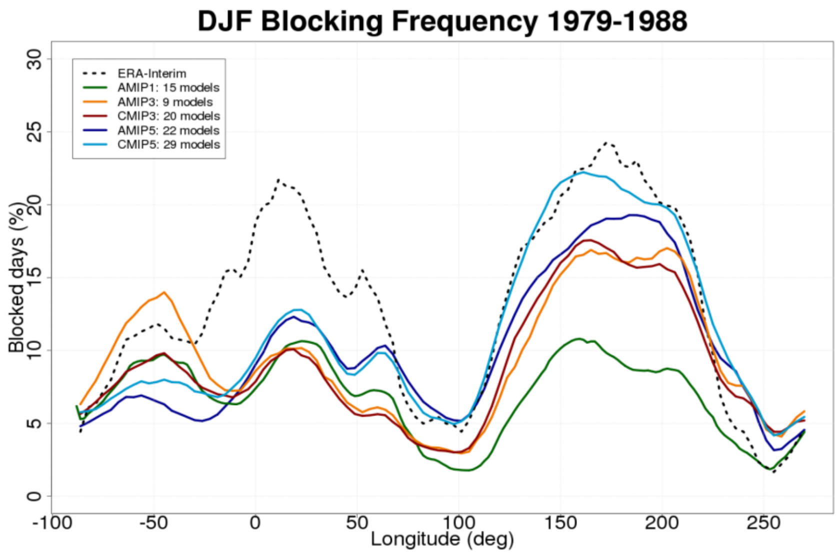


FIG. 3. 1979-1988 DJF multi-model ensemble mean (MMM) of the instantaneous blocking frequency. Black dashed line is the ERA-Interim Reanalysis.

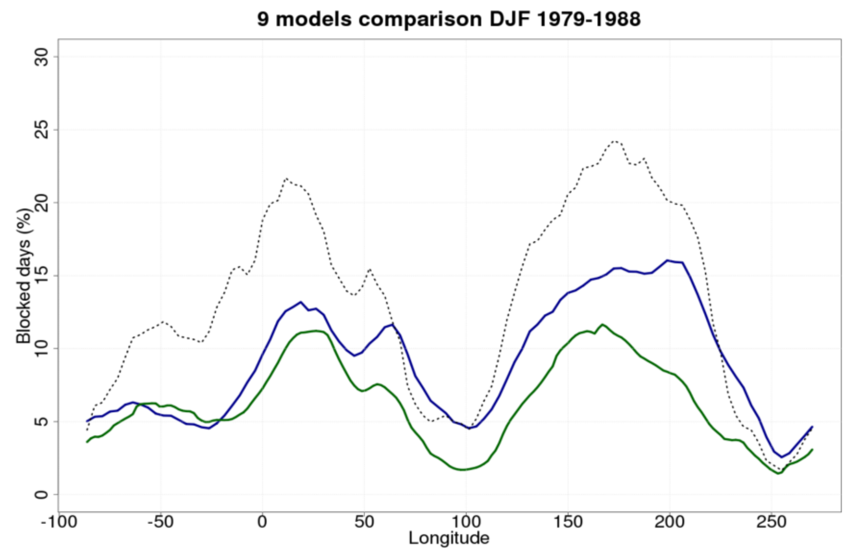
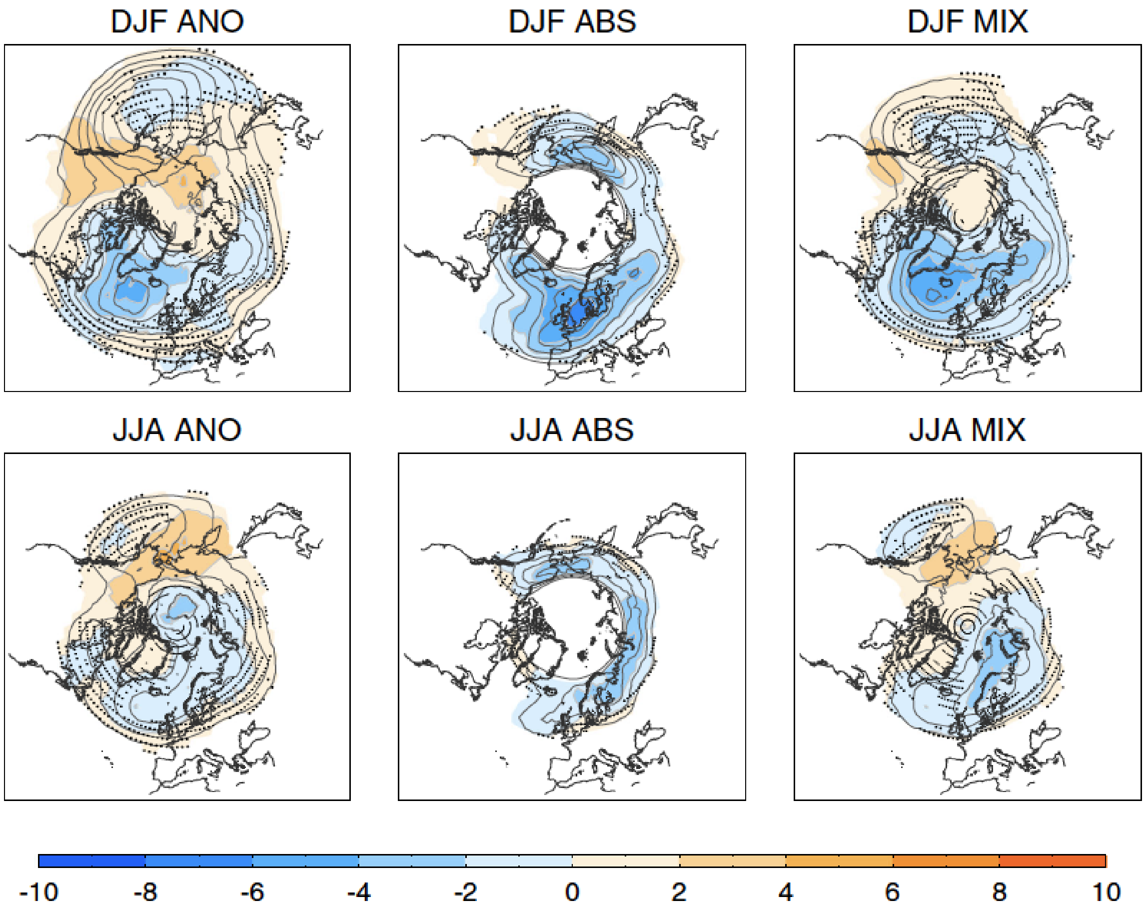


Figure A: MMM for the 9 AMIP1 and AMIP5 models shown in Figure 2 of the manuscript. Results are considerably similar to the one seen in the MMM of Figure 3.

Davini and D'Andrea 2016

Performance of Climate Models (b)



Woolings et al. 2018

Question 1

- Blocked conditions are, visually, quite special. But:

- **Are Blocked conditions associated to higher or lower instability of the atmosphere?**

Lyapunov Exponents (a)

Dynamical system of interest

$$\frac{dx}{dt} = F\left(x\right)$$

Tangent Linear Equation

$$\frac{dy}{dt} = \frac{\partial F\left(x\right)}{\partial x} y$$

Linear Propagator

$$y(t) = J_t\left(x\right) y$$

Lyapunov Exponents (b)

- Matrix $\Lambda(x) = \lim_{t\to\infty} (J_t^T(x) J_t(x))^{1/2t}$

- Eigenvalues: $\Lambda_i(x)$

- Lyapunov Exp.: $\lambda_i = \log \Lambda_i$

Finite-Time Lyapunov Exponents

- Matrix $\Lambda(x,t) = (J_t^T(x) J_t(x))^{1/2t}$
- Eigenvalues: $\Lambda_i(x,t)$
- Lyapunov Exp.: $\lambda_i(x,t)$

Indicators of Instability

- Kaplan Yorke Dimension

$$D_{KY} = m + \frac{\sum_{i=1}^{m} \lambda_i}{|\lambda_{m+1}|}$$

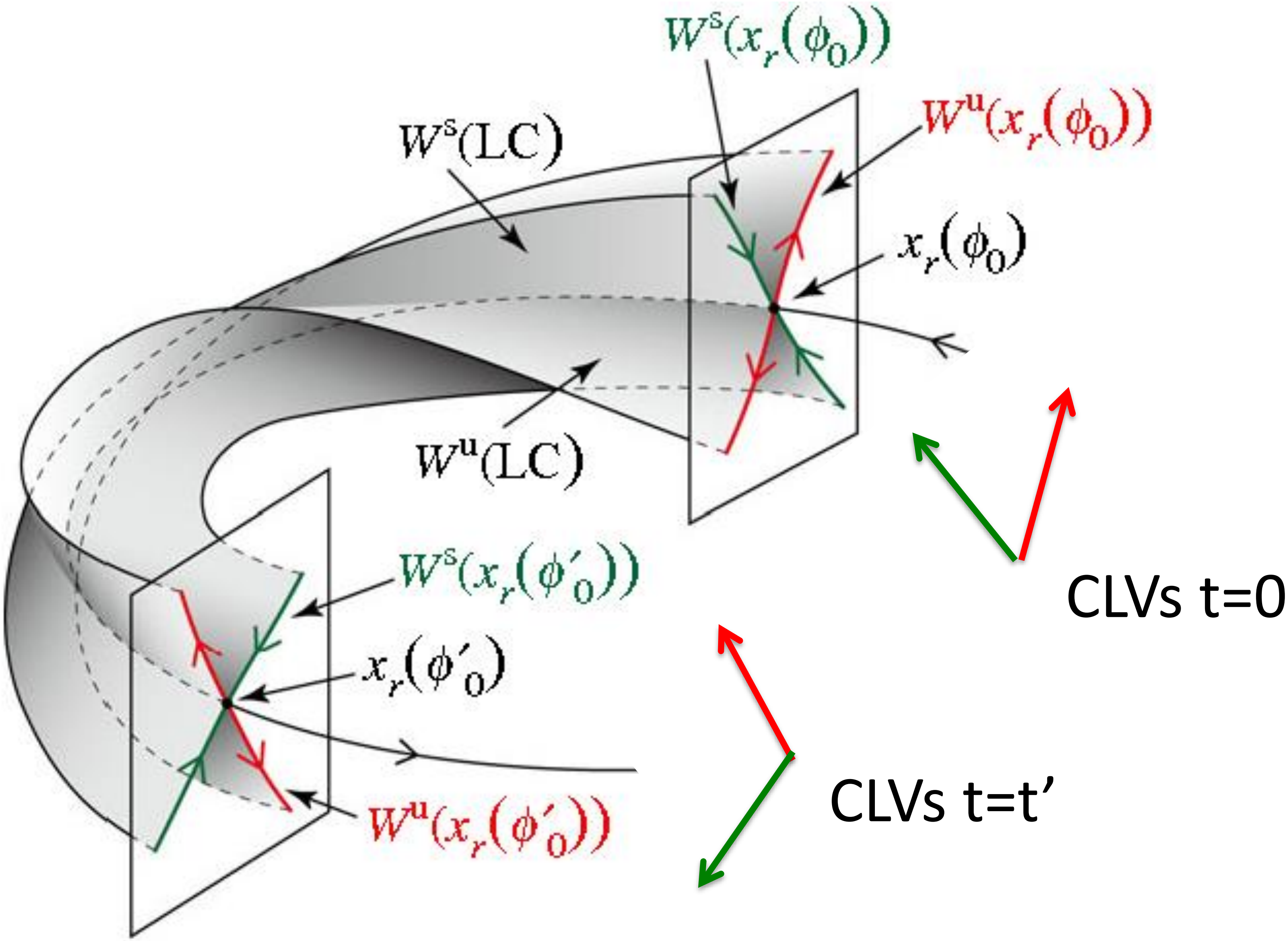
- Kolmogorov-Sinai entropy
  - (and its finite-time counterpart)

$$h_{KS} = \sum_{\lambda_i > 0} \lambda_i$$

Covariant Lyapunov Vectors

- Asymptotic grow with positive/negative Lyapunov exponents into the future/past
- Local counteparts of FTLEs as $l_i(x,t)$
- $n(x,t)$ - # unstable dimensions # $\{l_i(x,t>0\}$
- **Non-orthogonal** and **covariant** with respect to the Tangent Linear evolution
- Efficient algorithms by Ginelli et al. (2007) and Wolfe & Samelson (2007)

Geometrical idea



A QG 2-Layer Model

Vertical Structure

0.5 ≅ 0 hPa

1 ≅ 250 hPa

1.5 ≅ 500 hPa

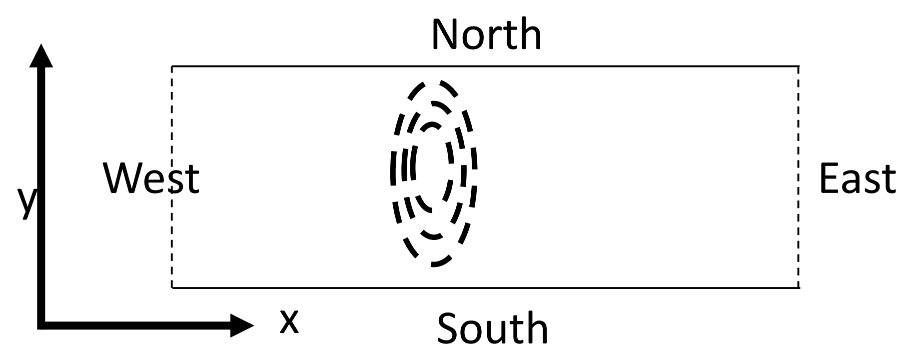
2 ≅ 750 hPa

2.5 ≅ 1000 hPa

Horizontal Structure

- no flux across N and S
- 2.5 Layer with/out orographic forcing:

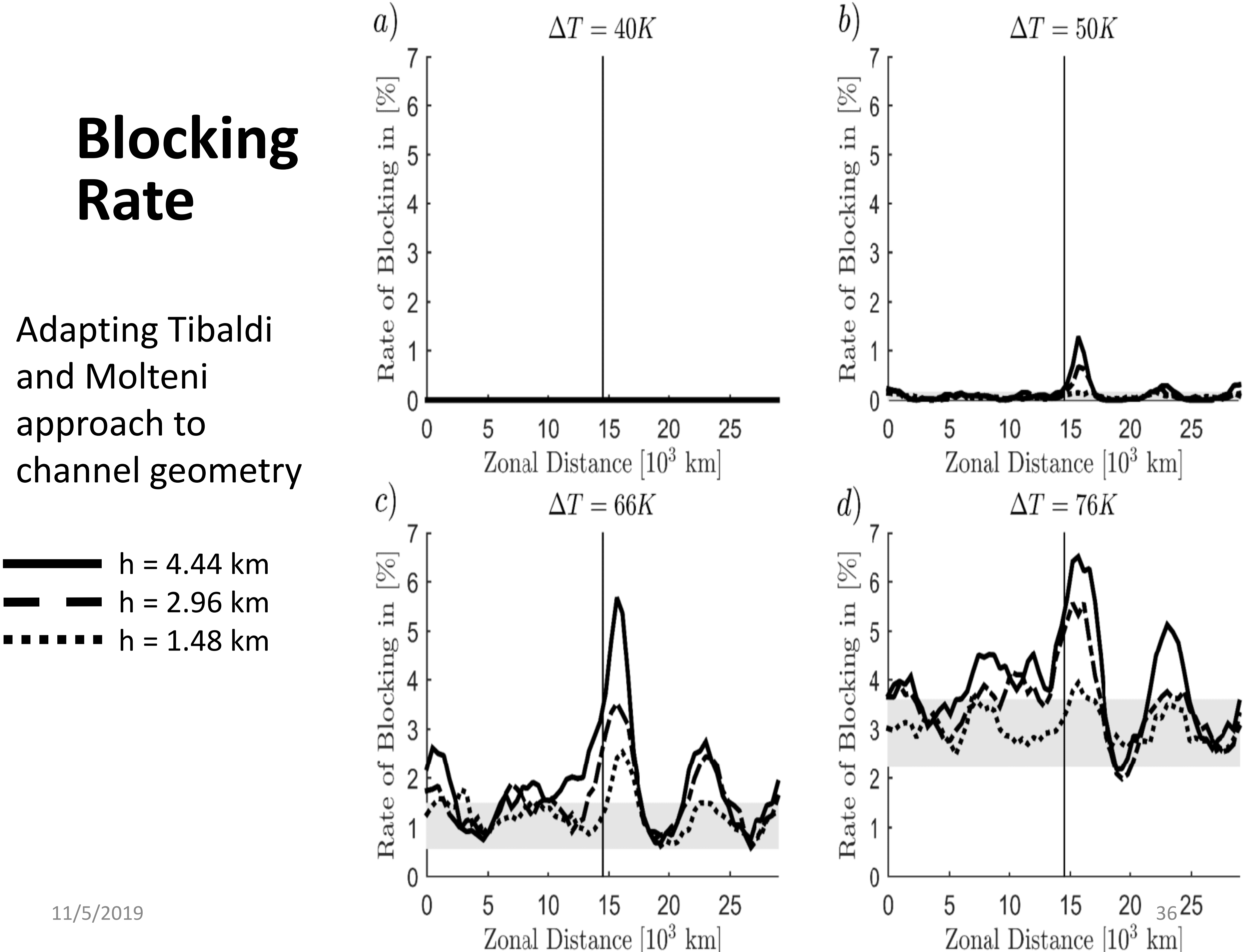
  Bump (Lx= 1000 km; Ly =2000 km)



Schubert and Lucarini QJRMS (2016)

Blocking Rate

Adapting Tibaldi and Molteni approach to channel geometry

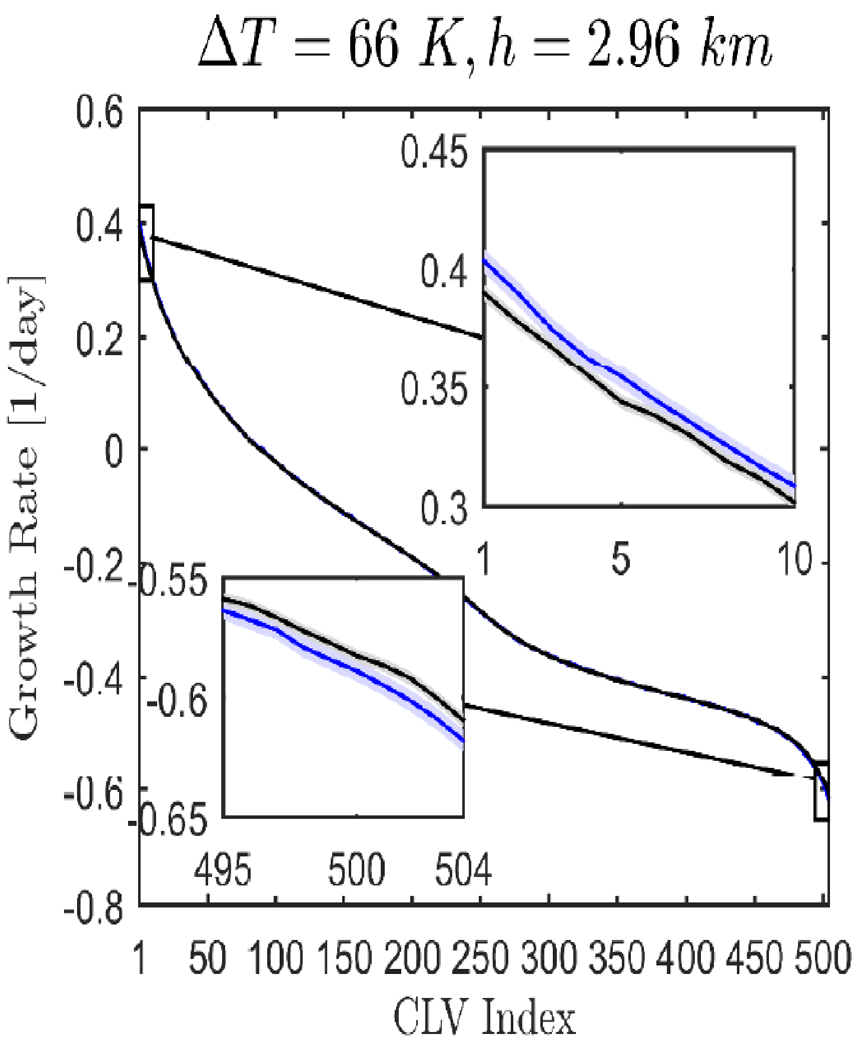


Lyapunov Exponents

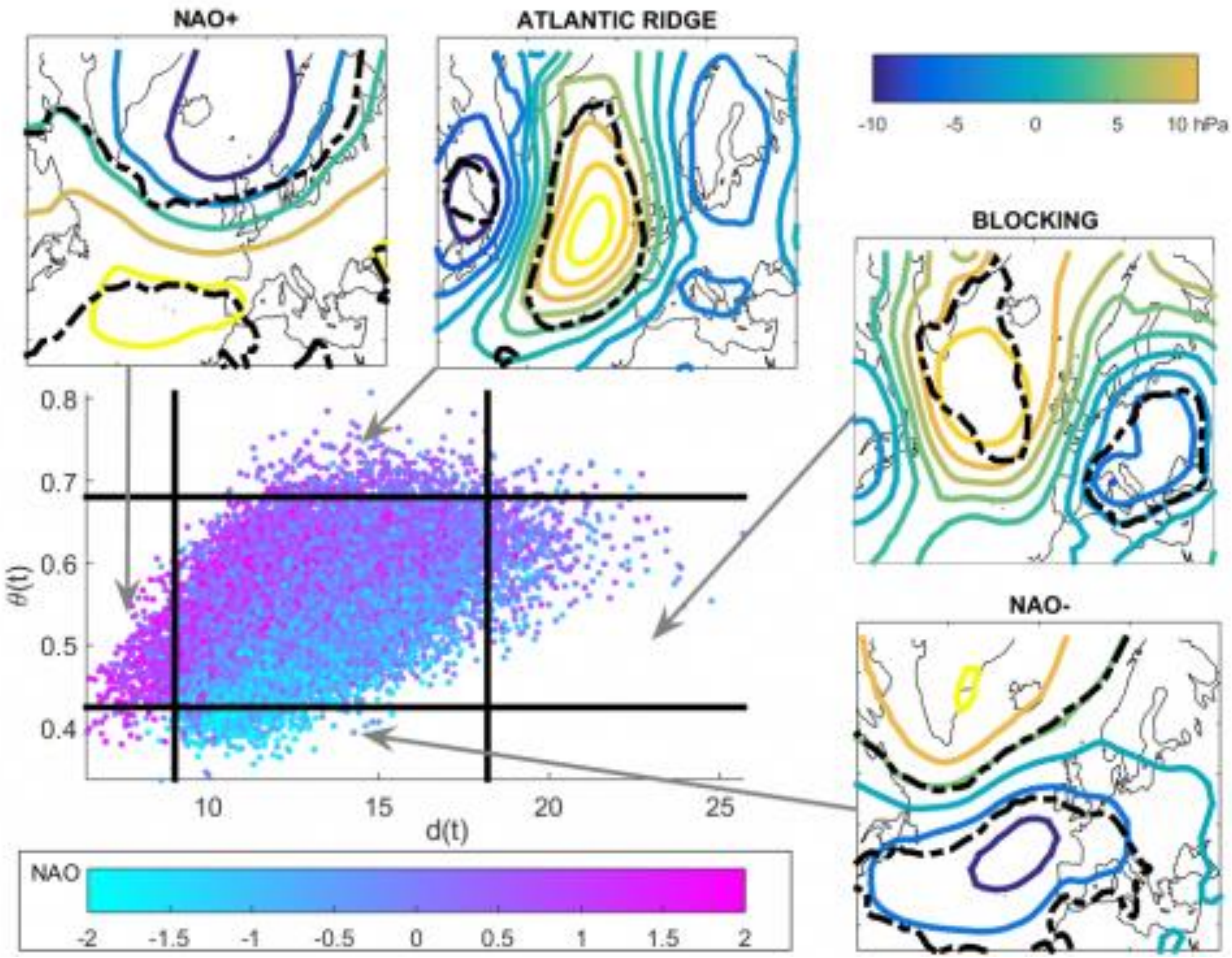
- **Non-Blocked**
- **Blocked**

Positive exponents are larger during blockings

Height increases



Blockings correspond to region of low predictability



EVT-based Analysis of atmospheric flow

Blocking: High values of the local dimension of the attractor

Faranda et al. Sci Rep. (2017)

Answer 1

- Blocking events are indeed associated to higher instability of the atmosphere
- Both Barotropic and Baroclinic instabilities are impacted
- Blocking events are associated to **lower predictability, higher dimensionality** of the atmosphere
- **Conjecture**: Predicting onset and decay of organised structures is very difficult

Question 2

- Do we see a signature of difficulties in predicting onset and decay of Blockings?

Marshall-Molteni 1993 Model (a)

Evolution Equations

$$\partial_t q_j + J(\psi_j, q_j) = -D_j + S_j, \qquad j = 1,2,3$$

Quasi-Geostrophic Potential Vorticity

$$q_1 = \Delta\psi_1 - (\psi_1 - \psi_2)/R_1^2 + f$$
$$q_2 = \Delta\psi_2 + (\psi_1 - \psi_2)/R_1^2 - (\psi_2 - \psi_3)/R_2^2 + f$$
$$q_3 = \Delta\psi_3 + (\psi_2 - \psi_3)/R_2^2 + f(1 + h/H_0)$$

Northern Hemisphere Domain

Marshall-Molteni 1993 Model (b)

125000 days of integration

Dissipation

$$-D_1 = (\psi_1 - \psi_2)/(\tau_R R_1^2) - R^8 \Delta^4 q'_1/(\tau_H \lambda_{max}^4)$$
$$-D_2 = -(\psi_1 - \psi_2)/(\tau_R R_1^2) + (\psi_2 - \psi_3)/(\tau_R R_2^2) - R^8 \Delta^4 q'_2/(\tau_H \lambda_{max}^4)$$
$$-D_3 = -(\psi_2 - \psi_3)/(\tau_R R_2^2) - EK_3 - R^8 \Delta^4 q'_3/(\tau_H \lambda_{max}^4)$$

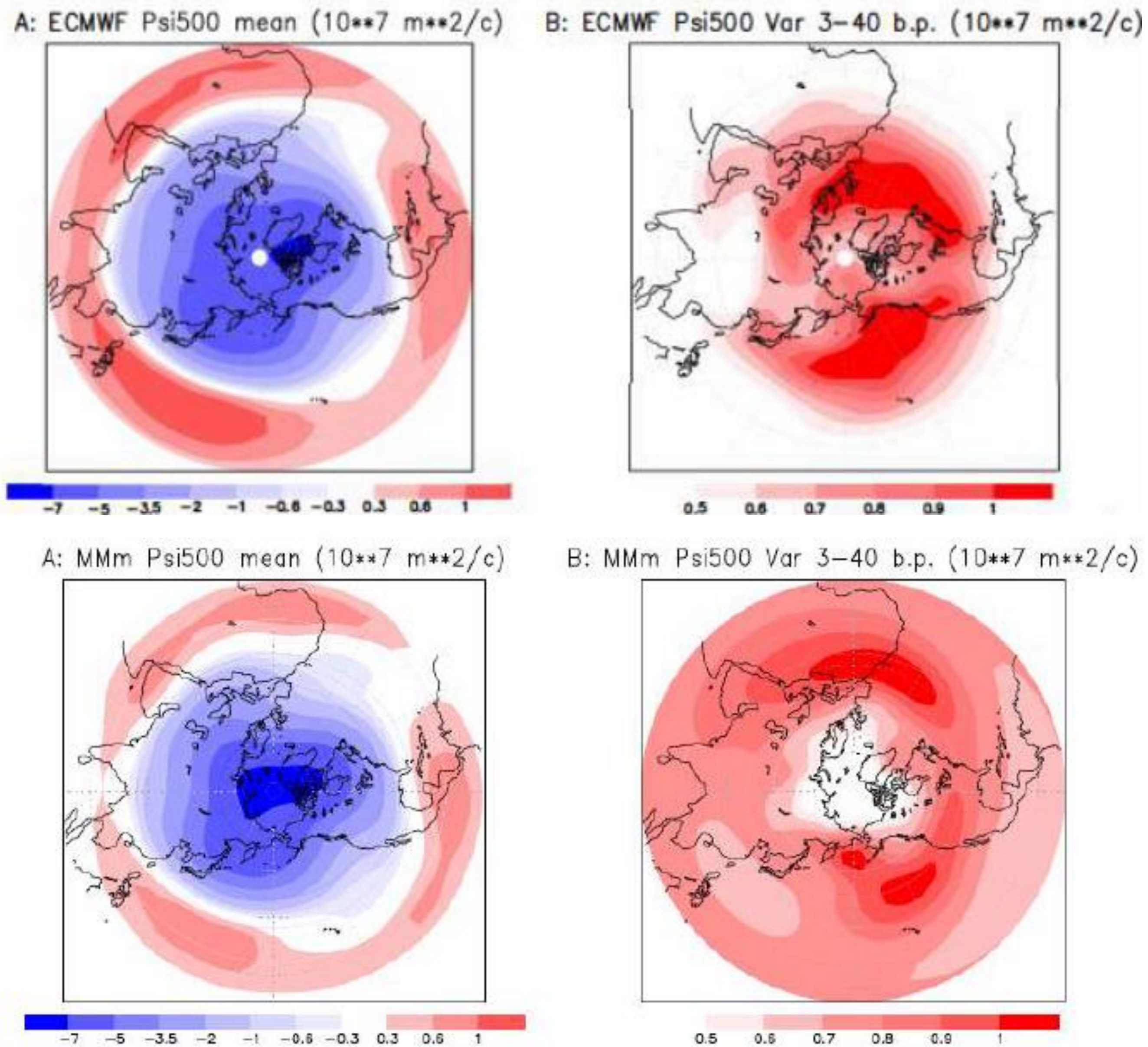
Forcing (taken from ECMWF DJF data)

$$S_j = \overline{J(\psi_j, q_j)} + \bar{D}_j \qquad j = 1,2,3$$

Winter Climatology

ECWMF

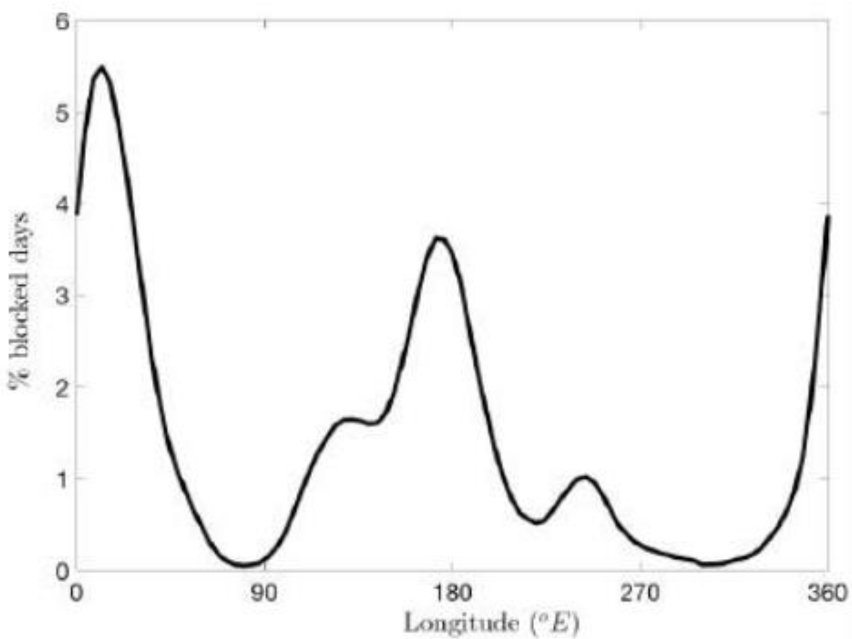
Model



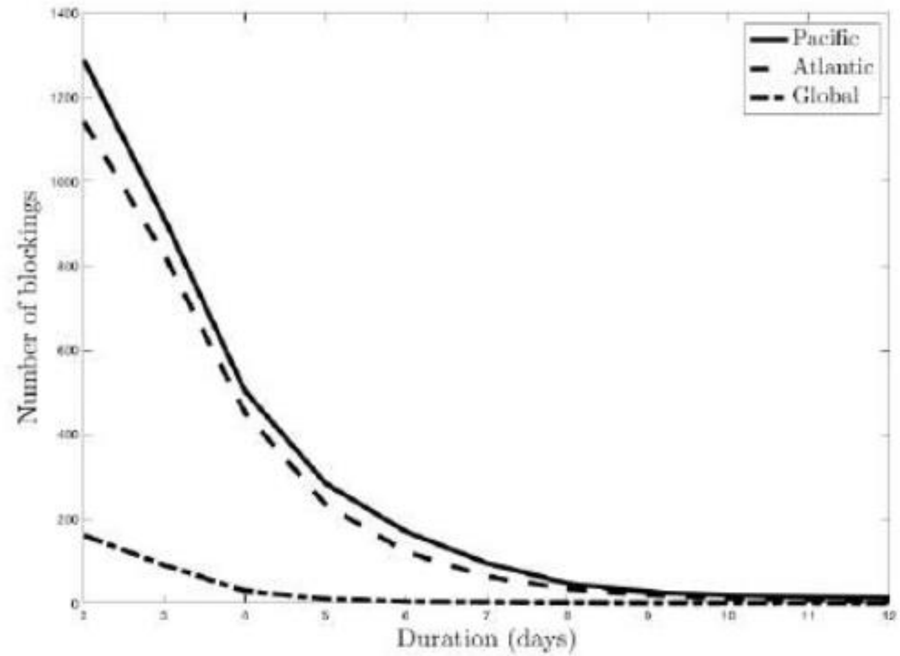
Statistics of Blockings

Location

Persistence

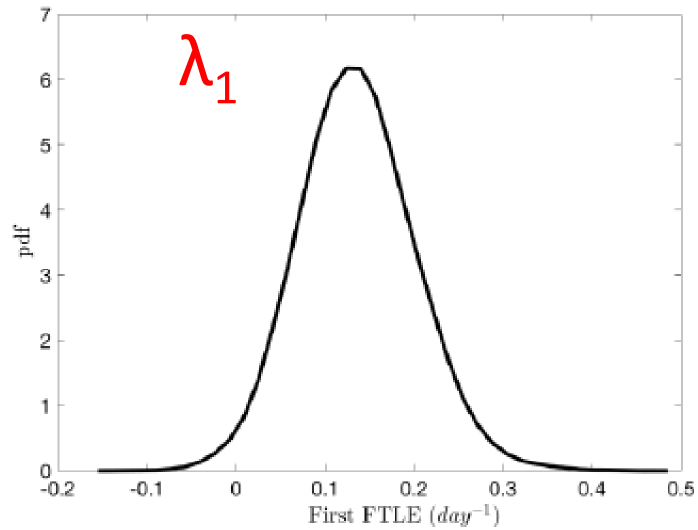


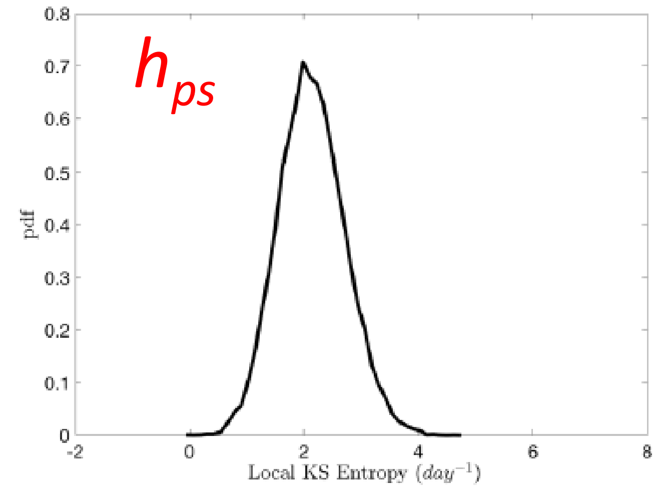
a)

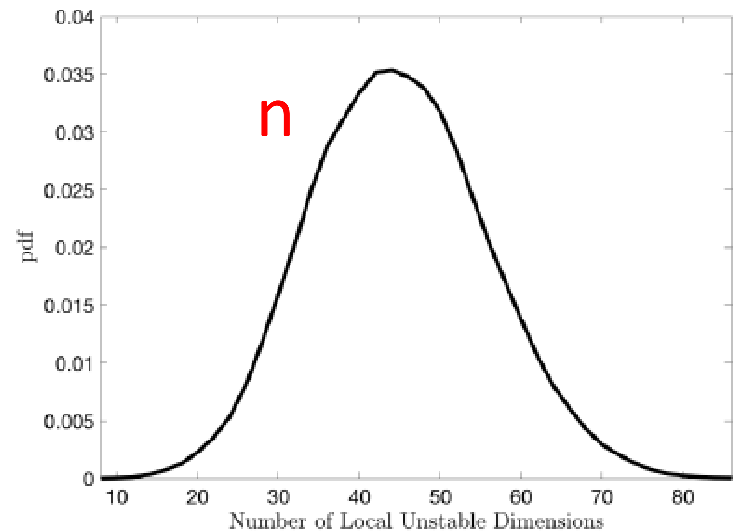


b)

Daily fluctuations in the instability

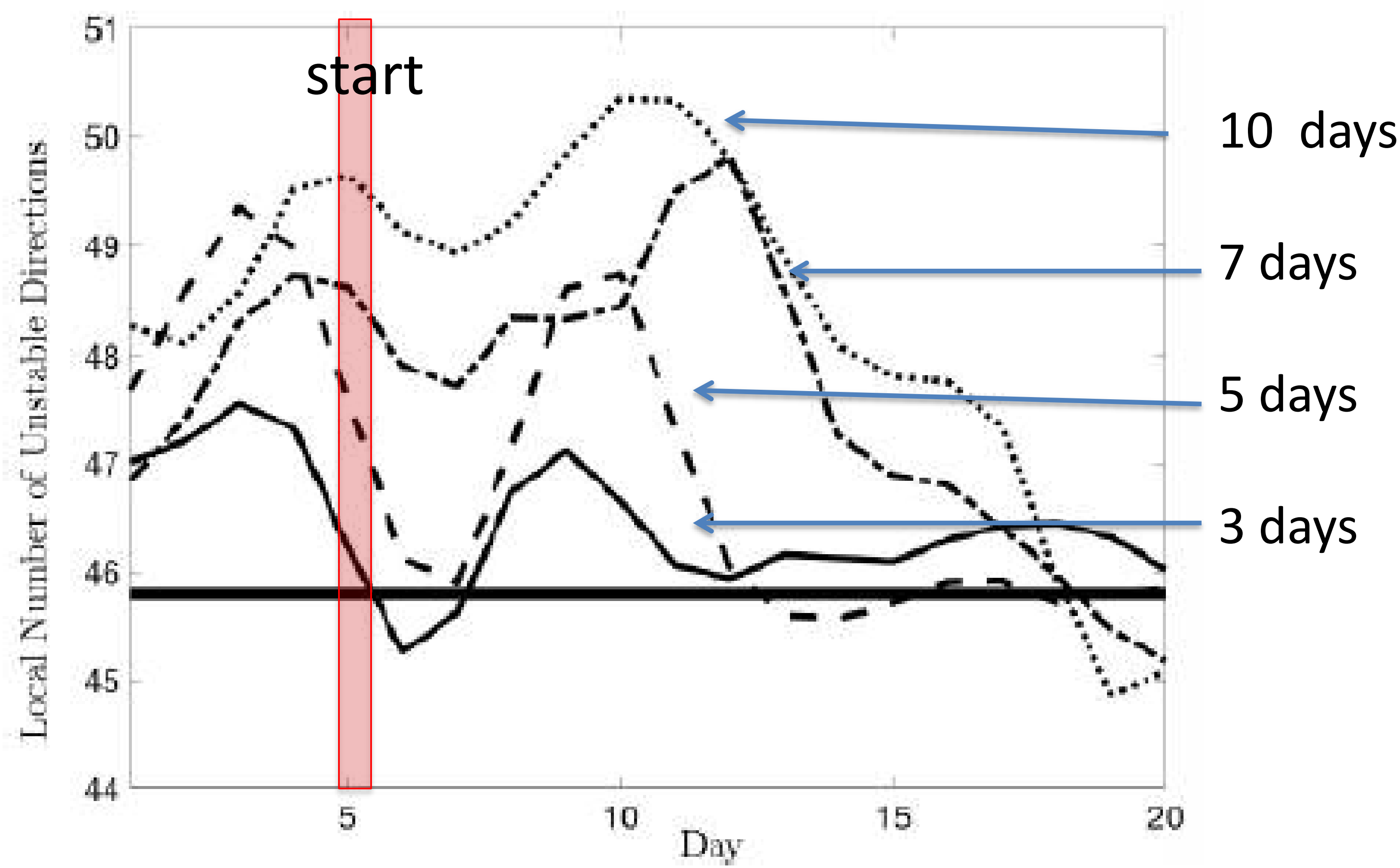






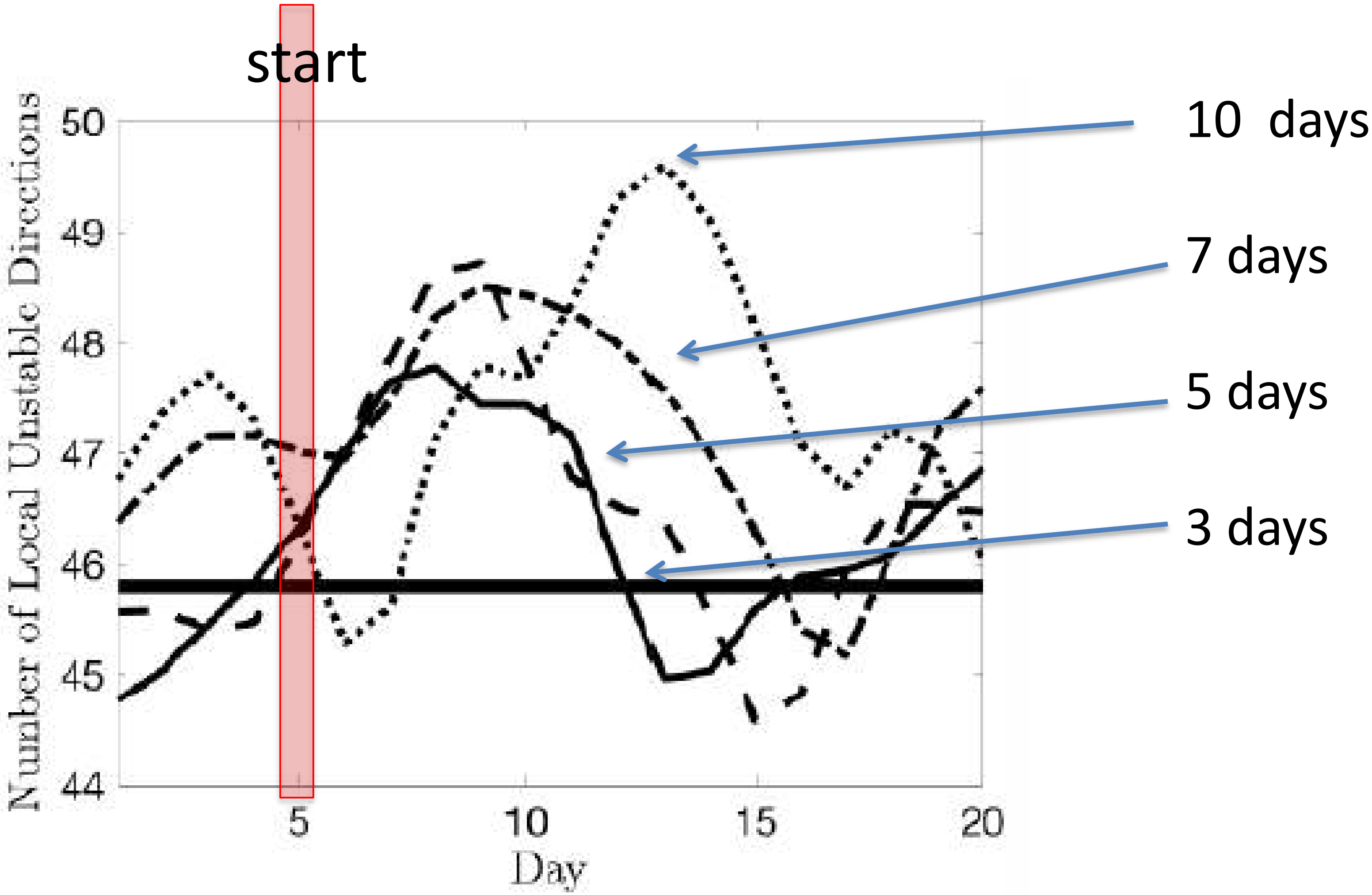
The attractor is extremely heterogeneous – predictability varies a lot

Atlantic Blockings



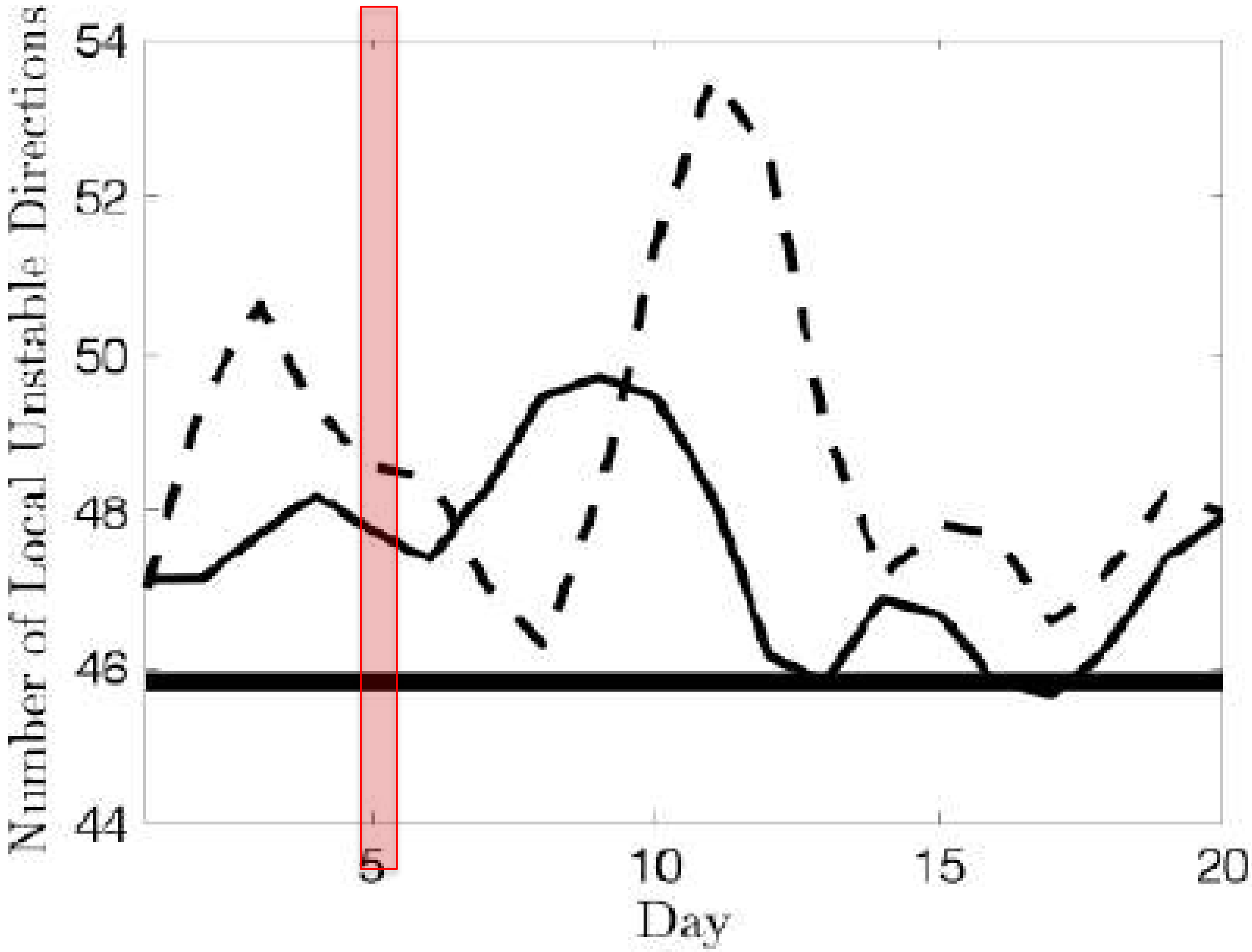
- Longer blockings are more unstable
- Instability peaks at the beginning and end of the blocking

Pacific Blockings



- Longer blockings are more unstable
- Instability peaks when blocking is mature

Global Blockings



- Longer blockings are more unstable
- Global Blockings are more unstable than sector ones

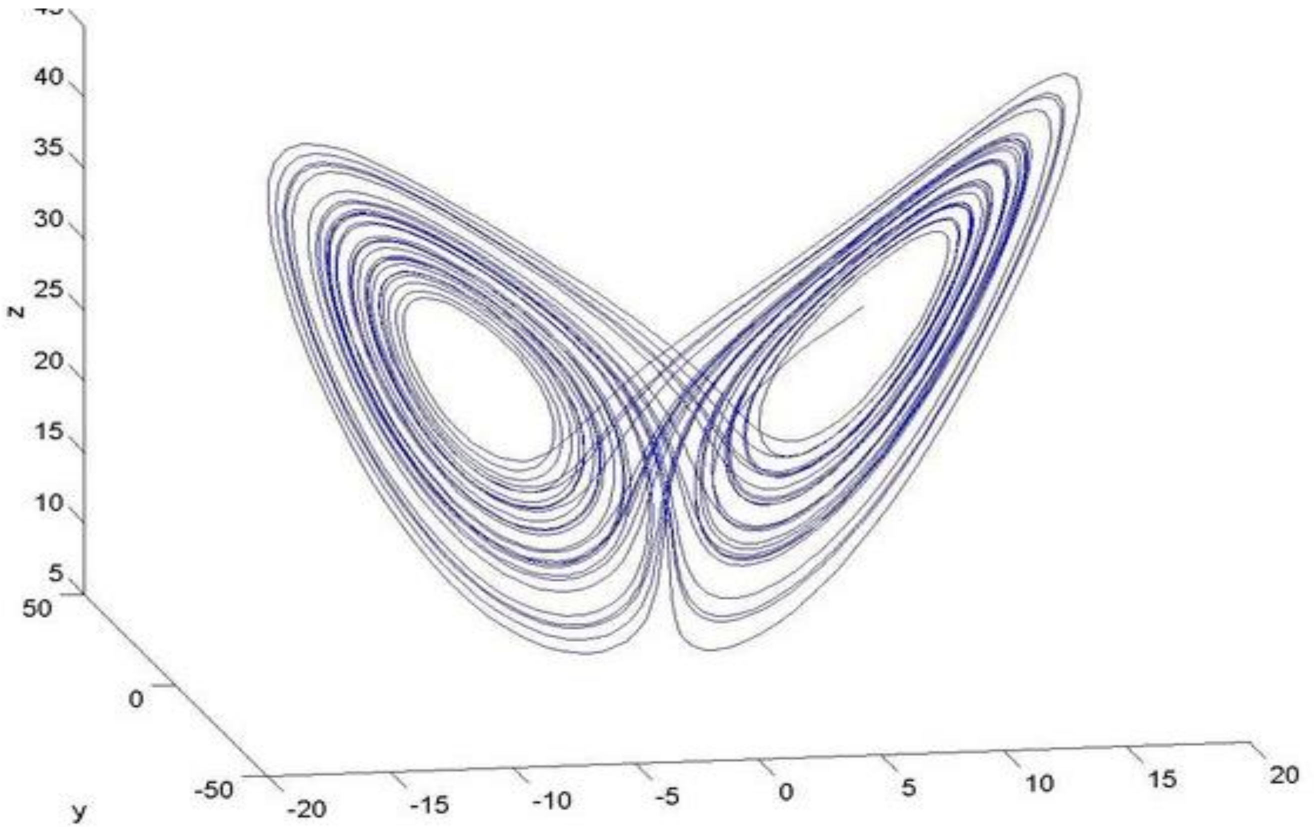
Answer 2

- The local predictability of the atmosphere follows the life cycle of the blockings
- In the case of Atlantic blockings, instability is lowest at the beginning and end of the blocking
- In general, longer blockings are more unstable than shorter ones
- Note: rarer is also more unstable; is it by chance?

Question 3

- “Why” and “How” are Blocked conditions associated to higher instability of the atmosphere?
- Is there something special about blockings?
- Are they associated to special modes of the atmosphere?
- What about transitions between blocked and zonal states?

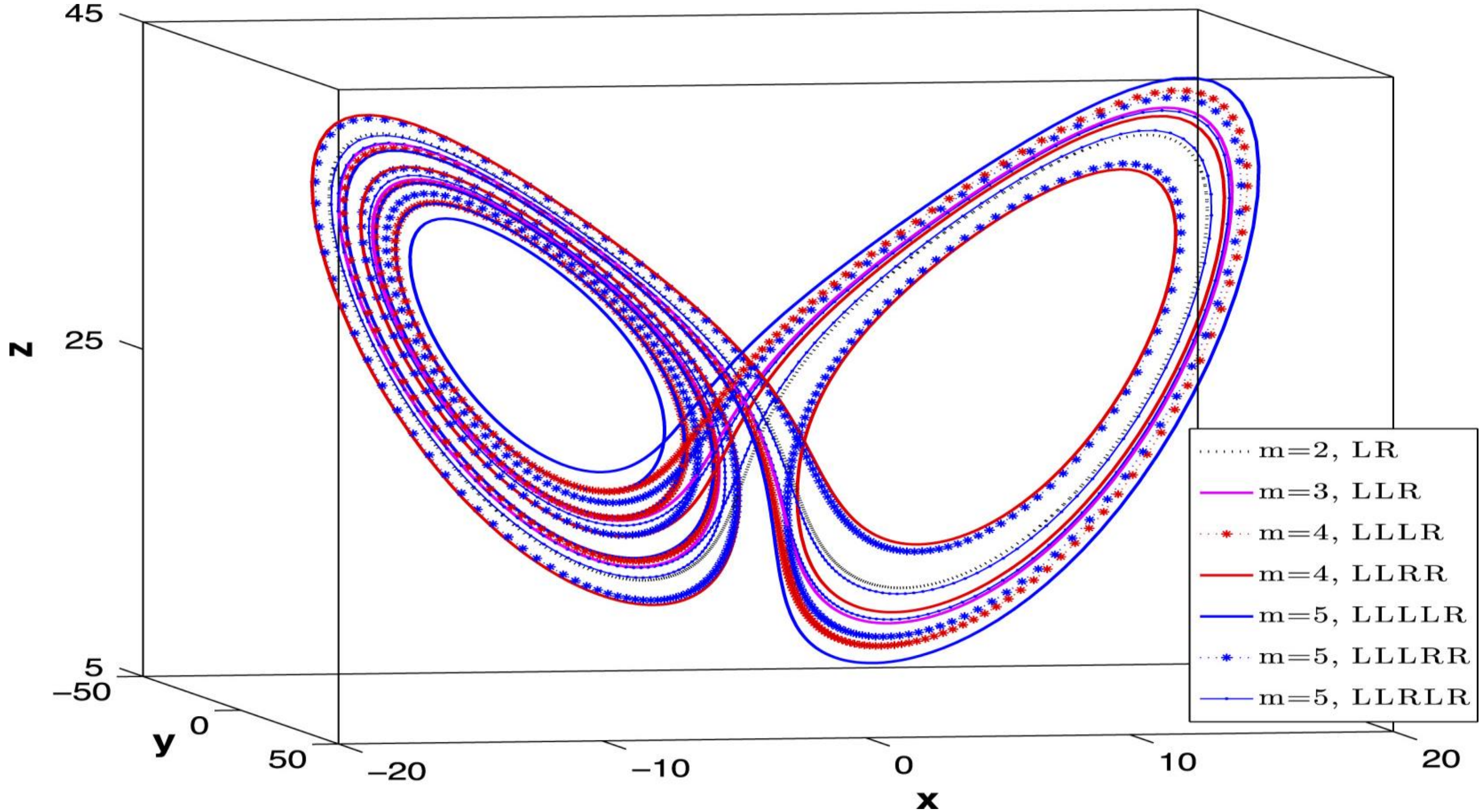
Regular Ergodic Average



$$\nu(\Phi) = \langle\Phi\rangle_0 = \int \nu\,(dx)\Phi(x) = \lim_{t\to\infty}\frac{1}{t}\int_0^t d\tau\,\Phi(S^\tau x)$$

Phase space average = Long time average

Unstable Periodic Orbits



- Attractor densely populated by unstable periodic orbits
- Average of observables: weighted average over UPOs
- Each UPO has its own weight

Weighting an UPO

- UPOs are classified according to their period

$$\nu(\Phi) = \lim_{t\to\infty} \frac{\sum_{\mathrm{U}^p, p\le t} w^{\mathrm{U}^p} \overline{\Phi^{\mathrm{U}^p}}}{\sum_{\mathrm{U}^p, p\le t} w^{\mathrm{U}^p}}$$

$$\overline{\Phi^{\mathrm{U}^p}}$$

average

- Longer Period, more unstable UPOs have lower weight

$$w^{\mathrm{U}^p} \propto \exp(-p h_{ks})$$

$$h_{KS} = \sum_{\lambda_i > 0} \lambda_i$$

- This is strictly true for hyperbolic systems…

Hopeless?

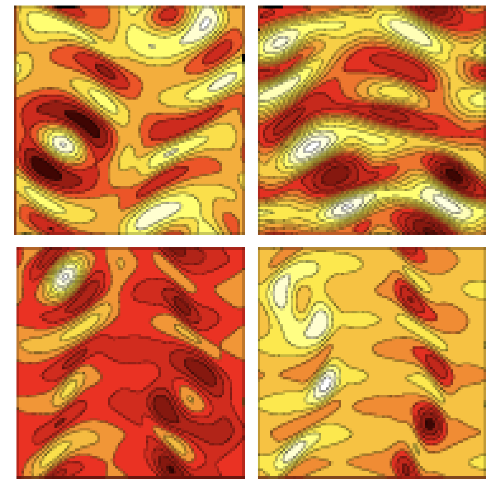
- Use of UPOs for reconstructing chaotci motions strongly advocated by Cvitanovic and co., see ChaosBook
- How can one reasonably hope to find even one UPO in high dimensional systems with turbulence etc?
- What about a “sufficient” number of UPOs?
- I first learnt about this approach in this room, in winter 2007
- I thought it was hopeless …

Focus on Fluids journals.cambridge.org/focus

Recurrent flows: the clockwork behind turbulence

Predrag Cvitanović†

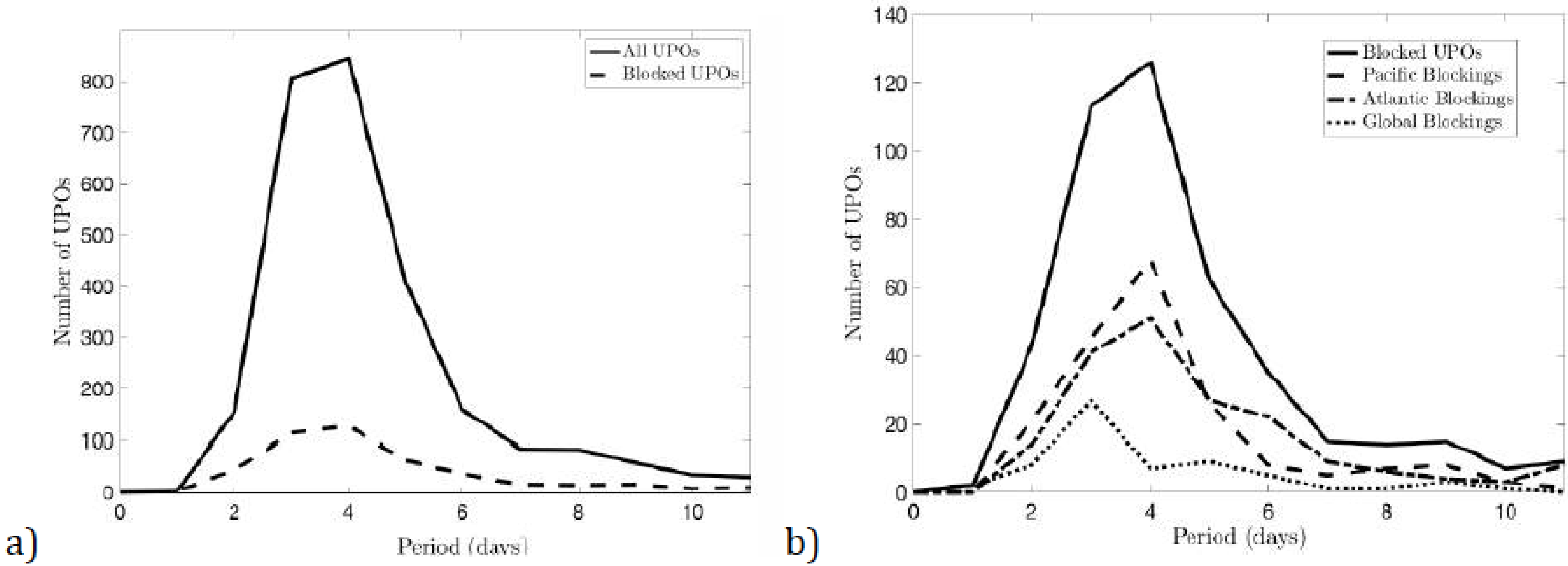
School of Physics, Georgia Institute of Technology, Atlanta, GA 30332, USA



J. Fluid Mechanics 2013, special issue

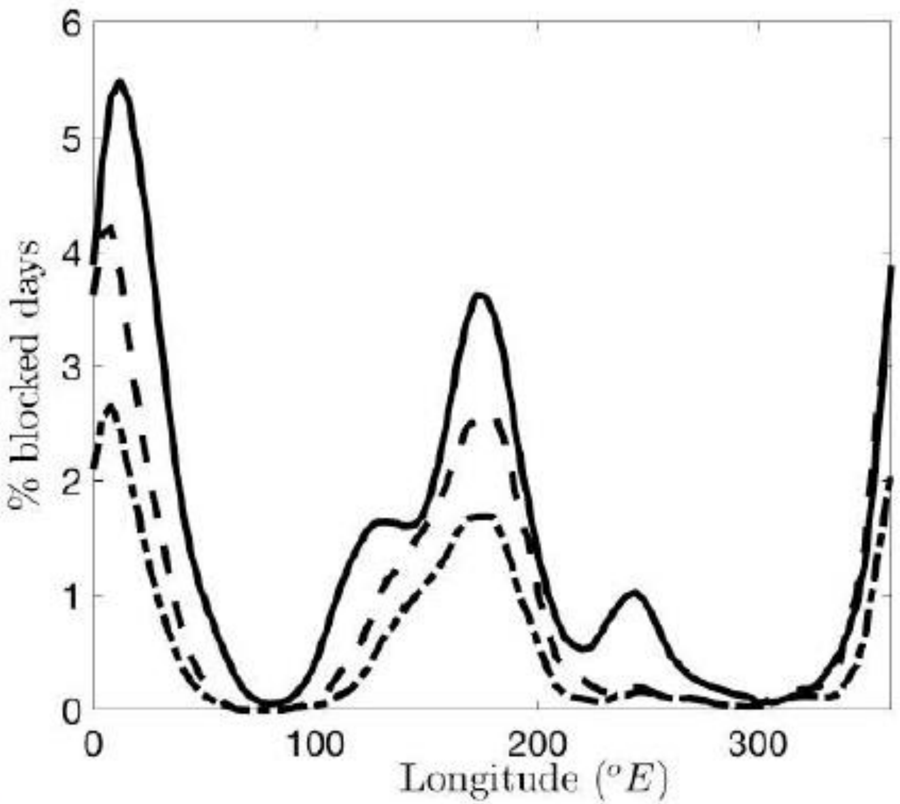
The understanding of chaotic dynamics in high-dimensional systems that has emerged in the last decade offers a promising dynamical framework to study turbulence. Here turbulence is viewed as a walk through a forest of exact solutions in the infinite-dimensional state space of the governing equations. Recently, Chandler & Kerswell (*J. Fluid Mech.*, vol. 722, 2013, pp. 554–595) carry out the most exhaustive study of this programme undertaken so far in fluid dynamics, a feat that requires every tool in the dynamicist's toolbox: numerical searches for recurrent flows, computation of their stability, their symmetry classification, and estimating from these solutions statistical averages over the turbulent flow. In the long run this research promises to develop a quantitative, predictive description of moderate-Reynolds-number turbulence, and to use this description to control flows and explain their statistics.

What we find – 2600 UPOs!

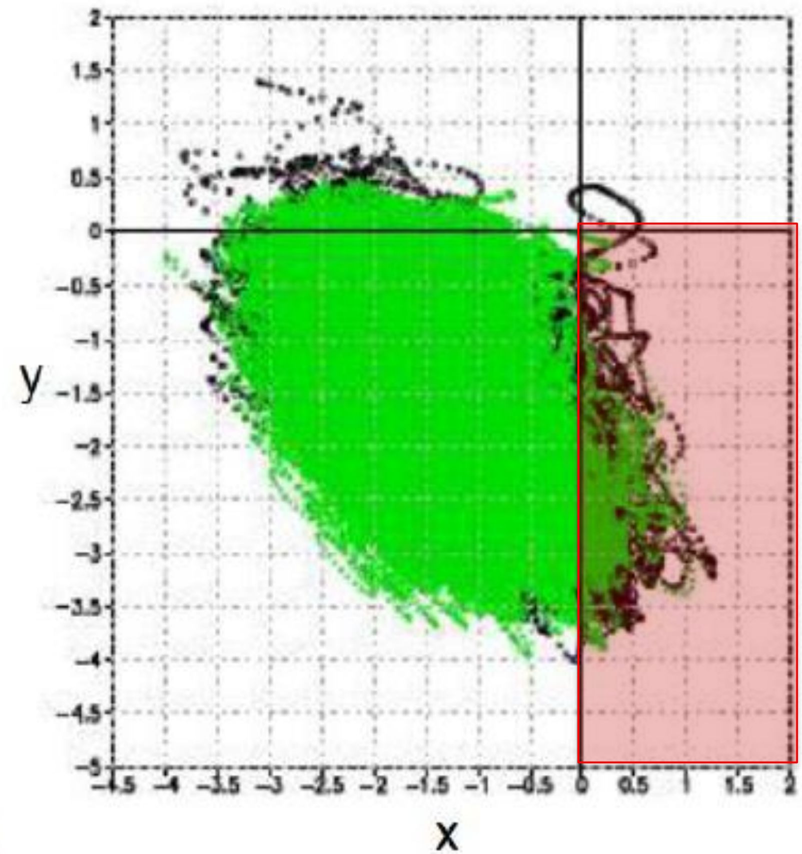


- It seems a lot, but is it enough?
- Number of UPOs grows exponentially with period
- Difficulty in detecting long period UPOs

Covering the Phase Space



a)

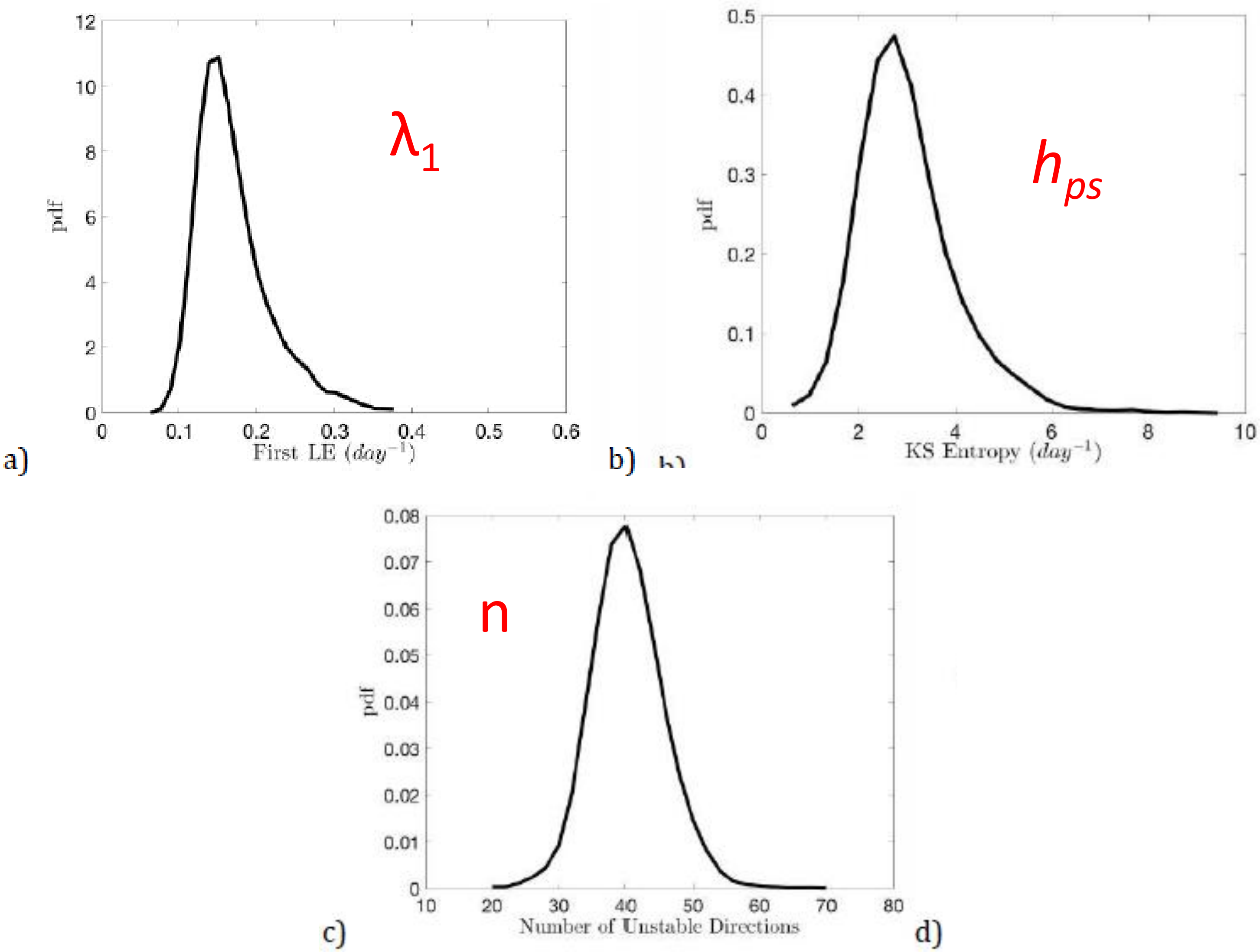


b)

Geographical pattern of blockings

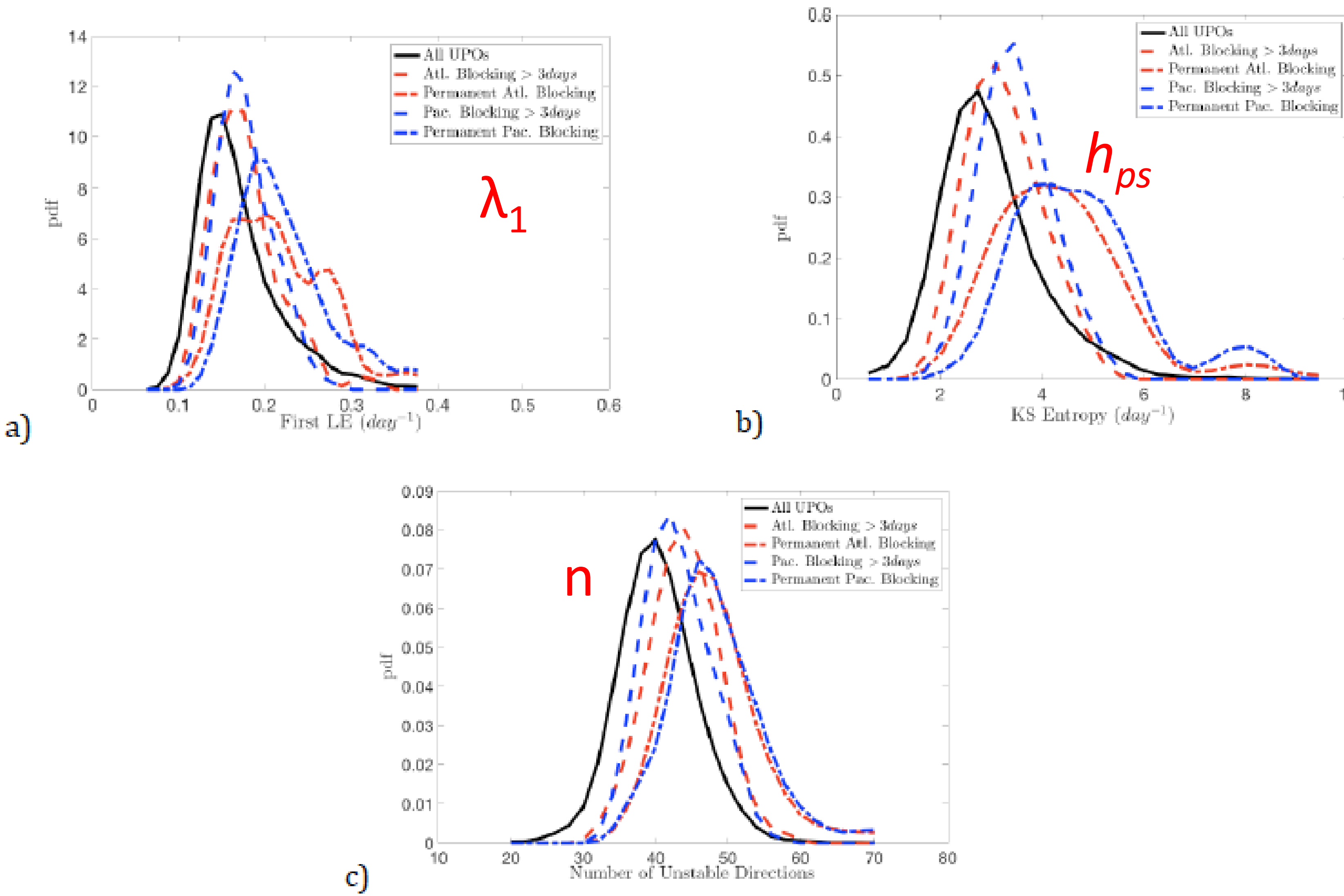
Y<0, X>0 is blocked

Heterogeneity of the Attractor



UPOs are building blocks of the dynamical landscape

UPOS and Blockings

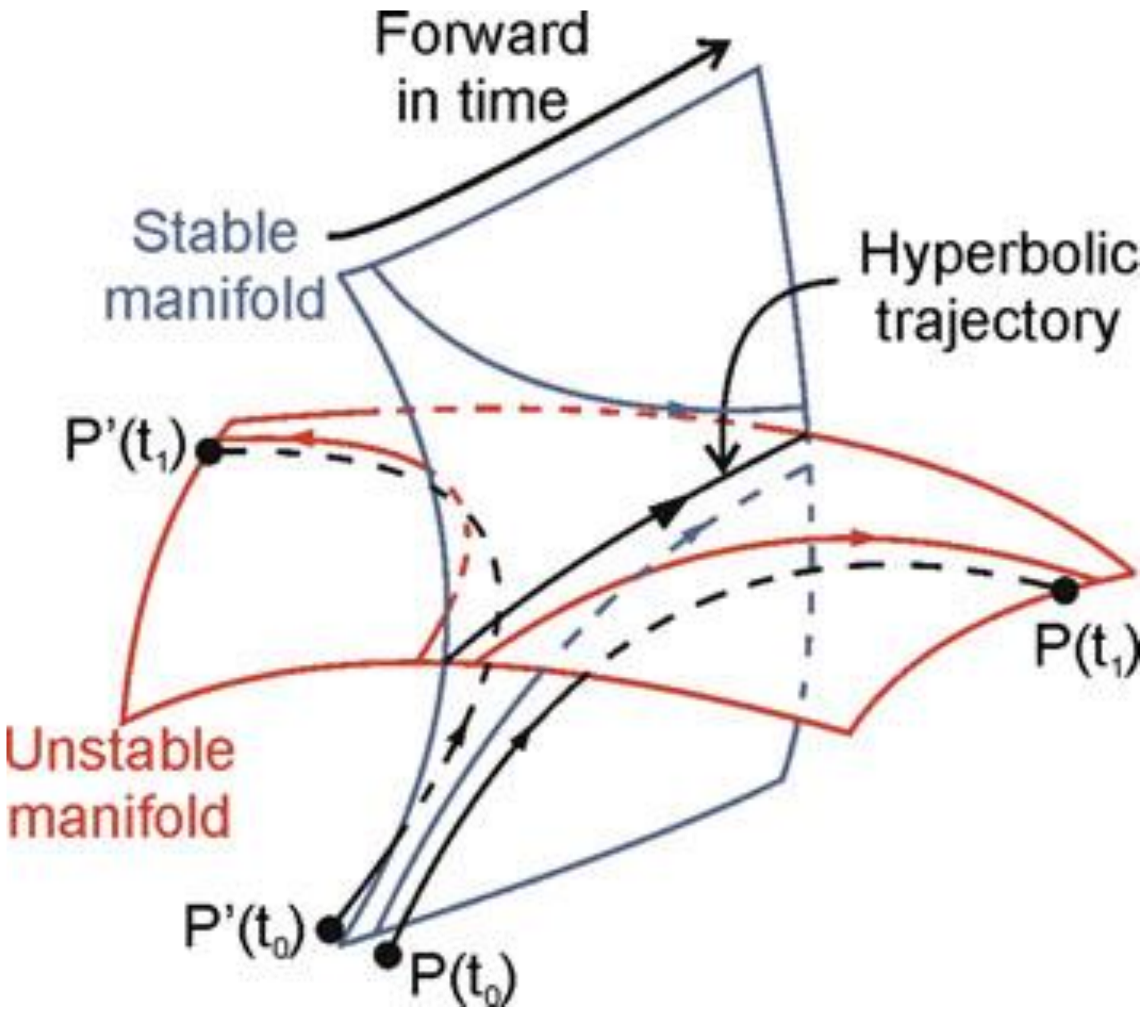


Blockings are associated to special, very unstable UPOs

Answer 3

- Blockings occur when the orbit is in the neighbourhood of special class of UPOs
- These UPOs are much more unstable of the typical one
- The longer the orbit is near then, the higher the instability it picks up
- Global blockings are associated to ultra unstable UPOs
- Markov chain models are coarse-grained versions of this fundamental dynamical processes

Hyperbolicity



- Gives you structural stability, shadowing, linear response, …

Bonus Result

- Variability in the number of unstable dimensions: no hyperbolicity
  - numerical models have hard time being near true trajectories for long time
- Might be an essential structural problem for geophysical fluid dynamical modelling
  - A good reason why blockings are so hard to model?
  - ‘’ why so much uncertainty in climate change impact on blockings?
- Relevant for algorithms relevant for data assimilation
  - Assimilation in the unstable manifold (Trevisan, Talagrand, Bouchet, Carrassi…)
  - One needs to add neutral and weakly stable directions (Grudzien et al. 2018)

Conclusions

- Combining the formalism of CLVs, finite-time Lyapunov Exponents, and, UPOS, we can address simultaneously and coherently several aspects of Blocking events
- Blockings are associated to more unstable conditions
- Indeed, Blockings are special in fundamental mathematical terms
- We see signature of onset and decay of the pattern
- Variability of the number of unstable directions is a key element of structural instability of GFD
- Obviously, more complex models need to be used. Anyone in the room interested?
- I tried to answer three questions, I have now new questions to be answered – happy!