Simulation of extreme European heat waves with rare event algorithms

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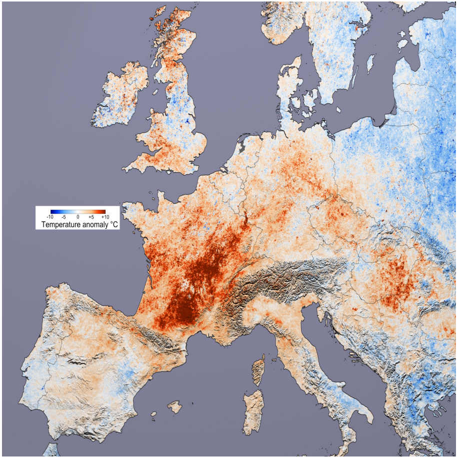
Outline

1. Extreme events sampling and motivation
2. Background
   - Importance sampling
   - Large deviation framework
   - Giardina-Kurchan-Tailleur-Lecomte algorithm
3. Application to heat waves with a GCM (Plasim)
   - Model setup
   - Results in large deviation limit
   - Results for seasonal time scales
4. Conclusions and perspectives

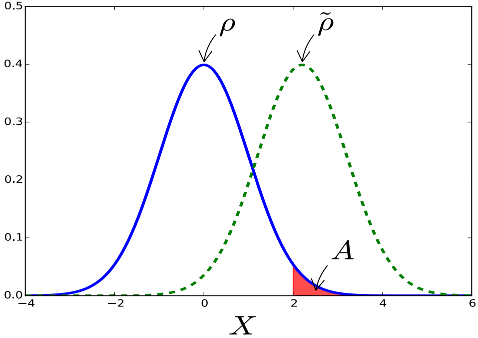
Extremely rare heat waves

- Europe 2003, Russia 2010: estimated return times of several decades or hundreds of years;
- To obtain reliable estimates we need very long, expensive simulations;
- Attribution to climate change uncertain (also) for lack of statistics.



European heat wave of summer 2003

Importance sampling for rare events

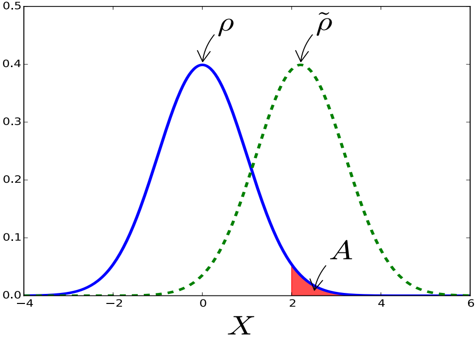


$$\gamma_A = \int dx \rho(x) 1_A(x) \;\rightarrow\; \hat{\gamma}_A = \frac{1}{N}\sum_{i=1}^{N} 1_A(X_i)$$

vs

$$\gamma_A = \int dx \tilde{\rho}(x) L(x) 1_A(x) \;\rightarrow\; \hat{\gamma}_A = \frac{1}{N}\sum_{i=1}^{N} L(\tilde{X}_i) 1_A(\tilde{X}_i)$$

Variance reduction



$$\hat{\gamma}_A = \frac{1}{N}\sum_{i=1}^{N} 1_A(X_i) \quad \rightarrow \quad \frac{1}{\gamma_A}\sqrt{\frac{Var(\hat{\gamma}_A)}{N}} \approx \frac{1}{\sqrt{\gamma_A N}}$$

vs

$$\hat{\gamma}_A = \frac{1}{N}\sum_{i=1}^{N} L(\tilde{X}_i) 1_A(\tilde{X}_i) \quad \rightarrow \quad \frac{1}{\gamma_A}\sqrt{\frac{Var(\hat{\gamma}_A)}{N}} = \sqrt{\frac{\mathbb{E}[L(x)1_A(x)] - \gamma_A^2}{\gamma_A{}^2 N}}$$

Rare event algorithms, how to choose them

- In a dynamical system we need to take into consideration the dynamics: importance sampling or other approaches must be applied at the level of the trajectories
- The choice of the algorithm must reflect the properties of the extreme event under consideration. Heat waves are a case of extreme event characterized by time persistence
- Statistically, we can approach this in (at least) two ways: clustering of extremes, or tails of distributions of time averages. Second approach leads us to large deviation theory

Donsker-Varadhan large deviation framework

- Large time asymptotics for time averaged observables

$$\rho\left(\frac{1}{T}\int_0^T f(x(t))\,dt = a\right) \underset{T\to\infty}{\asymp} C e^{-TI(a)}.$$

where $I(a)$ is called rate function (e.g. *Touchette 2009*).

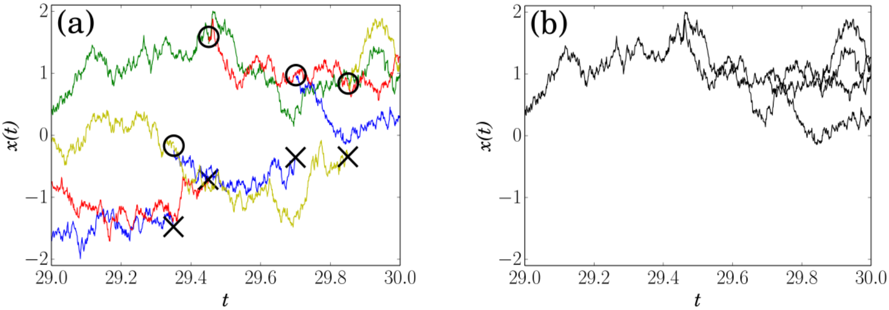
- Linked to the scaled cumulant generating function (SCGF)

$$\lambda(k) = \lim_{T\to+\infty} \frac{1}{T} log\left(\mathbb{E}[e^{k\int_0^T f(x(t))\,dt}]\right)$$

by the Gärtner-Ellis theorem $I(a) = \sup_a \{ka - \lambda(k)\}$.

- Giardina-Kurchan-Tailleur-Lecomte algorithm (GKTL): rare event algorithm designed to evaluate large deviation functions

Giardina-Kurchan-Tailleur-Lecomte (GKTL) algorithm



Ensemble of N trajectories $x_n(t)$, target observable $f(x(t))$. At times $t_j = j\tau$ each trajectory produces a number of copies of itself proportional to

$$w_n^{k^*}(t_j) = \frac{1}{R_j^{k^*}} e^{k^* \int_{t_{j-1}}^{t_j} f(x_n(t))dt}, \quad with \quad R_j^{k^*} = \frac{1}{N} \sum_{n=1}^{N} e^{k^* \int_{t_{j-1}}^{t_j} f(x_n(t))dt}$$

Effective ensemble reconstructed tracing back the history of the survivors.

GKTL algorithm and importance sampling

- The GKTL algorithm gives an unbiased estimator of the SCGF

$$\lambda(k^*) = \lim_{T_a \to +\infty} \frac{1}{T_a} \sum_{j=1}^{T_a/\tau} \log R_j^{k^*}$$

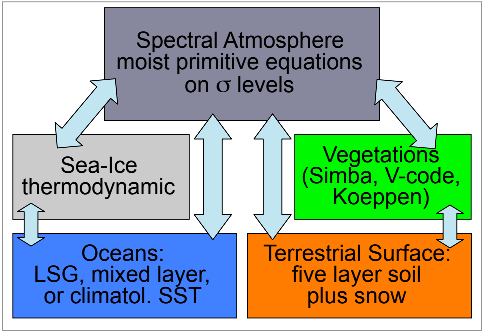
for arbitrary values of k^*, hence fluctuations a in large deviation limit.

- It generates importance sampling of trajectories

$$\mathbb{P}_{k^*}\left(\{x(t)\}_{0\leq t\leq T_a}\right) \underset{N\to\infty}{\sim} \frac{e^{k^* \int_0^{T_a} f(x(t))dt}}{\mathbb{E}\left[e^{k^* \int_0^{T_a} f(x(t))dt}\right]} \mathbb{P}_0\left(\{x(t)\}_{0\leq t\leq T_a}\right)$$

up to errors of order $1/\sqrt{N}$ on expectation values (*Del Moral 2004*).

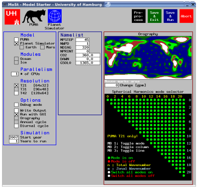
Plasim, advertisement slide

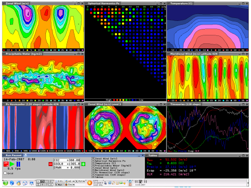


Key features

- portable
- fast
- open source
- parallel
- modular
- easy to use
- documented
- compatible

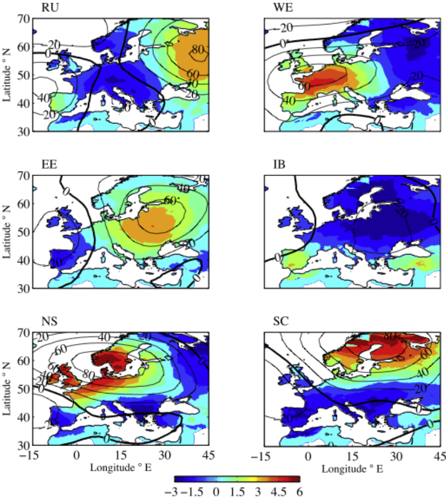
Model Starter
and
Graphic User Interface





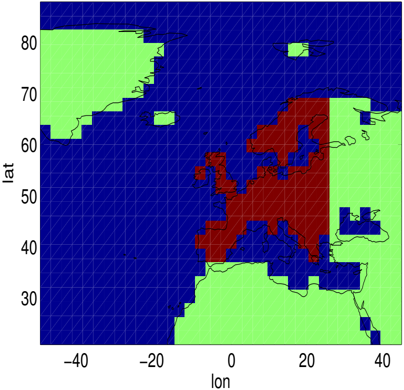
European heat waves classification

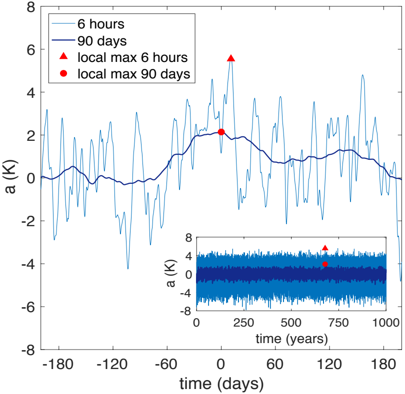
- Typically mid-latitude blocking, persistent anticyclonic anomaly leading to surface warming
- Spatial clustering and feedbacks at different scales (hydrological preconditioning, mesoscale effects, etc.)
- Complex picture: we simplify the question at first



Stefanon et. al 2012

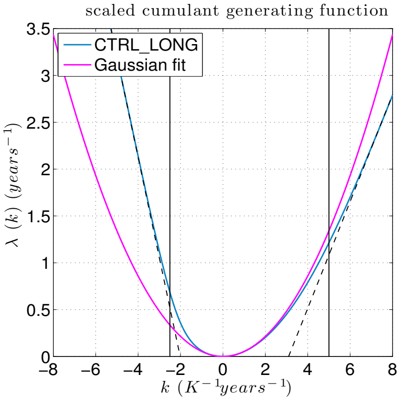
Model setup and experimental settings

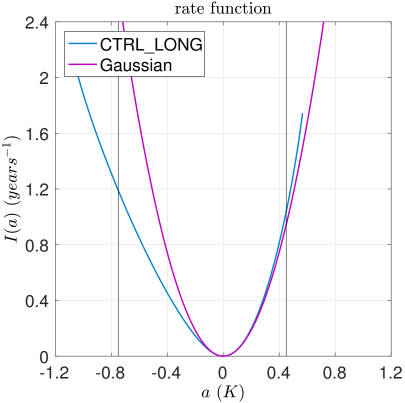




- T42 resolution, 10 vertical layers, $\sim 10^5$-10^6 degrees of freedom. Perpetual summer setup: no daily or seasonal cycle
- Simple indicator of heat waves: fluctuations of time averages of western European surface temperature over averaging time T

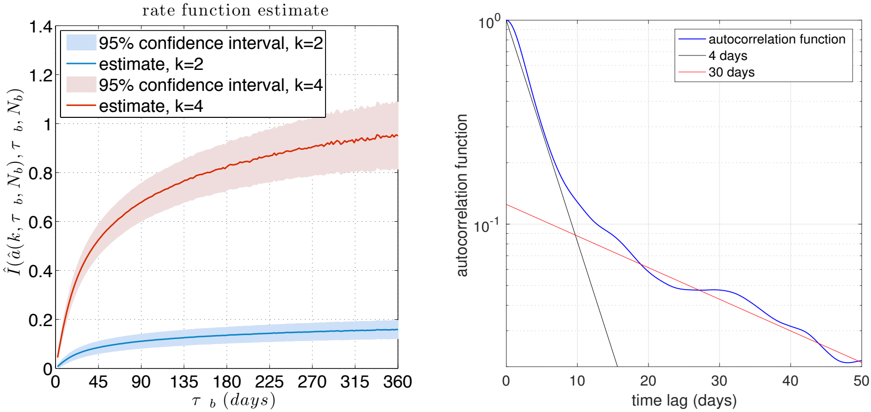
Direct estimate of large deviation functions





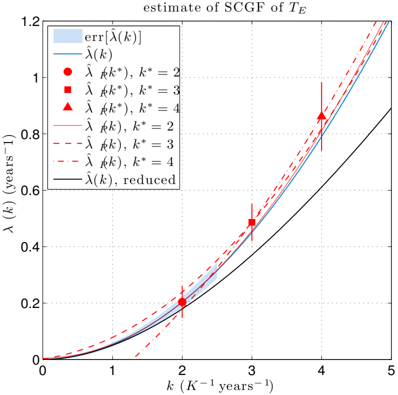
- Direct estimate from 1000 years control run as benchmark
- Barely outside of Gaussian regime for positive anomalies
- Problem: very long time to be in the large deviation limit, $T > 1$ year

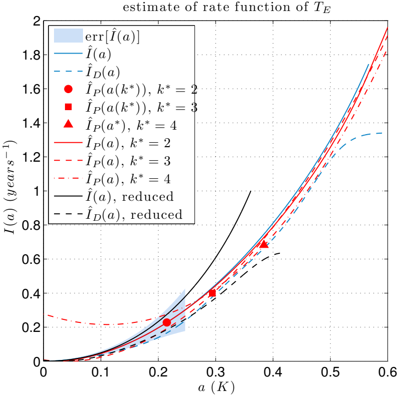
Problem: limit not reached for weekly/seasonal averages



- Slow convergence probably related to slow decay of the tail of the autocorrelation function
- Monthly time scale introduced by land surface processes (?)

Test of reconstruction of SCGF

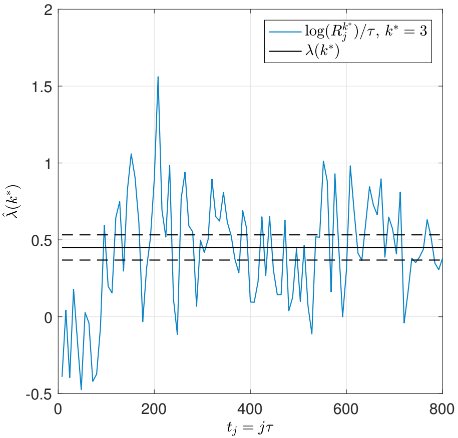




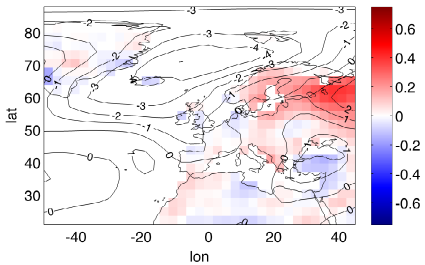
- Direct estimate from 1000 years control run as benchmark;
- GKTL gives correct results for smaller cost ($\sim$ 284 years);

What is the GKTL algorithm selecting?

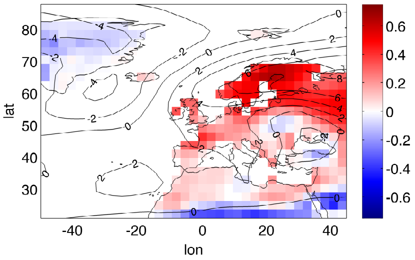
$$\lambda(k^*) = \lim_{T_a \to +\infty} \frac{1}{T_a} \sum_{j=1}^{T_a/\tau} \log R_j^{k^*}$$



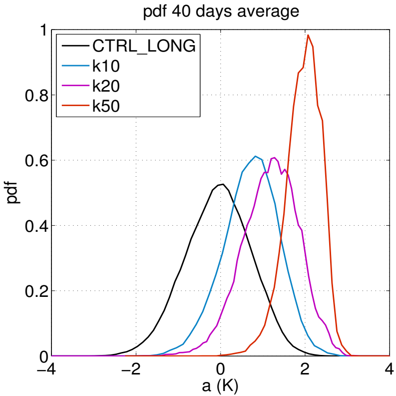
GK k*=3 - anomalies T_s and 500 hPa gph - days 1-80

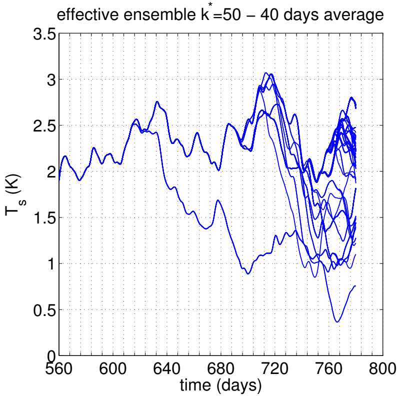


GK k*=3 - anomalies T_s and 500 hPa gph - days 721-800



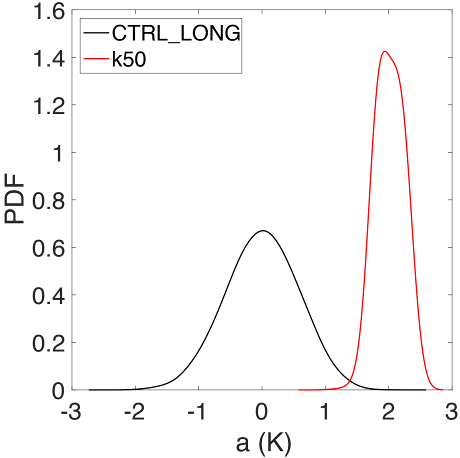
Statistics of shorter time averages





- Needs large values of k, because pdf widens
- For values of k too large, issue with multiplicity of trajectories

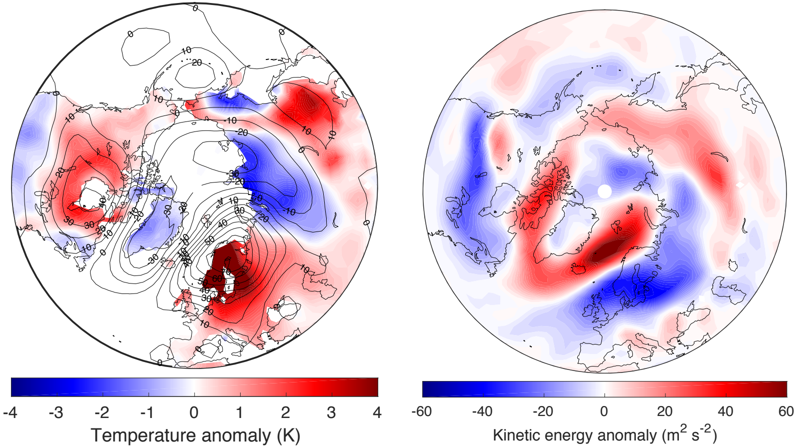
Importance sampling for seasonal time scales



Ragone, Wouters, Bouchet (2018)

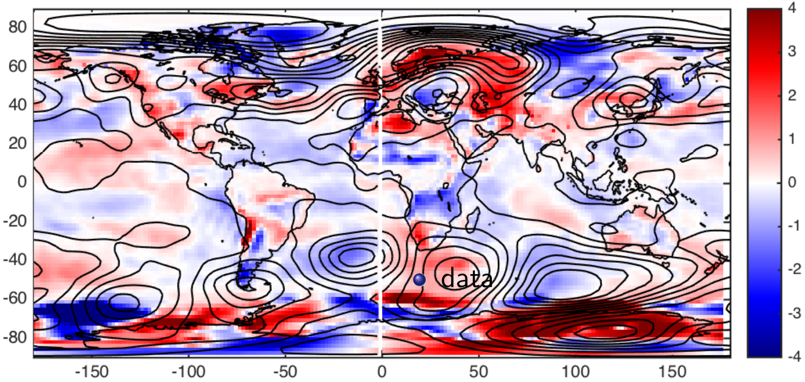
- $T = 90$ days, finite time average not in the large deviation limit
- Compromise between averaging time, length of the trajectories (short) and number of trajectories (large)
- Finding the right balance importance sampling seems to work very well

Composite analysis for $a > 2\ K$, teleconnection patterns



Ragone, Wouters, Bouchet (2018)

- Precise estimate of conditional expectation values (composites);
- "Teleconnection" patterns with northward displacement of jet stream.


80
60
40
20
0
-20
-40
-60
-80
-150
-100
-50
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data
4
3
2
1
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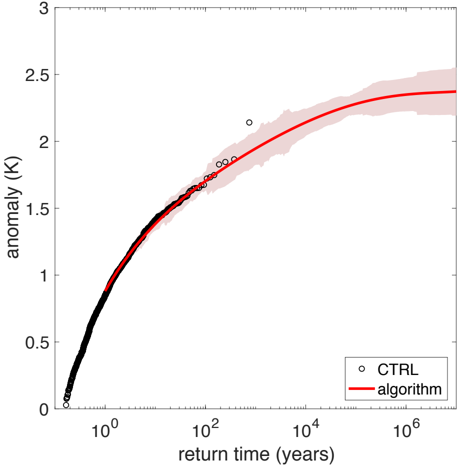
-1
-2
-3
-4

Return times of 90 days average temperature anomalies



- Return time of 90 days average European temperature;
- Same computational cost (1000 years) for direct and GKTL;
- Precise estimates of return times of millions of years with costs orders of magnitude smaller.

Ragone, Wouters, Bouchet (2018)

Conclusions and work in progress

Conclusions

- The GKTL algorithm seems to be effective to oversample heat waves
- This could open a new range of dynamical studies with GCMs

Work in progress

- Extend to systems with time dependent forcings (seasonal cycle!)
- Dynamical studies: relation with blocking events? Which physical feedbacks?

Next

- Combination with other methods, like analogues?
- This is ok for time persisting phenomena, but what about instantaneous extremes?

References

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Heat waves

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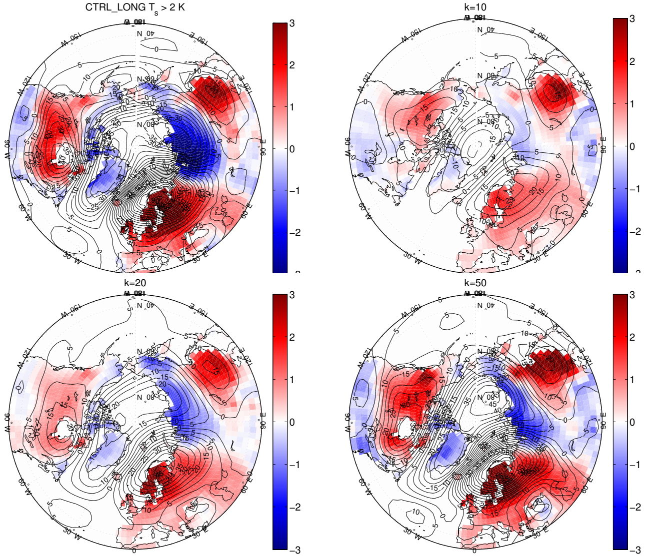
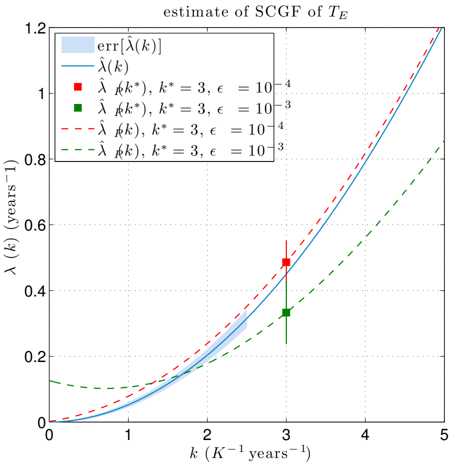
- Stefanon, D'Andrea, Drobinski (2012), Heatwave classification over Europe and the Mediterranean region. Environ Res Lett 7:014023

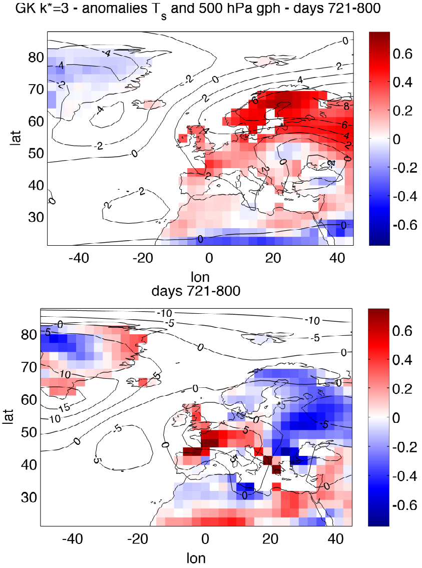
Composite average for $T_s > 2\ K$ and GKTL climate



Sensitivity: increased perturbation intensity

We set $\epsilon = 10^{-3}$ instead of $\epsilon = 10^{-4}$





Sensitivity: changing the cloning time? (to do)