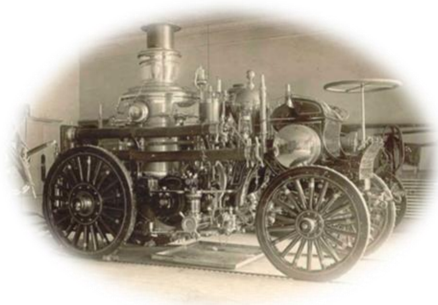
Laboratoire Interdisciplinaire des Energies de Demain

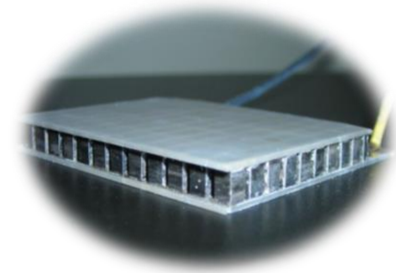
Paris 7 Interdisciplinary Energy Research Institute

LIED

PIERI



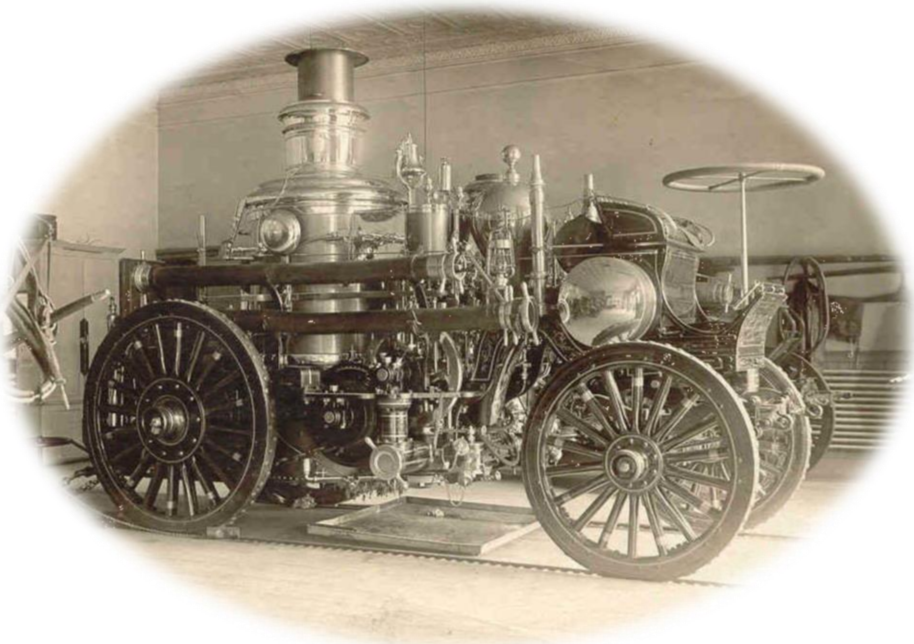
$$\left(P,T\right)\Leftrightarrow(\mu,T)$$



From thermoelectricity to finite time thermodynamics: a closed loop approach to thermodynamics

C. Goupil, H. Ouerdane, E. Herbert, Y. D’Angelo and Ph. Lecoeur

Fluid and engine



Un corps solide, une barre métallique, par exemple, alternativement chauffée et refroidie, augmente et diminue de longueur, et peut mouvoir des corps fixés à ses extrémités. Un liquide alternativement chauffé et refroidi augmente et diminue de volume et peut vaincre des obstacles plus ou moins grands opposés à sa dilatation.

RÉFLEXIONS

SUR LA

PUISSANCE MOTRICE

DU FEU

ET

SUR LES MACHINES

PROPRES A DÉVELOPPER CETTE PUISSANCE.

PAR S. CARNOT,

ANCIEN ÉLÈVE DE L'ÉCOLE POLYTECHNIQUE.

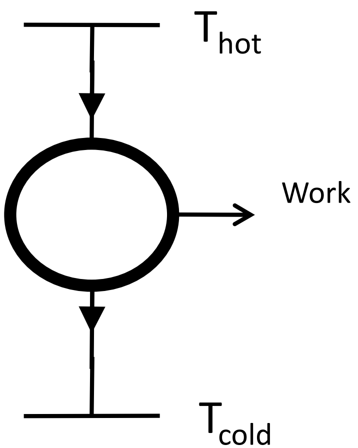
A PARIS,

CHEZ BACHELIER, LIBRAIRE,

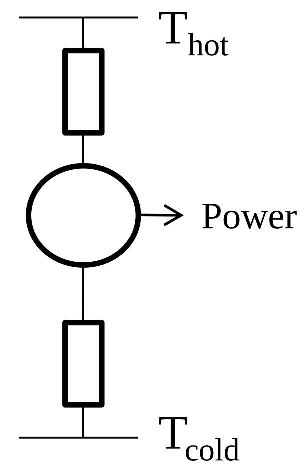
QUAI DES AUGUSTINS, N°. 55.

1824.

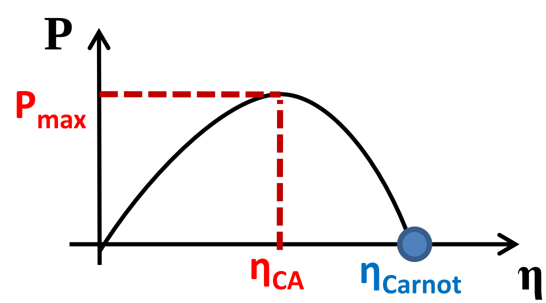
Endoreversible: OYCNCA



Finite Time
Thermodynamics
FTT



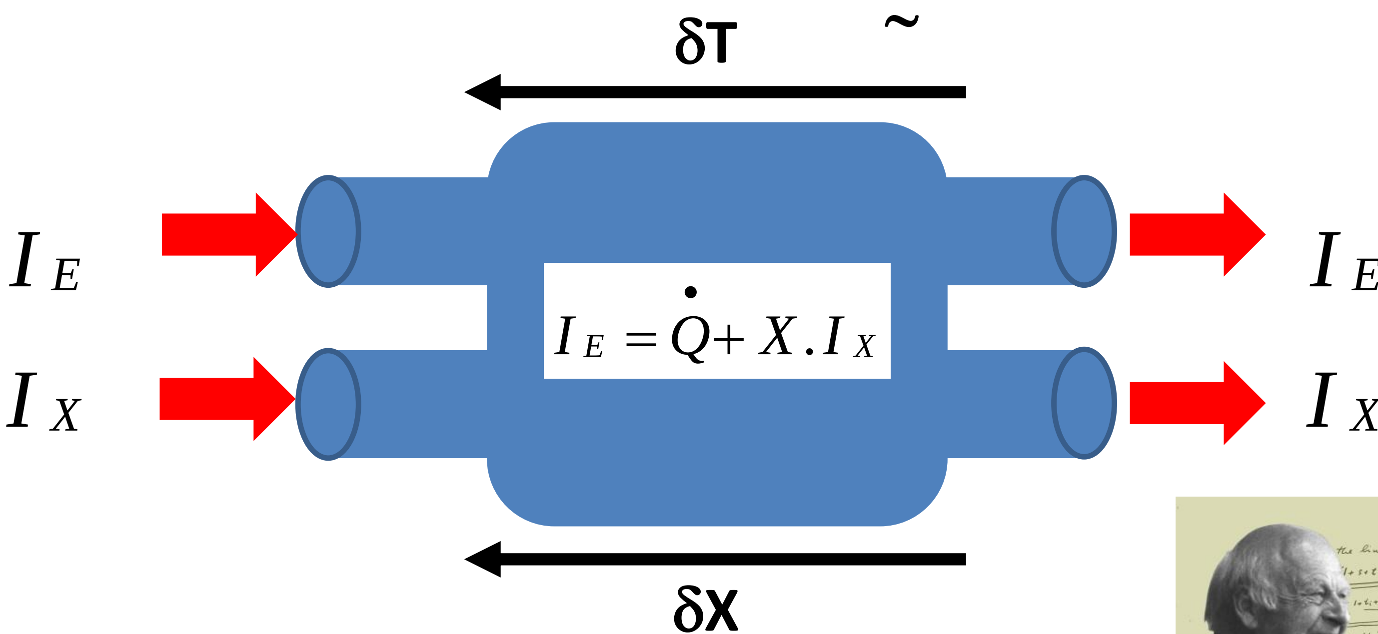
$$\eta_C = \frac{W}{Q_{in}} = 1 - \frac{T_{cold}}{T_{hot}}$$

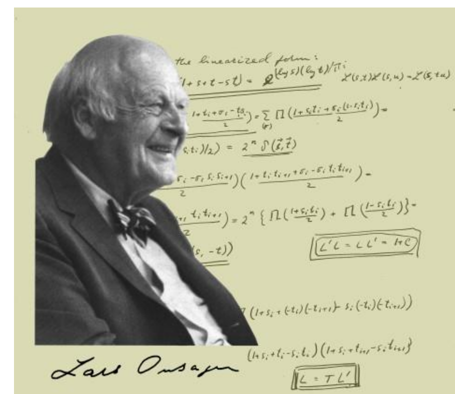


$$\eta_{CA} = \frac{\dot{W}}{\dot{Q}_{in}} = 1 - \sqrt{\frac{T_{cold}}{T_{hot}}}$$

- **H. T. Odum & R. C. Pinkerton,** TIME'S SPEED REGULATOR: THE OPTIMUM EFFICIENCY FOR MAXIMUM POWER OUTPUT IN PHYSICAL AND BIOLOGICAL SYSTEMS, American Scientist, Vol. 43, No. 2 (APRIL **1955**), pp. 331-343
- **J. Yvon,** The saclay Reactor: Two Years of Experience in the Use of a Compresed gas as a Heat Transfer Agent, Proceedings of the International Conference on the Peaceful Uses of Atomic Energy (**1955**)
- **P. Chambadal** L*es centrales nucléaires*. Armand Colin, Paris, France, 4 1-58, (**1957**)
- **I.I. Novikov,** Efficiency of an Atomic Power Generation Installation, Atomic Energy 3 (**1957**)
- **F.L. Curzon & B. Ahlborn**, Efficiency of a Carnot Engine at Maximum Power Output, Am. J. Phys. 43 (**1975**)

Non endoreversible: Onsager (1931)




Lars Onsager 27 November , 1903 -- October 5, 1976

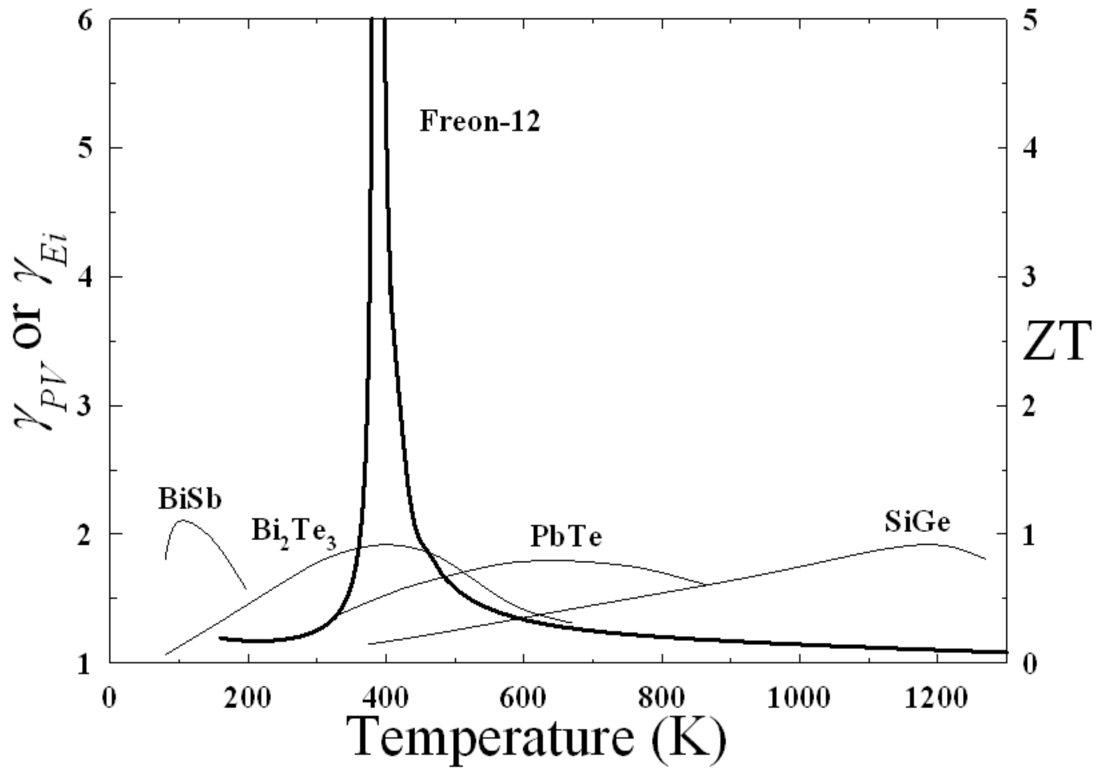
$$\begin{pmatrix} I_X \\ I_E \end{pmatrix} = \begin{pmatrix} L_{NN} & L_{NE} \\ L_{EN} & L_{EE} \end{pmatrix} \begin{pmatrix} \delta\left(-\frac{X}{T}\right) \\ \delta\left(\frac{1}{T}\right) \end{pmatrix}$$

$$\begin{bmatrix} dN \\ dS \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial N}{\partial \mu}\right)_T & \left(\frac{\partial N}{\partial T}\right)_\mu \\ \left(\frac{\partial S}{\partial \mu}\right)_T & \left(\frac{\partial S}{\partial T}\right)_\mu \end{bmatrix} \begin{bmatrix} d\mu \\ dT \end{bmatrix}$$

- Two currents (energy & matter)
- Two conductivities. (viscosities)
- One coupling coefficient

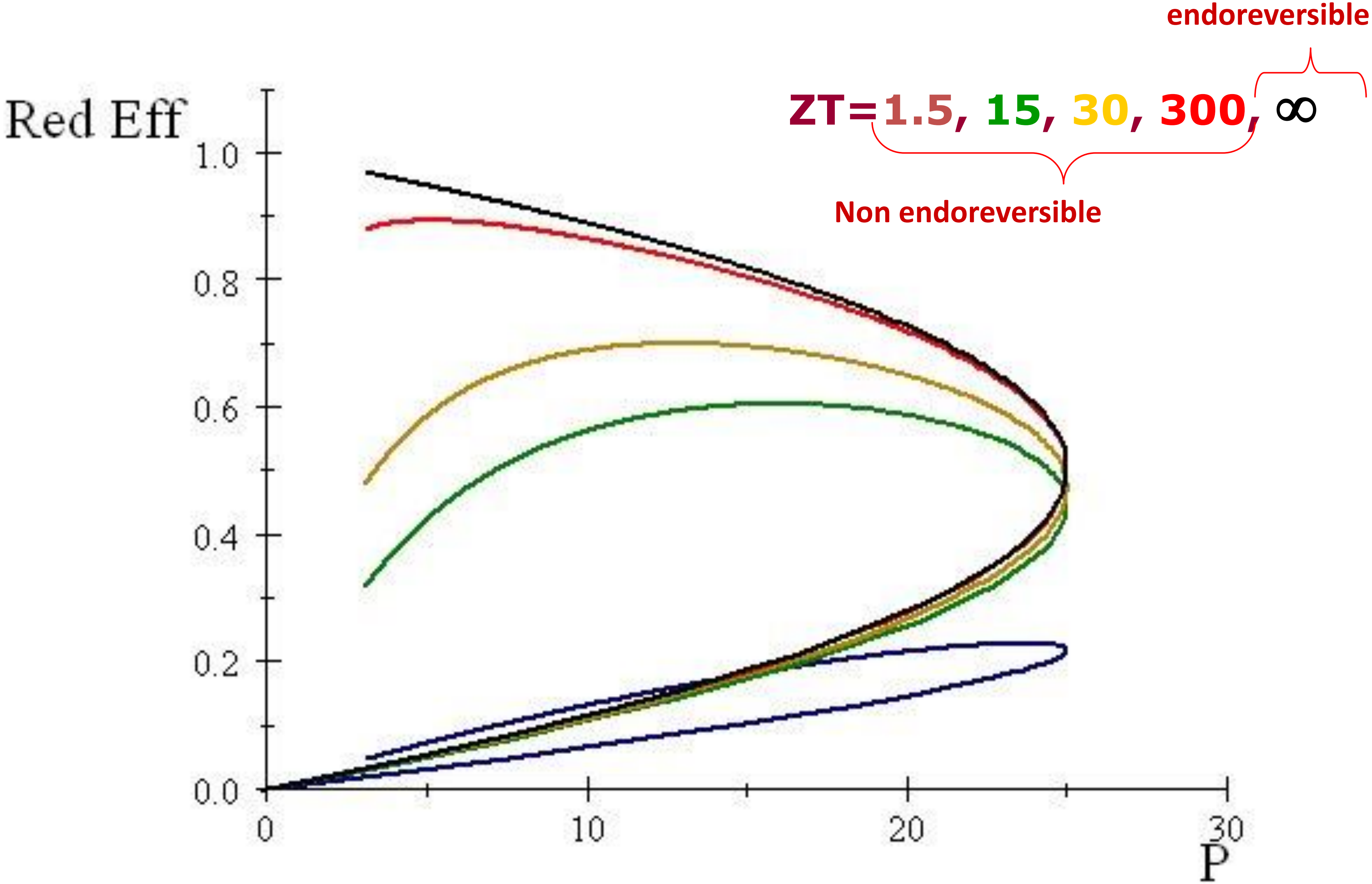
Thermodynamic fluid

$$\frac{\kappa_E}{\kappa_0} = 1 + \frac{\alpha^2 \sigma_T}{\kappa_0} T \qquad \frac{\sigma_Q}{\sigma_T} = 1 + \frac{\alpha^2 \sigma_T}{\kappa_0} T \qquad \frac{C_\mu}{C_N} = 1 + \frac{{S_N}^2 \chi_T}{C_N} T$$

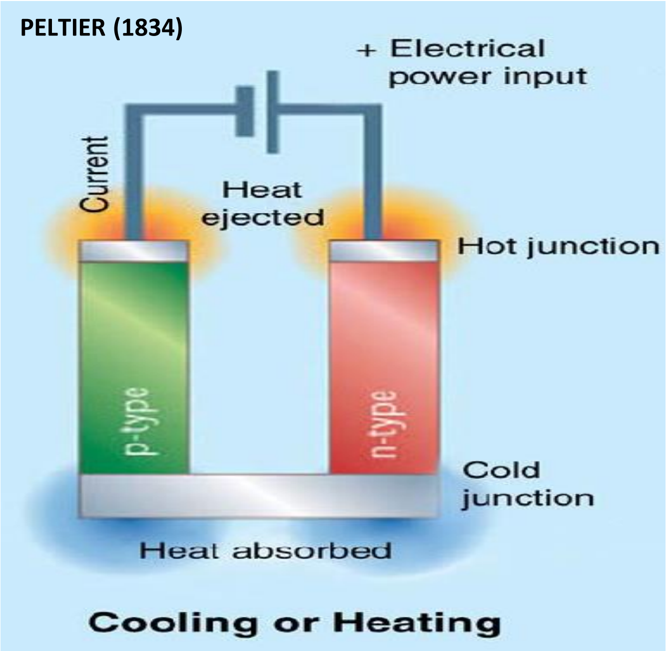


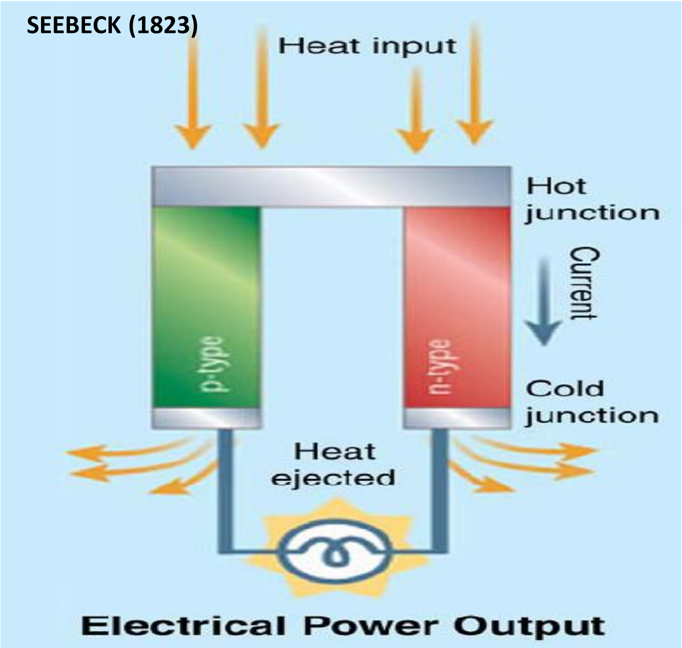
Vining 1997

Efficiency or Power



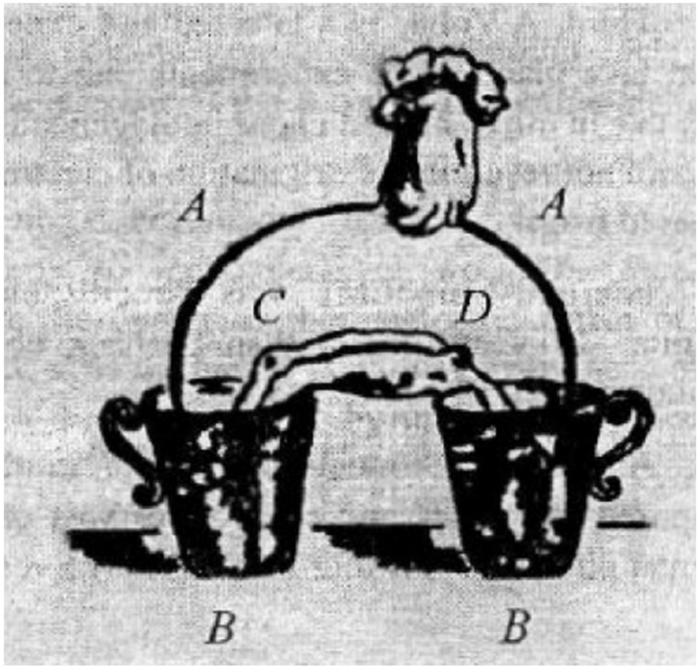
Thermoelectricity





"Coupling Ohm's law and Fourier's law"

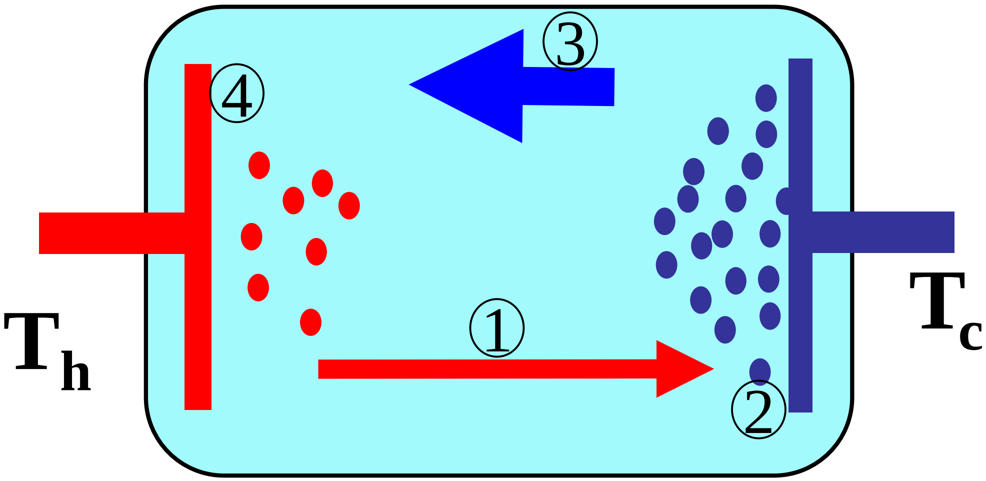
The Volta Story



Alessandro Volta
18 February 1745 – 5 March 1827)

1794 --- 1795: Letter to professor Antonio Maria Vassalli (accademia delle scienze di torino) "... I immersed for a mere 30 seconds the end of such arc into boiling water, removed it and allowing no time for it to cool down, resumed the experiment with two glasses of cold water. It was then that the frog in the water started contracting, and it happened even two, three, four times on repeating the experiment till one end of the iron previously immersed into hot water did not cool down".

Thermal & Electrical coupling.



- 1: Adiabatic.
- 2: Isothermal.
- 3: Adiabatic.
- 4: Isothermal.

- Reversible adiabatic transport of the carriers. (isentropic)
- The convective part contribute to the entropy transport…
- … but not the conductive part .(leak)

- => Reduce the conductive part and increase the convective part .

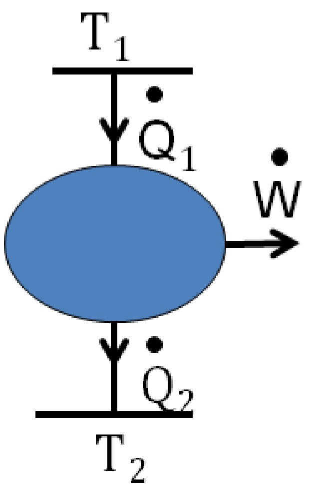
- The electical conductivity shoud be large.

•=> Yes but by increasing mobility only.

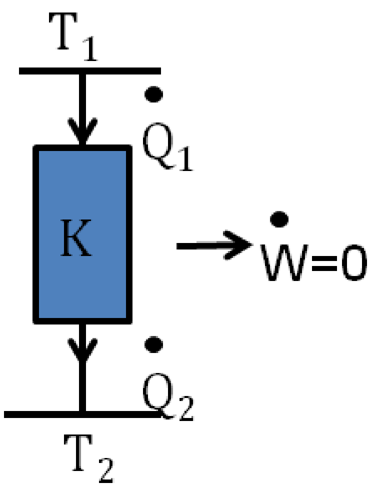
What is a good TE system?

$$P = \dot{W} = \dot{Q}_1 - \dot{Q}_2$$

Fully reversible



Fully irreversible

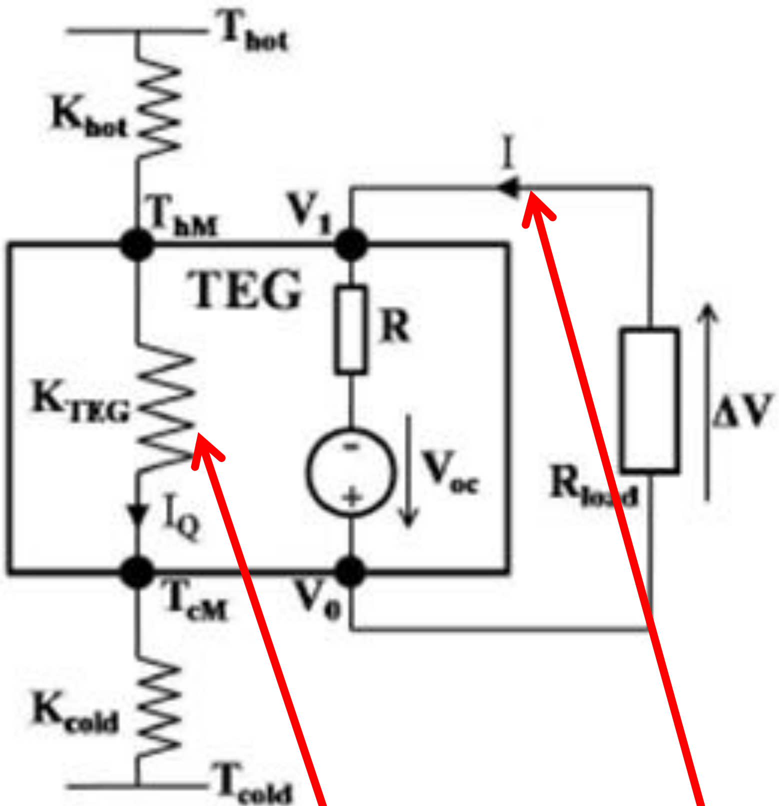


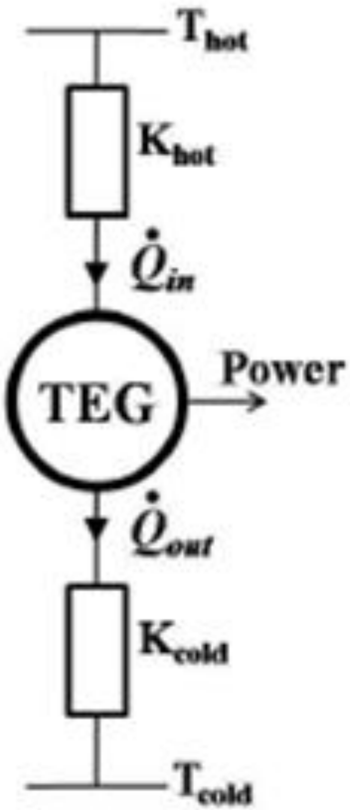
$$\dot{S}_1 = \dot{S}_2 = \frac{\dot{Q}_1}{T_1} = \frac{\dot{Q}_2}{T_2} \Rightarrow \dot{W} = \dot{Q}_1\left(1 - \frac{T_2}{T_1}\right)$$

$$\dot{Q}_1 = \dot{Q}_2 \Rightarrow \dot{W} = \dot{Q}_1 - \dot{Q}_2 = 0$$

The perfect engine : when entropy becomes conservative.

Nodal model: presentation

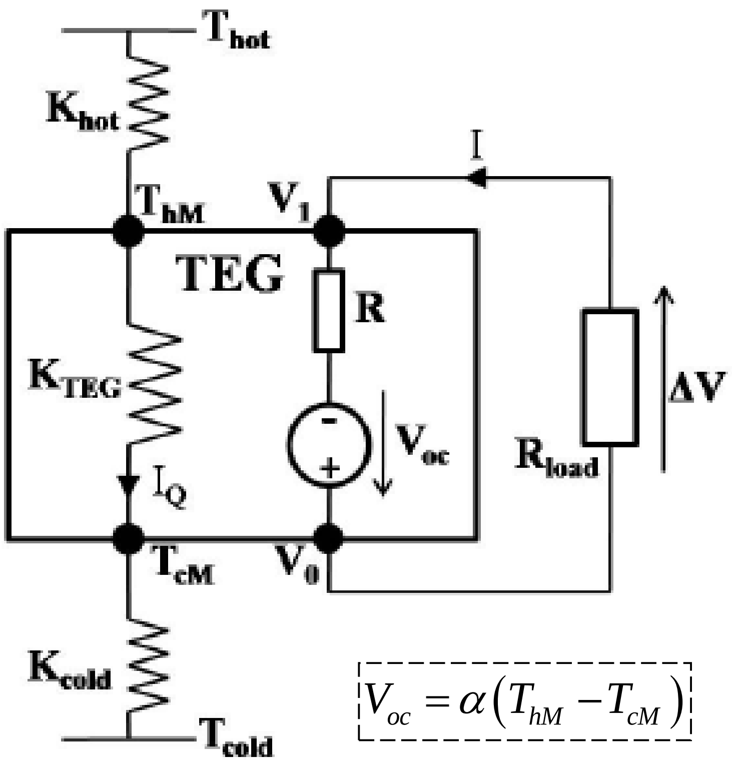




$$K_{TEG}(I) = K_0\left[1 + \frac{I}{I_{CC}} ZT\right]$$

$$\frac{\kappa_E}{\kappa_0} = 1 + \frac{\alpha^2 \sigma_T}{\kappa_0} T$$

Nodal model: Onsager description



$$V_{oc} = \alpha\left(T_{hM} - T_{cM}\right)$$

Force-Flux : $$\begin{bmatrix} I \\ I_Q \end{bmatrix} = \begin{bmatrix} \frac{1}{R} & \frac{\alpha}{R} \\ \frac{\alpha.T}{R} & \frac{\alpha^2.T}{R} + K_0 \end{bmatrix} \cdot \begin{bmatrix} \Delta V \\ T_{hM} - T_{cM} \end{bmatrix}$$

$$I_Q = \underset{advection}{\alpha T.I} + \underbrace{K_0.(T_{hM} - T_{cM})}_{conduction}$$

$$I_Q = \underbrace{\left(\frac{\alpha^2 T}{\underbrace{R + R_{load}}_{K_{adv}}} + K_0\right)}_{K_{TEG}} (T_{hM} - T_{cM})$$

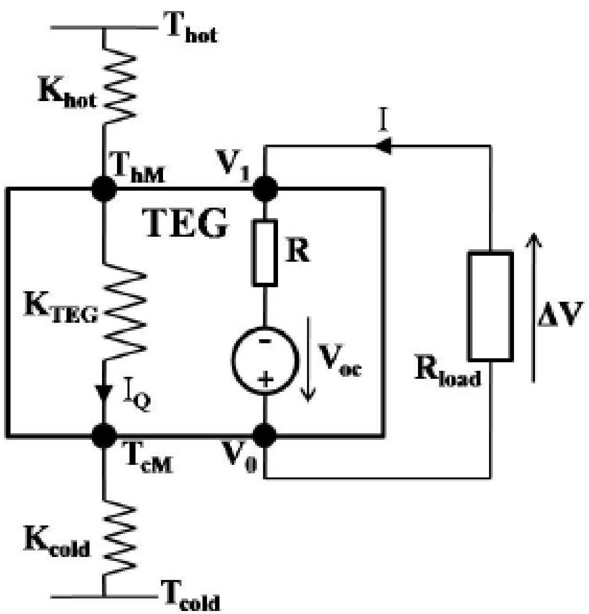
Effective thermal conductance !

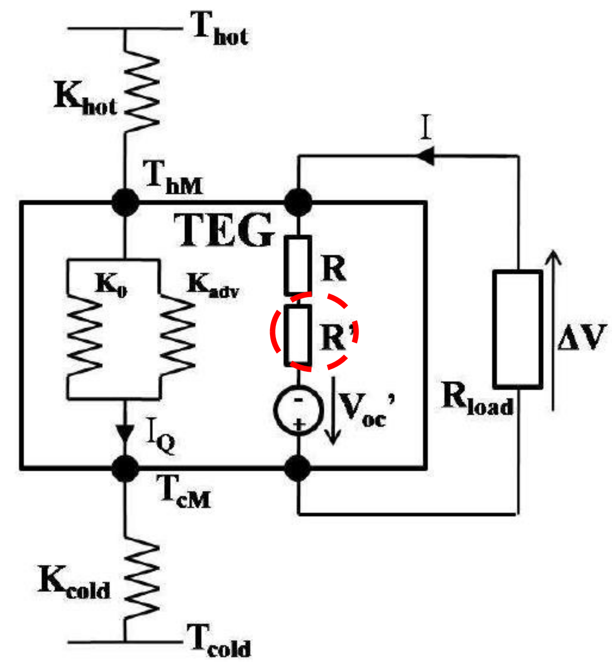
advection

Conduction

Y. Apertet, H. Ouerdane, O. Glavatskaya, C. Goupil et Ph. Lecoeur, EPL 97 (2012)

Nodal model: resulting picture





$$\boxed{V_{oc} = \alpha\left(T_{hM} - T_{cM}\right)}$$

Is a function of R_{load}!

Thevenin model:

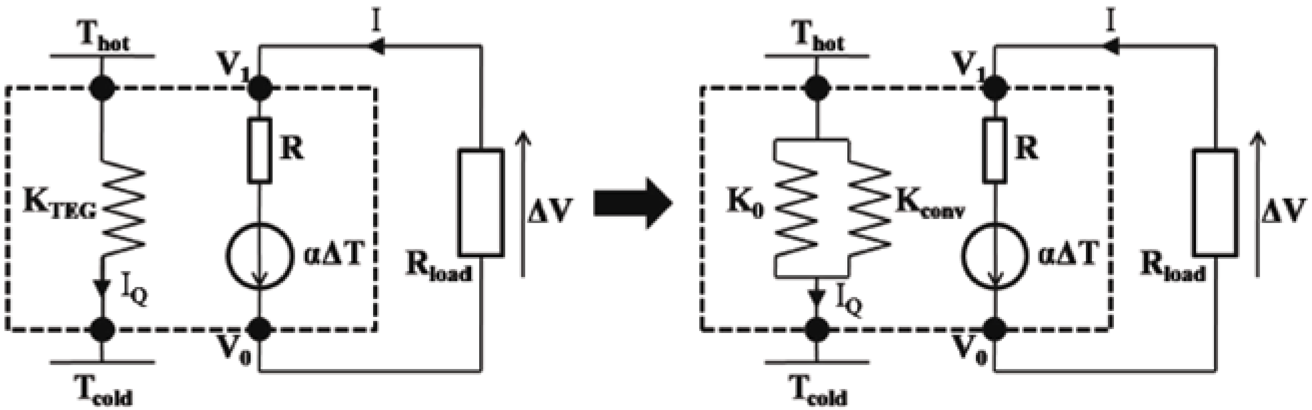
$$V_{oc} = \underbrace{\alpha\Delta T \frac{K_{contact}}{K_{contact} + K_0}}_{V_{OC}\,'} - I \underbrace{\frac{\alpha^2 T}{K_{contact} + K_0}}_{R'}$$

- The feedback comes from K_c
- No R_{TE} if the coupling is perfect, $K_c \Rightarrow \infty$

$$K_{TEG}(I) = K_0\left[1 + \frac{I}{I_{CC}} ZT\right]$$

Y. Apertet, et al. EPL 97 (2012)

Thermoelectric Prandtl number



$$K_{TEG} = K_0 + \frac{\alpha^2 T}{(R + R_{load})}$$

$$\sigma_P = \frac{\text{Kinematic viscosity}}{\text{Thermal diffusivity}} \equiv \frac{\text{momentum diffusion}}{\text{heat diffusion}}$$

Influence of the fluid velocity profile on the temperatures profile.

$$\sigma_{P_{TE}} = \frac{\mathrm{K}_{\mathrm{conv}}\Delta T}{\mathrm{K}_0 \Delta T} = \frac{\alpha^2 T}{\mathrm{K}_0\left(R + R_{load}\right)} = \frac{ZT}{1 + R_{load}/R}$$

Then — The thermoelectric Prandtl number is controled by the electrical load !
(Electrical feedback on the dissipative properties, including thermal)

So — The Prandtl number is maximal in short_circuit (or more…).

But — The maximal power is obtained for half this value : $\sigma_{P_{TE\,(R=R_{load})}} = \frac{ZT}{2}$

Impedance matching

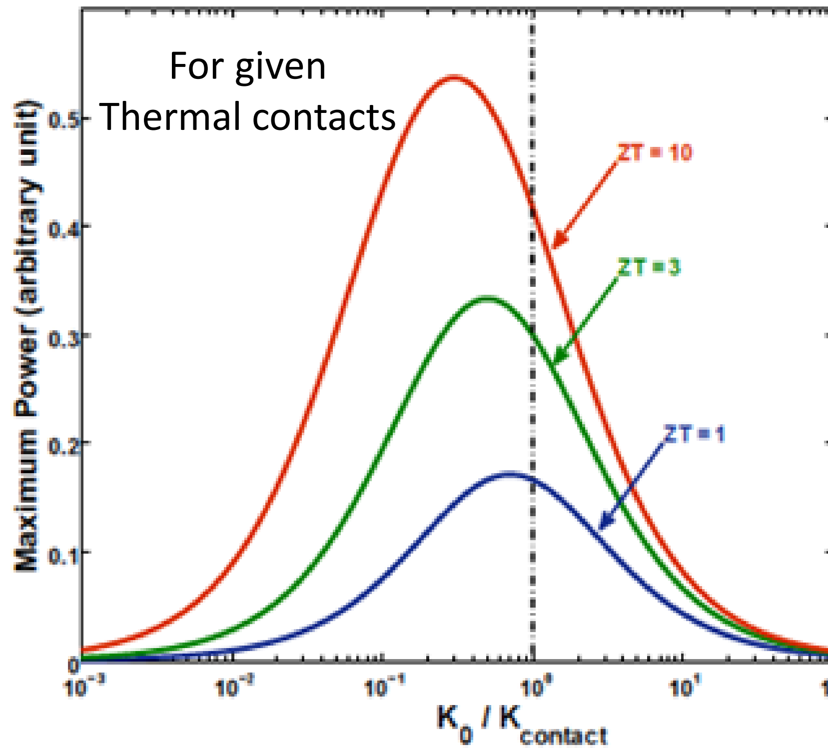
<u>Electric adaptation</u>

$$R_{\mathrm{load}} = R_{\mathrm{TEG}}$$

<u>Thermal adaptation</u>

$$K_{\mathrm{contact}} = K_{\mathrm{TEG}}.$$

$$\begin{cases} K_{contact}/K_0 = \sqrt{ZT+1} \\ R_{load}/R = \sqrt{ZT+1} \end{cases}$$



The thermal adapatation is fundamental for correct working conditions!

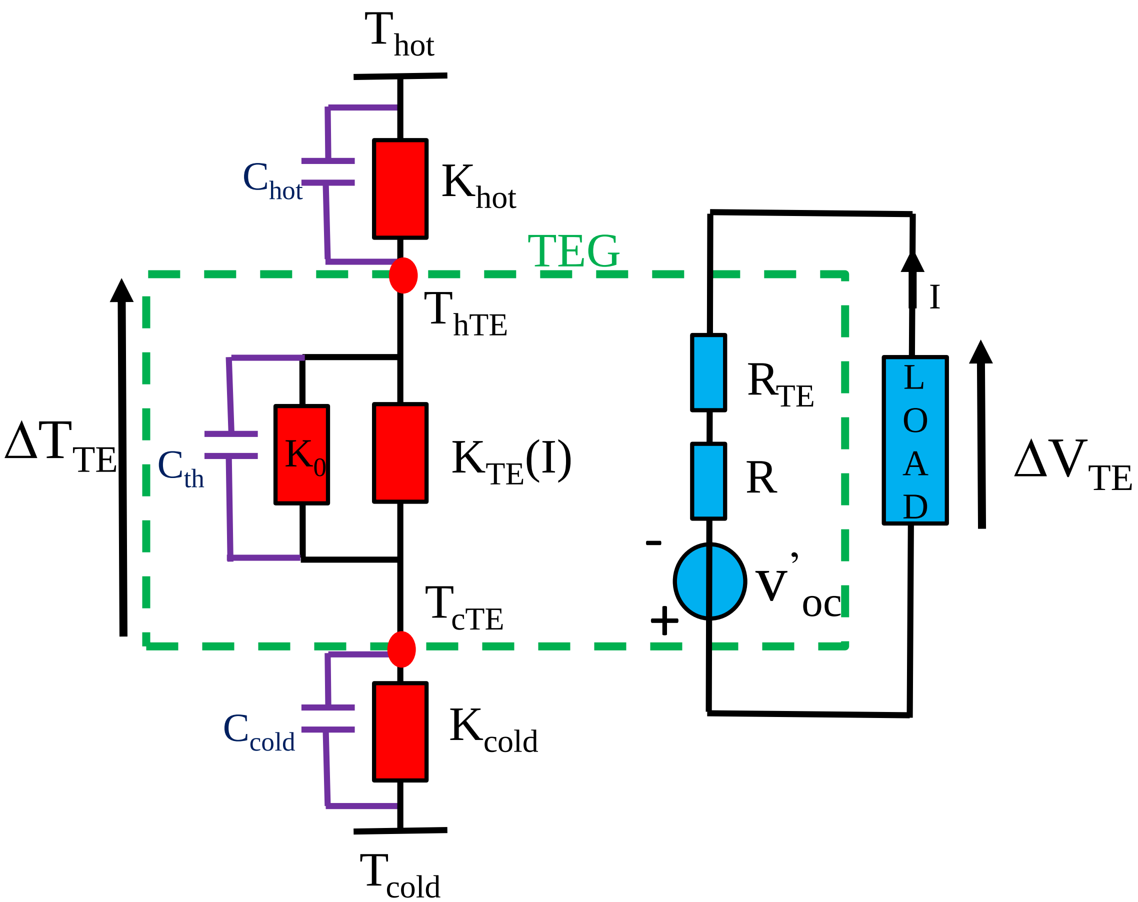
See also:

[M. Freunek et al., J. Elec. Mat. **38** (2009)]

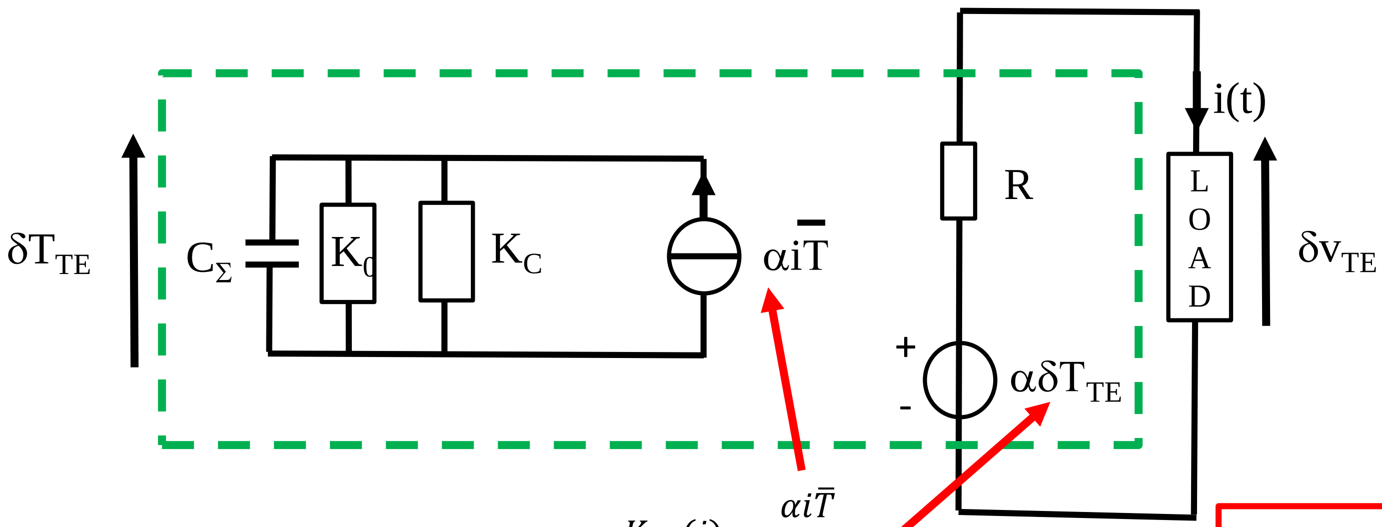
[K. Yazawa et A. Shakouri, JAP **111** (2012)]

[Y. Apertet, et al. EPL 97 (2012)]

1rst order harmonic response



Small signal analysis



$$C_\Sigma = C_{th} + C_{hot} + C_{cold}$$

$$K_C = \frac{K_{hot} K_{cold}}{K_{hot} + K_{cold}}$$

$$K_{TE}(i) = \frac{\alpha i \bar{T}}{\delta T_{TE}}$$

$$\boxed{\delta T_{TE}(i) = \frac{\alpha i T}{j C_\Sigma \omega + K_C + K_0}}$$

Small signal analysis

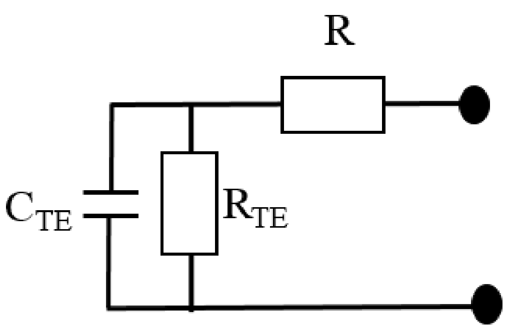
$$\alpha\delta T_{TE} = \frac{\alpha^2 iT}{jC_{Th}\omega + K_c + K_0} = \frac{i}{\dfrac{jC_{Th}\omega}{\alpha^2 T} + \dfrac{1}{R_{TE}}}$$

$$\frac{\alpha\delta T_{TE}}{i} = \frac{R_{TE}}{1 + j\omega \dfrac{C_{Th}}{\alpha^2 T} R_{TE}}$$

$$\begin{cases} R_{TE} = \dfrac{\alpha^2 T}{K_C + K_0} \\ C_{TE} = \dfrac{C_\Sigma}{\alpha^2 T} \\ \tau_{TE} = R_{TE} C_{TE} \end{cases}$$

Total ouput impedance

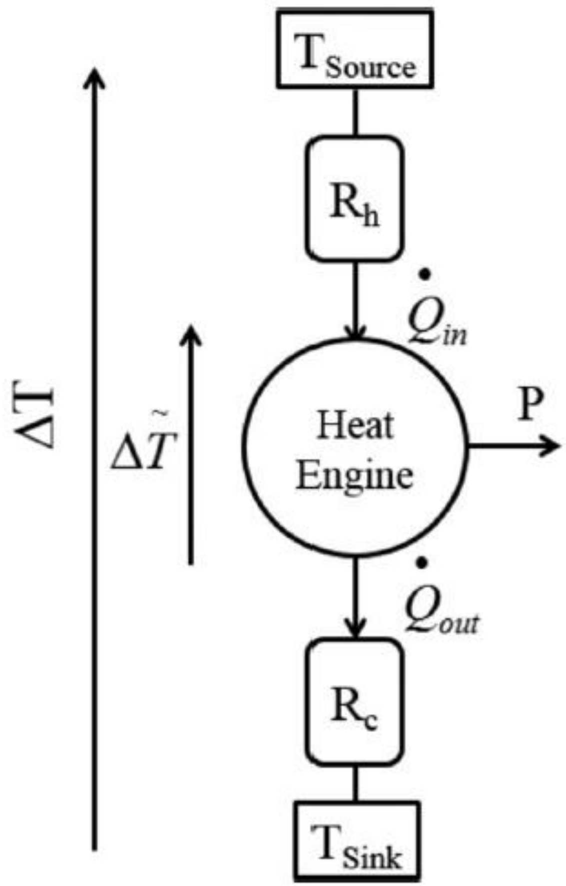
$$\boxed{Z(j\omega) = R + \frac{R_{TE}}{1 + j\omega\tau_{TE}}}$$

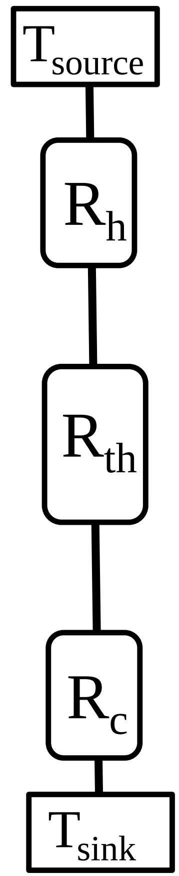


Maximal output power if impedance matching => Better @ non zero frequency?

Generalization: Closed loop

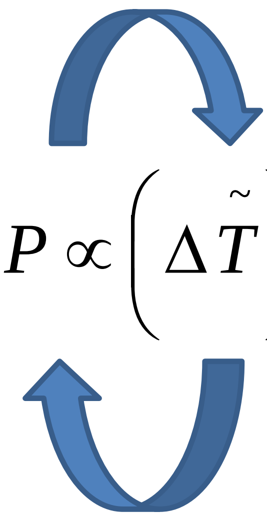
General system: non endoreversible





$$R_{th} = \frac{\Delta\tilde{T}}{\dot{Q}_{Avg}}$$

$$\Delta\tilde{T} = \Delta T\, R_{\mathrm{th}}/(R_{\mathrm{th}} + R_{\theta})$$

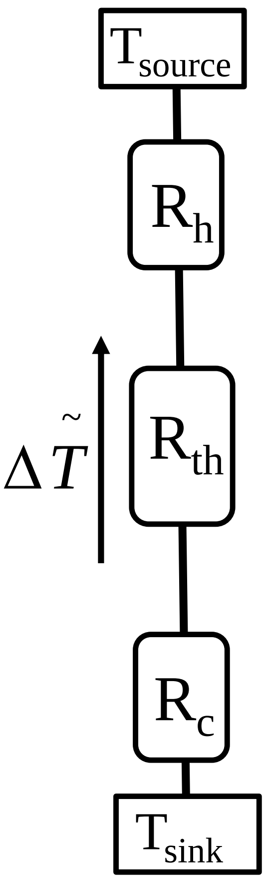


$$P \propto \left(\Delta\tilde{T}\right)^2$$

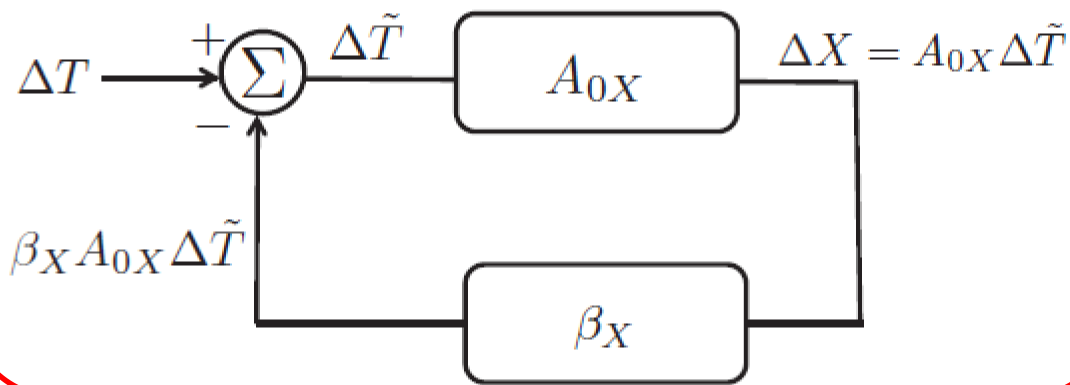
The system is mainly driven by its thermal boundary conditions which defines the feedback factors.

C. Goupil et al.
*PHYSICAL REVIEW E **94**, 032136 (2016)*

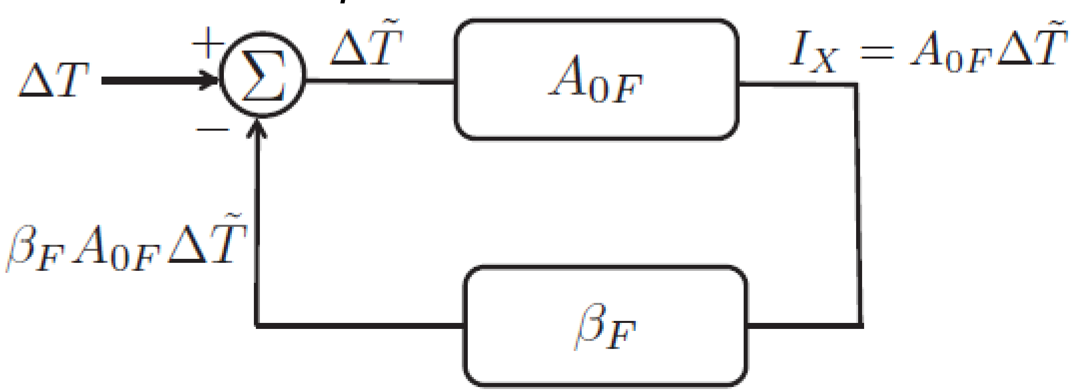
Feedback effects



temperature-potential feedback



temperature-flux feedback



$$A_{0X} = \Delta X / \Delta \tilde{T}.$$

$$A_{0F} = I_X / \Delta \tilde{T}.$$

$$\frac{\Delta \tilde{T}}{\Delta T} = \frac{1}{1 + A_0 \beta}$$

$$\boxed{A_{CL} = \frac{A_0}{1 + A_0 \beta}}$$

$$\boxed{A_0 \beta = \frac{R_h + R_c}{R_{th}}}$$

Working modes

Power $$P = \Delta X I_X = A_{0X} A_{0F} (\Delta \widetilde{T})^2 = A_{0X} A_{0F} \left(\frac{\Delta T}{1 + A_0 \beta} \right)^2$$

Efficiency $$\eta = \frac{P}{\dot{Q}_{\text{in}}} = \frac{A_{0X} A_{0F} \Delta T}{R_{\text{th}}} \left(\frac{1}{1 + A_0 \beta} \right)^3$$

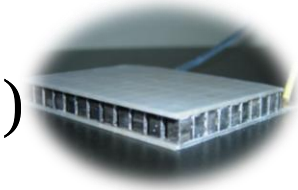
Conversion, feedback, and gain	A_{0X}	A_{0F}	β_X	β_F	A_{cl}
Zero load	$A^{\star}_{0X}$	0	$\frac{R_\theta}{\alpha R_{\text{th}}}$	∞	$\frac{A^{\star}_{0X}}{1+\frac{R_\theta}{R_{\text{th}}}}$
Blocking load	0	$\frac{\alpha}{\beta^{\star}_F} \frac{R_\theta}{R_{\text{th}}}$	∞	$\beta^{\star}_F$	0

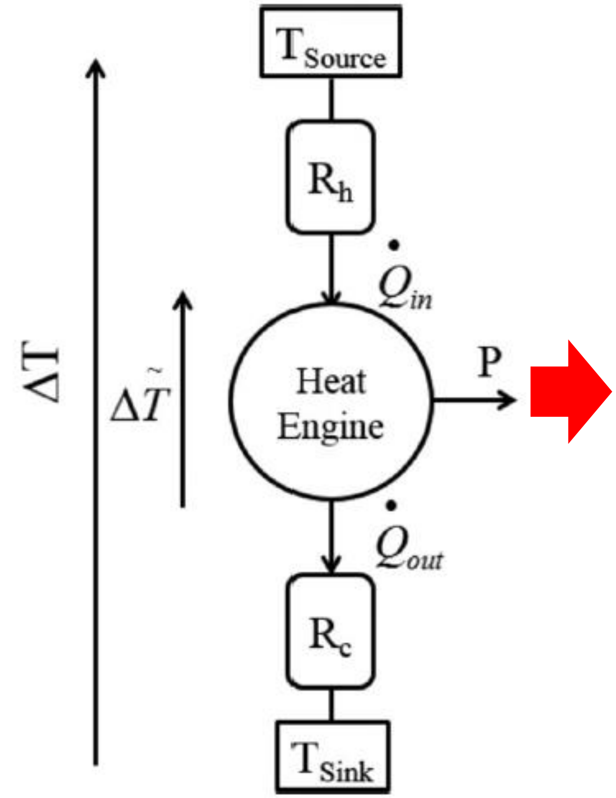
$$A_{CL} = \frac{A_0}{1 + A_0 \beta}$$

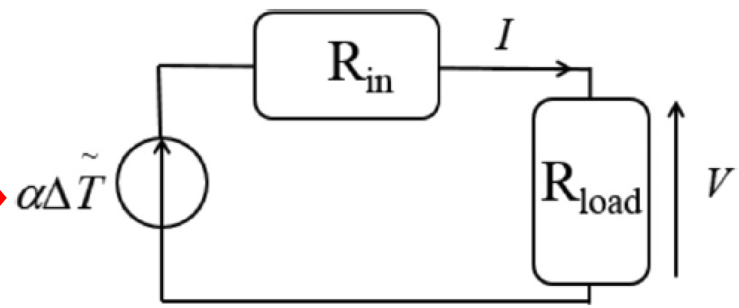
Possible oscillations of the system!

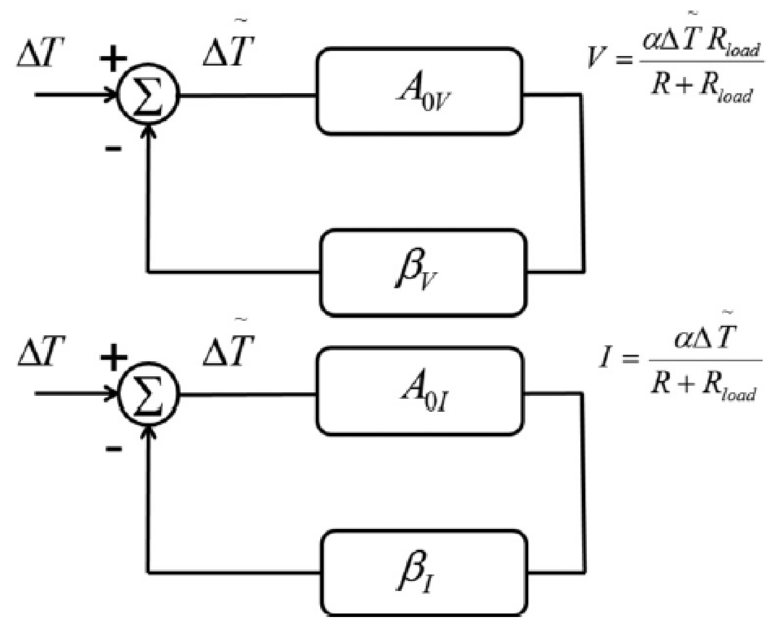
Application: Thermoelectricity

$$\left(P,T\right) \Leftrightarrow \left(\mu,T\right)$$







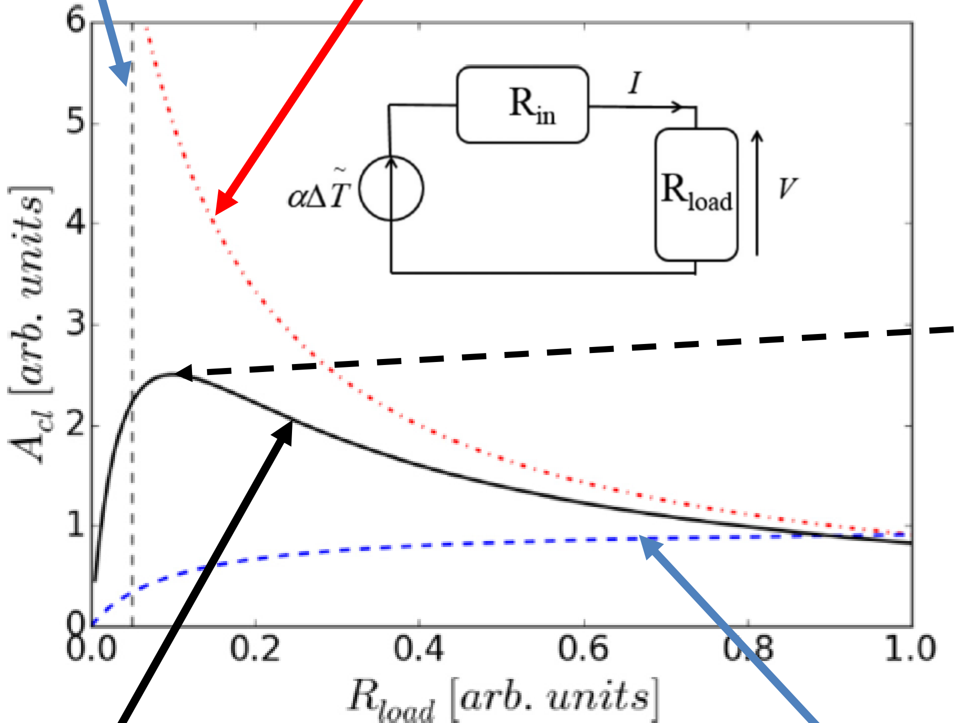


$$A_{0V} = A_{0I}\,R_{\mathrm{load}} = \alpha\,\frac{R_{\mathrm{load}}}{R_{\mathrm{in}} + R_{\mathrm{load}}},$$

$$\beta_V = \frac{\beta_I}{R_{\mathrm{load}}} = \frac{R_\theta}{A_{0V}\,R_{\mathrm{th}}} = \frac{R_\theta}{R_{\mathrm{th}}}\,\frac{R_{\mathrm{in}} + R_{\mathrm{load}}}{\alpha\,R_{\mathrm{load}}}.$$

Application: Thermoelectricity

$R_{Load} = R_{in}$ $A_{CL}(current)$



$A_{CL}(power)$ $A_{CL}(voltage)$

The maximal power is not obtained for

$R_{Load}=R_{in}$

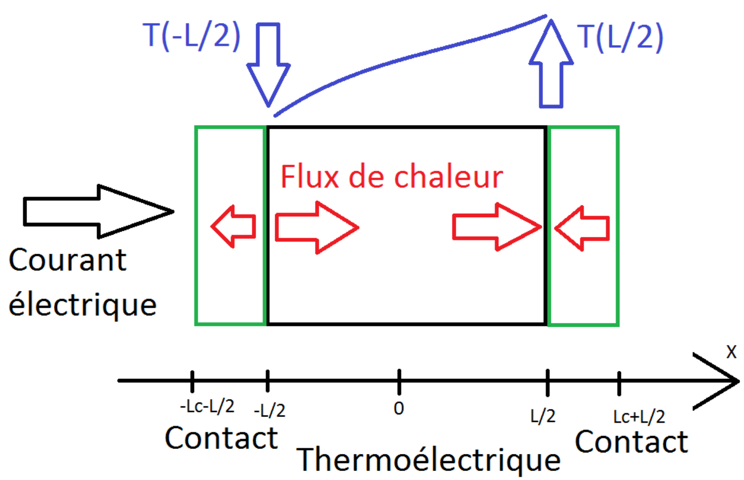
But for

$R_{load} = R_{in} + \boxed{R_{TE}}$

Feedback

Y. Apertet et al. Physical Review B vol. 85, 033201 (2012)
Y. Apertet et al. Europhysics Letters vol. 97, 28001 (2012)

Experimental: Thermoelectricity



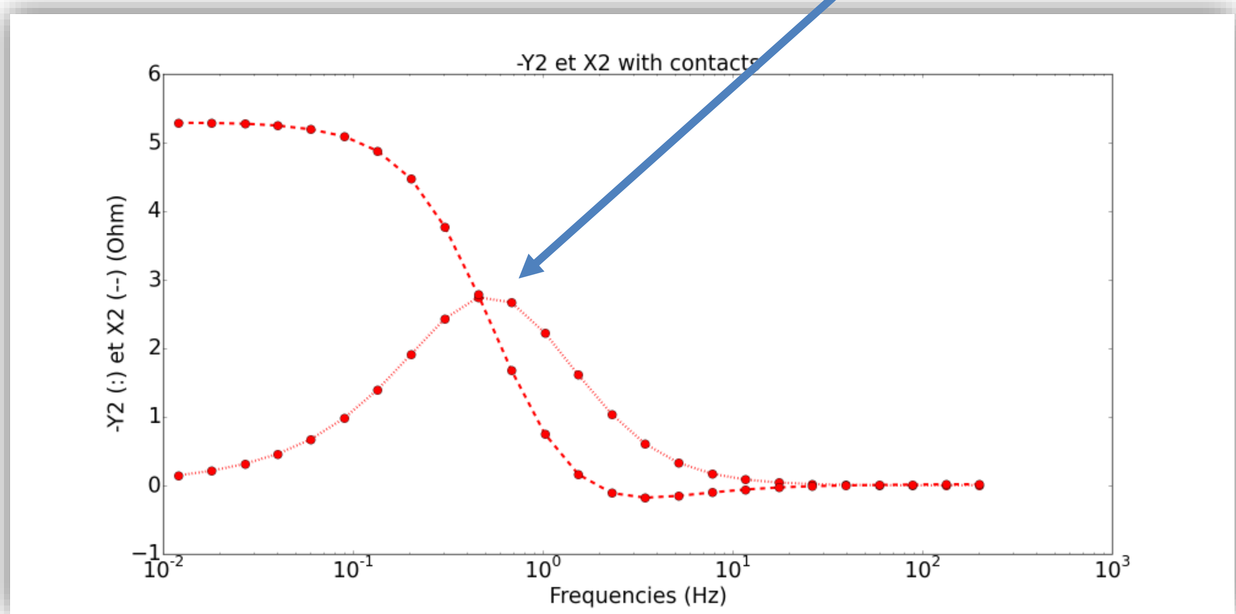
$$R_{TE} = \frac{\alpha^2 T}{K_C + K_0}$$

Extra dissipation

$$C_{TE} = \frac{C_\Sigma}{\alpha^2 T}$$

$$\tau_{TE} = R_{TE} C_{TE}$$

T-μ time constant

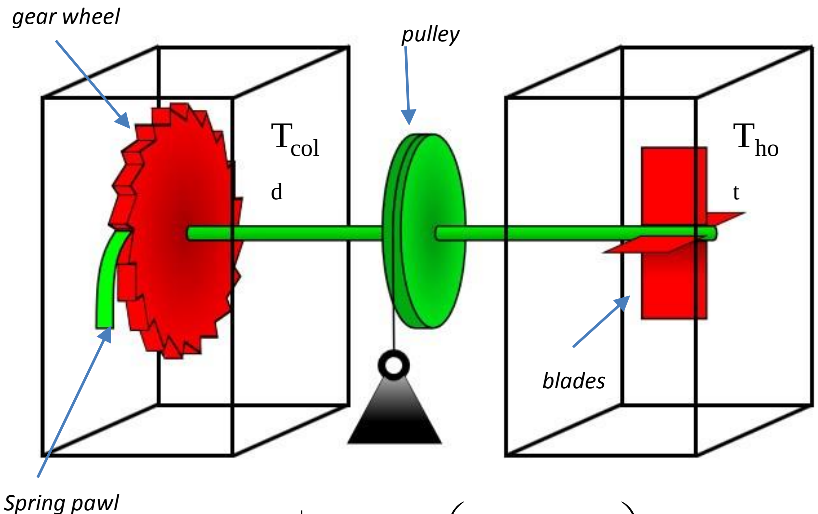


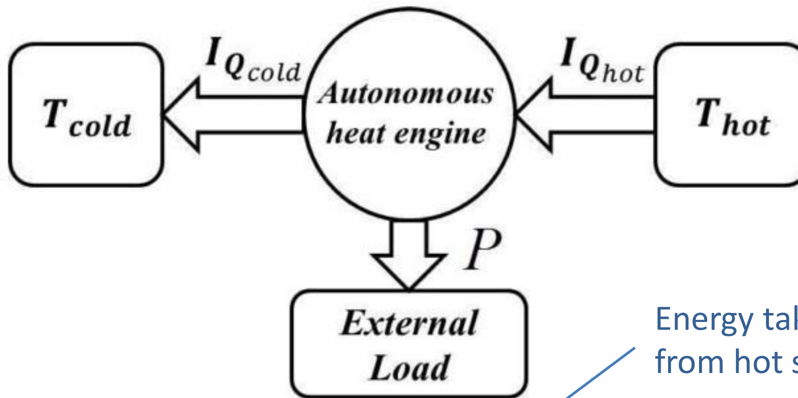
Experimental evidence of the Thermal to Electrochemical potential feedback coupling.
(Etienne Thiebaut's PhD)

Ratchet approah

Feynman ratchet model

R.P. Feynman, R.B. Leighton, and M. Sands,
The Feynman Lectures on Physics I
Addison-Wesley, Reading, (1963), Chap. 46.





$$\dot{N}^{+} = \nu \exp\left(-\frac{\xi + L\theta}{k_B T_{hot}}\right)$$

torque

$$\dot{N}^{-} = \nu \exp\left(-\frac{\xi}{k_B T_{cold}}\right)$$

spring energy

escape frequency

$$\dot{N}_{eff} = \dot{N}^{+} - \dot{N}^{-}$$

effective « current »

Energy taken from hot side

$$I_{Q_{hot}} = \left(\xi + L\theta\right)\dot{N}_{eff}$$

Energy delivered to the cold side

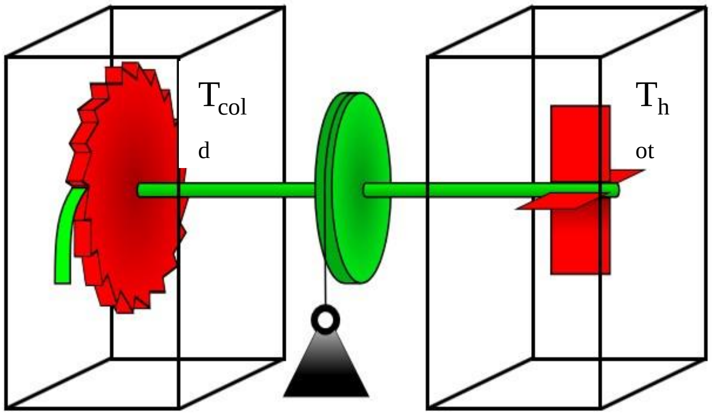
$$I_{Q_{cold}} = \xi \, \dot{N}_{eff}$$

$$P = I_{Q_{hot}} - I_{Q_{cold}} = L\theta \, \dot{N}_{eff}$$

Strong coupling configuration

Velasco at al. J. Phys. D: Appl. Phys.34 (2001) 1000–1006

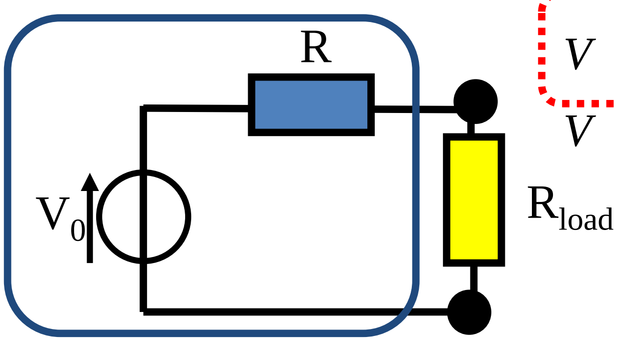
Linearization & thermoelectric model



$$\dot{N}^{+} = \nu\left(1 - \frac{\xi + L\theta}{k_B T_{hot}}\right)$$

$$\dot{N}^{-} = \nu\left(1 - \frac{\xi}{k_B T_{cold}}\right)$$

$$\dot{N}_{eff} = \frac{\nu}{k_B T_{hot}}\left(\frac{\xi \Delta T}{T_{cold}} + L\theta\right) = \frac{\nu}{k_B T_{hot}}\left[L_0\theta - L\theta\right]$$



$$\boxed{V = \alpha \Delta T - R\,I \iff L\theta = L_0\theta - \frac{k_B T_{hot}}{\nu}\dot{N}_{eff}}$$

$$V = R_{load} I$$

$$L_0\theta = \frac{\xi \Delta T}{T_{cold}} : stopping\ force\ (open\ voltage)$$

$$L\theta = 0 : (short\ circuit)$$

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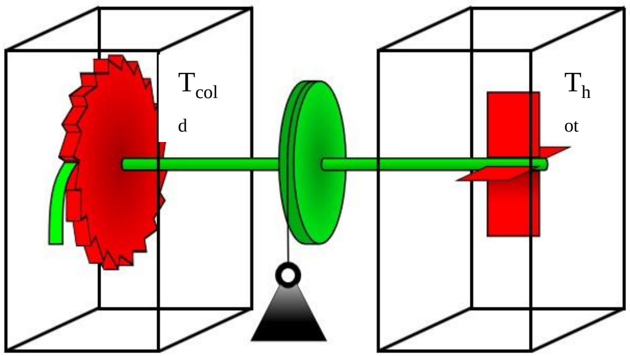
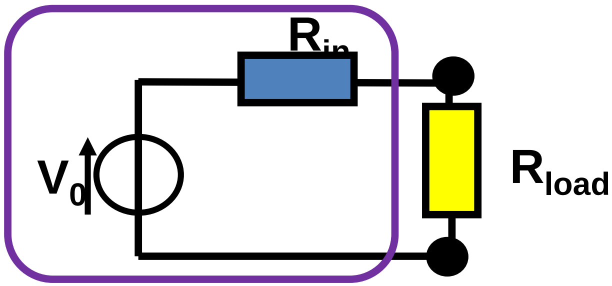
Power budget



$$I_{Q_{hot}} = (\xi + L\theta)\dot{N}_{eff} = \boxed{\left(\frac{\xi}{T_{cold}}\right)} T_{hot}\,\dot{N}_{eff} - \frac{k_B T_{hot}}{\nu}\dot{N}_{eff}^{\,2}$$

$$I_{Q_{cold}} = \left(\frac{\xi}{T_{cold}}\right) T_{cold}\,\dot{N}_{eff}$$

« entropy per tooth »
(Seebeck like: α_{FR})



$$I_{Q_{hot}} = \alpha T_{hot} I + K\Delta T - \frac{1}{2} R I^2$$

$$I_{Q_{cold}} = \alpha T_{cold} I + K\Delta T + \frac{1}{2} R I^2$$

dissipative
resistance

$$P = \left[\left(\frac{\xi}{T_{cold}}\right)\Delta T - \boxed{\frac{k_B T_{hot}}{\nu}}\dot{N}_{eff}\right]\dot{N}_{eff}$$

$$P = \left[\alpha \Delta T - \boxed{R} I\right] I$$

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Extension to non-linear model

$$I_{Q_{hot}} = (\xi + L\theta)\dot{N}_{eff} = \alpha_{FR} T_{hot} \dot{N}_{eff} - [L_0\theta - L\theta]\dot{N}_{eff}$$

$$I_{Q_{cold}} = \left(\frac{\xi}{T_{cold}}\right) T_{cold} \dot{N}_{eff} = \alpha_{FR} T_{cold} \dot{N}_{eff}$$

$$P = L\theta \dot{N}_{eff} = \alpha_{FR} \Delta T \dot{N}_{eff} - \frac{[L_0\theta - L\theta]}{\dot{N}_{eff}} \dot{N}^2_{eff}$$

$$R_{dissip} = \frac{L_0\theta - L\theta}{\dot{N}_{eff}} = \frac{L_0\theta}{\dot{N}_{eff}} - R_{load}$$

$$R_{dyn} = \frac{k_B T_{hot}}{\nu}$$

In the non linear case the dissipative resistance and the linear resistance do not coincide.

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Non-linear model & efficiency

<u>Parameters</u> :

- x : Force
- I : Flux

- $R_{dyn} = -\frac{dx}{dI}$
- $R_{dissip} = \frac{x_0 - x}{I}$
- γ : coefficient de répartition de la chaleur dissipée

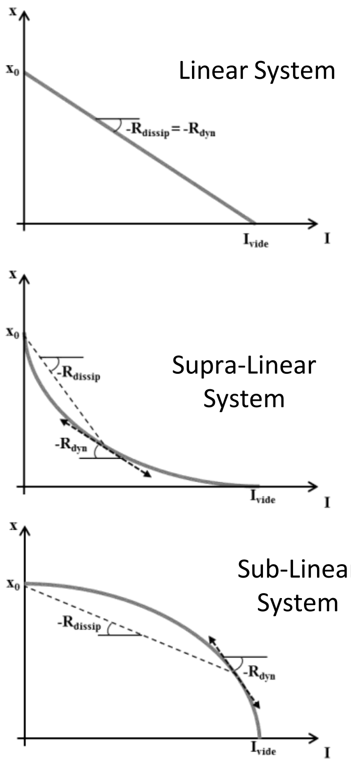
In a non linear system these parameters are functions of the working point!

<u>Maximum power</u> :

$$x_{MP} = x_0 \frac{R_{dyn}}{R_{dyn} + R_{dissip}}$$

<u>Efficiency at maximum power</u> :

$$\boxed{\eta_{MP} = \frac{\eta_C}{\frac{R_{dissip}}{R_{dyn}}(1 - \gamma\eta_C) + 1}}$$



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