Latitudinal heat flux distribution of thermal anelastic convection in a rotating spherical shell

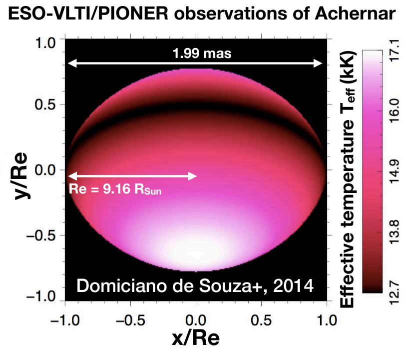
Raphaël Raynaud (CEA-Dap), Michel Rieutord (IRAP),
Ludovic Petitdemange (LERMA), Thomas Gastine (IPGP)

Groupe de travail Climat - MécaStat
Saclay, September 16th, 2019

Table of contents

Astrophysical motivation

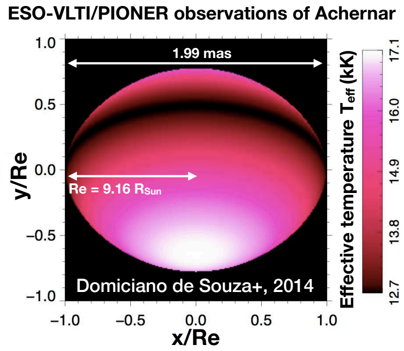
Gravity darkening



$P_{\rm rot} \sim 37\,\mathrm{h}$, $r_{pole} \sim 7\,\mathrm{R}_\odot$, $M \sim 6\,\mathrm{M}_\odot$

Astrophysical motivation

Gravity darkening



$P_{\mathrm{rot}} \sim 37\,\mathrm{h}$, $r_{pole} \sim 7\,\mathrm{R}_{\odot}$, $M \sim 6\,\mathrm{M}_{\odot}$

Von Zeipel theory (1926)

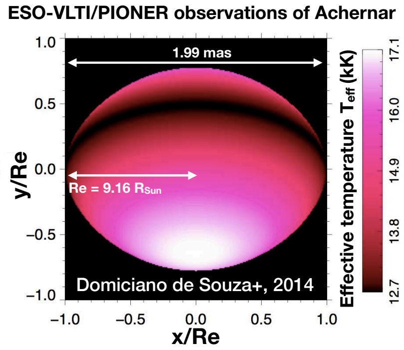
radiative equilibrium of oblate stars

$$T_{\mathrm{eff}} \propto g_{\mathrm{eff}}^{\beta}$$

$\beta =$ gravity-darkening exponent

Astrophysical motivation

Gravity darkening



$P_{\mathrm{rot}} \sim 37\,\mathrm{h}$, $r_{pole} \sim 7\,\mathrm{R}_\odot$, $M \sim 6\,\mathrm{M}_\odot$

Von Zeipel theory (1926)

radiative equilibrium of oblate stars

$$T_{\mathrm{eff}} \propto g_{\mathrm{eff}}^{\beta}$$

$\beta =$ gravity-darkening exponent

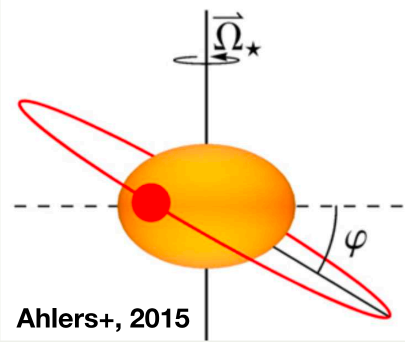
What about convective envelopes ?

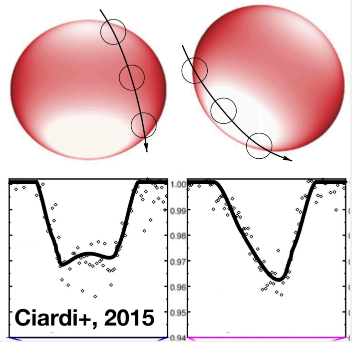
1. investigate the energy transport by convection in a rotating spherical shell
2. study the latitudinal variations of the heat flux at the outer surface

Potential applications

Photometry of binary systems

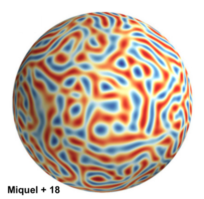
1. eclipsing binaries
2. transit of exoplanets



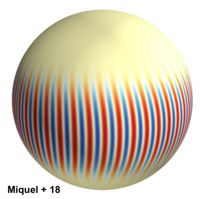


Effects of rotation

Slow rotation



Fast rotation



Onset

1. geostrophic (z-invariant)
2. symmetric (Busse 70)
3. $\ell \propto E^{1/3}$
4. $Ra_c \propto E^{-4/3}$

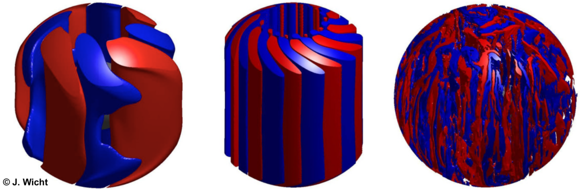
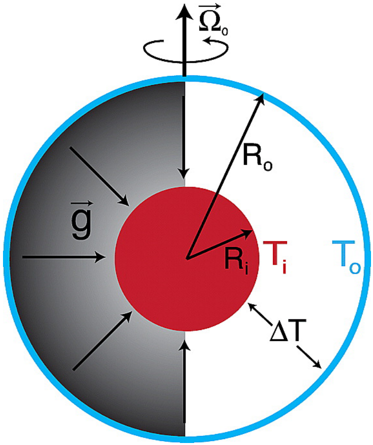


Table of contents

Numerical set up



King *et al.*, 2010, GGG, Q06016

Setup

- rotating spherical shell
- perfect gas with constant
  - kinematic viscosity ν
  - temperature/entropy diffusivity κ

Boundary conditions

- stress-free b. c. for the velocity field
- fixed temperature/entropy

How to model the fluid flow ?

The convective approximations

1. retain the essential physics with a minimum complexity
2. filter out sound waves

The Boussinesq approximation

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\rho' = f(T)$$

$$M \ll 1 \quad \text{and} \quad d/H_p \ll 1$$

- numerical MHD benchmark: Christensen + 2001
- common approximation for laboratory experiments

How to model the fluid flow ?

The convective approximations

1. retain the essential physics with a minimum complexity
2. filter out sound waves

The Boussinesq approximation

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\rho' = f(T)$$

$$M \ll 1 \quad \text{and} \quad d/H_p \ll 1$$

- numerical MHD benchmark: Christensen + 2001
- common approximation for laboratory experiments

The Anelastic approximation

$$\nabla \cdot (\rho_a \, \boldsymbol{u}) = 0$$

$$\rho' = f(T, P)$$

$$M \ll 1$$

- numerical MHD benchmark: Jones + 2001
- developped for atmospheric convection (Ogura & Philipps 62, Gough 69)

The anelastic reference state

Mechanical quasiequilibrium

hydrostatic balance

$$-\nabla P_a + \rho_a \boldsymbol{g} = 0$$

Thermal quasiequilibrium

"well mixed" state

$$\nabla S_a = 0$$

Polytropic solution

$$T_a = w(r), \quad \rho_a = w^n, \quad P_a = w^{n+1}$$

Remark: we recover the adiabatic gradient

$$\nabla T_a = \left(\frac{\partial T}{\partial P}\right)_S \nabla P_a = \frac{\alpha T_a}{C_p}\boldsymbol{g} = \alpha^S \boldsymbol{g}$$

The anelastic approximation

Resulting system

$$\nabla \cdot (\rho_a \boldsymbol{V}) = 0$$

$$D_t \boldsymbol{V} = -\nabla \left(\frac{P_c}{\rho_a} \right) - \alpha^S S_c \boldsymbol{g} + \boldsymbol{F}^\nu$$

$$\rho_a D_t S_c + \nabla \cdot \left(\frac{\boldsymbol{I}^q}{T_a} \right) = \frac{Q}{T_a} + \boldsymbol{I}^q \cdot \nabla T_a^{-1}$$

with

$$\begin{cases} \boldsymbol{F}^\nu = \nabla \cdot \tau & \text{viscous force} \\ Q = \tau_{ij} \nabla_j V_i & \text{viscous heating;} \, \tau_{ij} = 2\mu \left[\frac{1}{2} (\partial_i v_j + \partial_j v_i) - \frac{1}{3} \partial_k v_k \delta_{ij} \right] \\ \boldsymbol{I}^q & \text{heat flux} \end{cases}$$

Braginsky, S. I. & Roberts, P. H., 1995, GAFD, 79, 1

Mean field approach for the heat transfer equation

Solve the equation for average quantities

$$f = \langle f \rangle + f^t \quad (1)$$

This creates a turbulent entropy flux term

$$\boldsymbol{I}^{St} = \rho_a \langle S^t \boldsymbol{V}^t \rangle \propto \nabla \langle S \rangle \quad (2)$$

Strong simplification

- the usual molecular term $\boldsymbol{I}^q = -k \nabla T$ is then neglected in the heat transfer equation
- $\Longrightarrow$ temperature is removed from the problem !

Mean field approach for the heat transfer equation

Solve the equation for average quantities

$$f = \langle f \rangle + f^t \tag{1}$$

This creates a turbulent entropy flux term

$$\boldsymbol{I}^{St} = \rho_a \langle S^t \boldsymbol{V}^t \rangle \propto \nabla \langle S \rangle \tag{2}$$

Strong simplification

- the usual molecular term $\boldsymbol{I}^q = -k\nabla T$ is then neglected in the heat transfer equation
- $\Longrightarrow$ temperature is removed from the problem !

Drawbacks

- argument valid far from the onset of convection only
- P2 of thermodynamics not under warranty anymore

Numerical implementation

$$[r] = d = r_o - r_i, \quad [t] = d^2/nu, \quad [S] = \Delta S, \quad [\rho_a] = \rho_a(r_o), \quad [T_a] = T_a(r_o)]$$

$$\nabla \cdot (\rho_a \boldsymbol{v}) = 0$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\,\boldsymbol{v} = -\frac{1}{E}\nabla\left(\frac{P'}{\rho_a}\right) + \frac{Ra}{Pr}\frac{S}{r^2}\boldsymbol{e}_r - \frac{2}{E}\boldsymbol{e}_z \times \boldsymbol{v} + \boldsymbol{F}_\nu$$

$$\frac{\partial S}{\partial t} + (\boldsymbol{v} \cdot \nabla)\,S = \frac{(\rho_a T_a)^{-1}}{Pr}\nabla \cdot (\rho_a T_a \nabla S) + \frac{Di}{T_a}Q_\nu$$

magic-sph.github.io

1. poloidal/toroidal decomposition
   $\nabla \cdot \boldsymbol{B} = 0 \iff \boldsymbol{B} = \nabla \times \nabla \times B_P \boldsymbol{e}_r + \nabla \times B_T \boldsymbol{e}_r$
2. angular decomposition: spherical harmonics
3. radial decomposition: Chebyshev polynomials
4. OpenMP/MPI parallelisation

Control parameters

Name		Definition	Sun	DNS
aspect ratio	χ	r_i/r_o	~ 0.7	0.35 / 0.7
Rayleigh number	Ra	$\frac{GMd\Delta S}{\nu\kappa c_p}$	$\mathcal{O}(10^{20})$	$\lessapprox 10^7$
Prandtl number	Pr	$\frac{\nu}{\kappa}$	$\mathcal{O}(10^{-6})$	1
Ekman number	E	$\frac{\nu}{(\Omega d^2)}$	$\mathcal{O}(10^{-15})$	$\sim 10^{-4}$
density scale height	N_ϱ	$\ln(\varrho_i/\varrho_o)$	$\mathcal{O}(10)$	≤ 8

Output parameters

Nusselt number (total/conductive heat flux)

$$\overline{Nu}^{\varphi}(r,\theta) = \frac{(1-e^{-N_\varrho})\rho_a T_a r^2}{n c_1 \rho_a(r_o)} \frac{1}{2\pi}\int_0^{2\pi}\left(Pr(S-S_c)u_r - \frac{\partial S}{\partial r}\right)\mathrm{d}\varphi$$

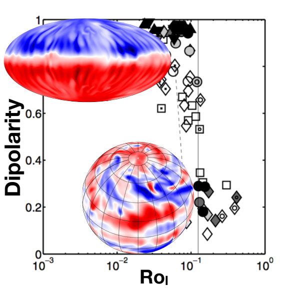
Local Rossby number

$$Ro \sim \frac{\text{inertia}}{\text{Coriolis}}$$

$$Ro_\ell = \frac{v^{nz}}{\Omega \ell}$$

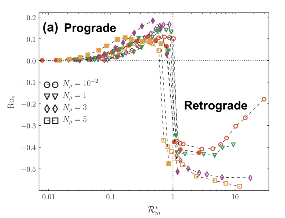
ℓ = typical convective length scale

Magnetic topolgy



Christensen & Aubert, 2006

Zonal flows



Gastine & Wicht, 2012

Table of contents

Impact of the stratification at the onset of convection

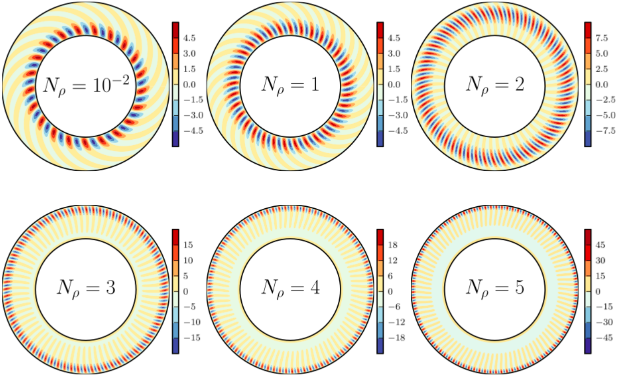


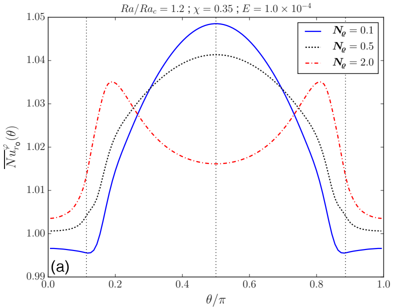
Figure 2: Radial component of the velocity u_r in the equatorial plane for different density stratifications very close to onset of convection (less than 5% over the critical Rayleigh number). Outward flows are rendered in red, inward flows in blue, dimensionless radial velocities are expressed in terms of Reynolds numbers.

Gastine T. & Wicht J., 2012, Icarus, 219, 428–442
Jones C. A. *et al.*, 2009, JFM, 634, 291–319

Heat flux at the onset of convection

Large shell

$\chi = 0.35$

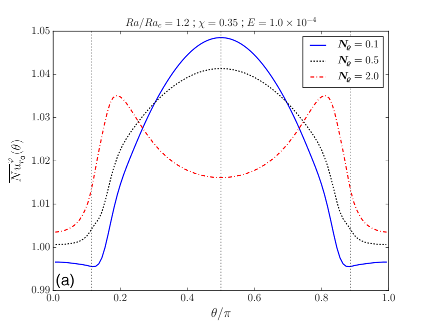


Nusselt number as a function of colatitude

Heat flux at the onset of convection

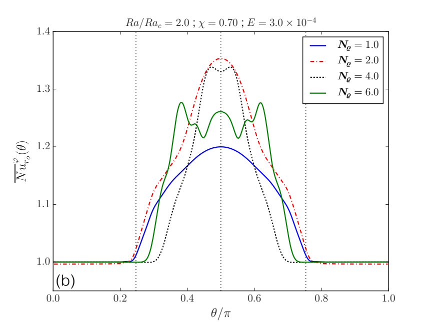
Large shell

$\chi = 0.35$



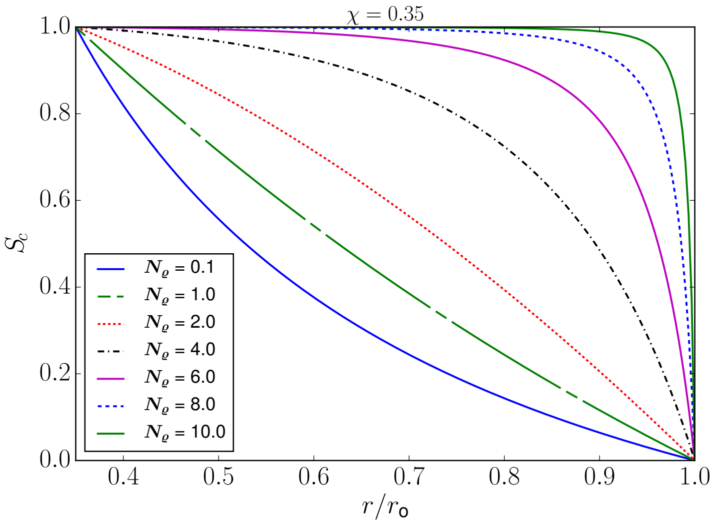
Thin shell

$\chi = 0.7$

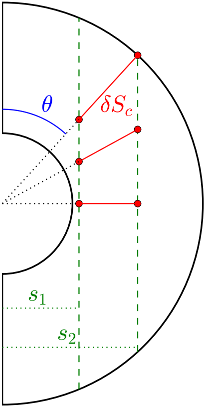


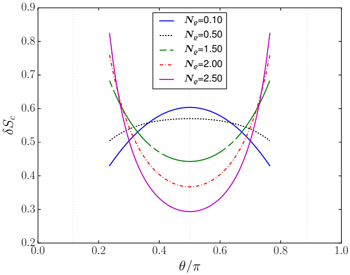
Nusselt number as a function of colatitude

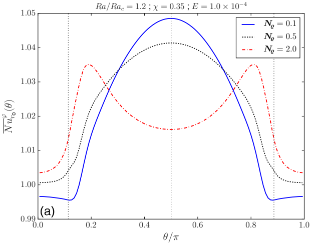
Conductive entropy profile S_c



A simple explanation

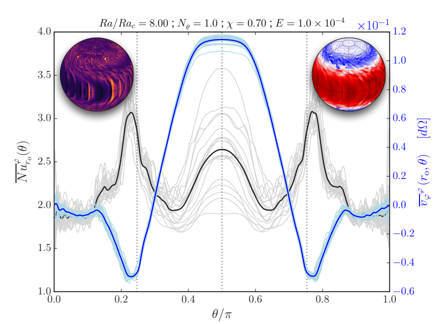






Increasing the Rayleigh number for moderate stratification

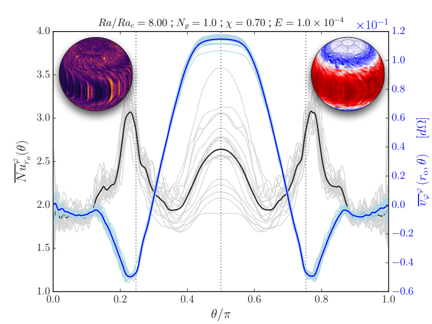
$Ra/Ra_c = 8$



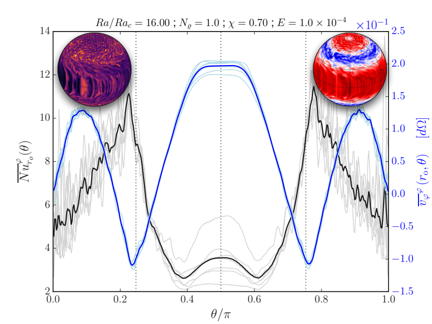
Nusselt (black) and zonal velocity (blue) latitudinal profiles

Increasing the Rayleigh number for moderate stratification

$Ra/Ra_c = 8$



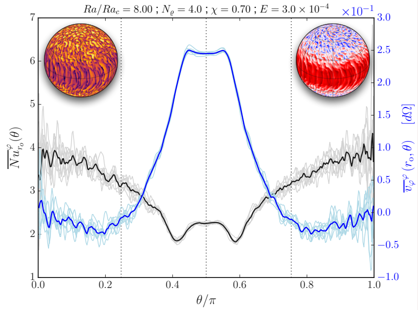
$Ra/Ra_c = 16$



Nusselt (black) and zonal velocity (blue) latitudinal profiles

Increasing the density stratification

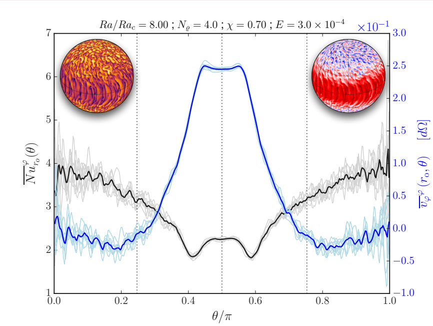
$Ra/Ra_c = 8$, $N_\varrho = 4$



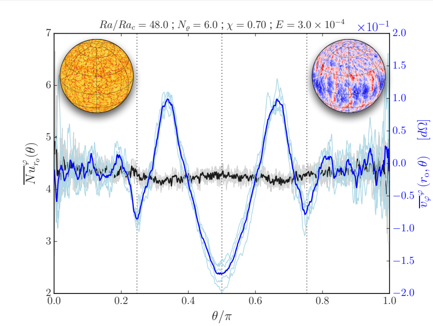
Nusselt (black) and zonal velocity (blue) latitudinal profiles

Increasing the density stratification

$Ra/Ra_c = 8$, $N_\varrho = 4$

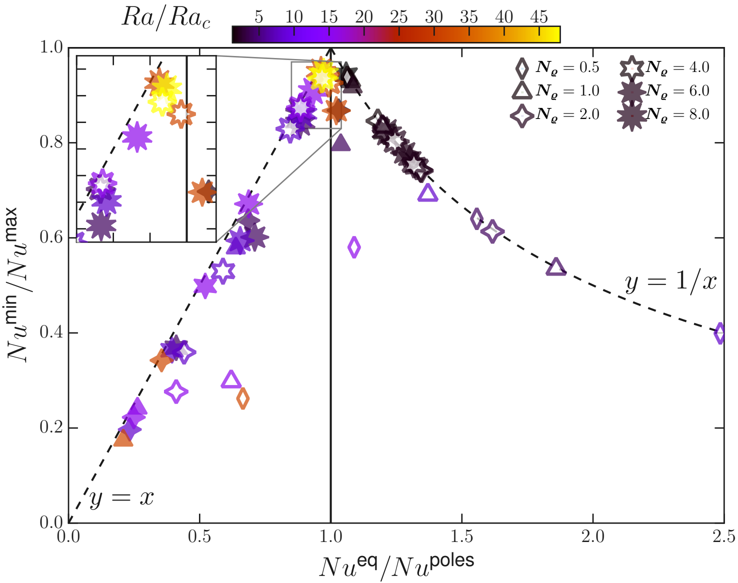


$Ra/Ra_c = 48$, $N_\varrho = 6$



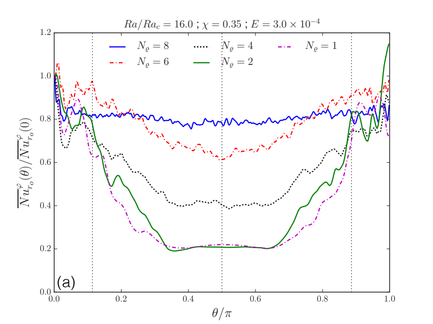
Nusselt (black) and zonal velocity (blue) latitudinal profiles

Inversion of the pole-equator contrast



Impact of the density stratification

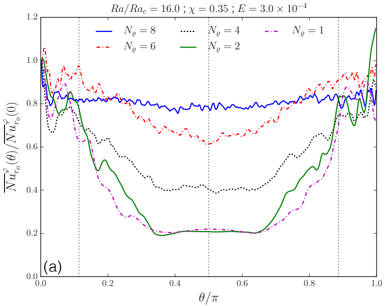
Latitudinal Nusselt profiles



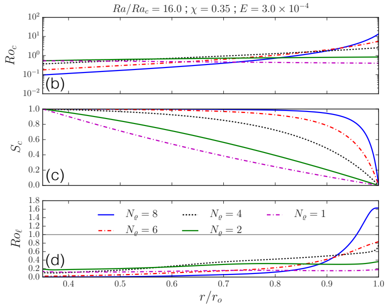
$$Ra/Ra_c = 16, \ \chi = 0.35$$

Impact of the density stratification

Latitudinal Nusselt profiles

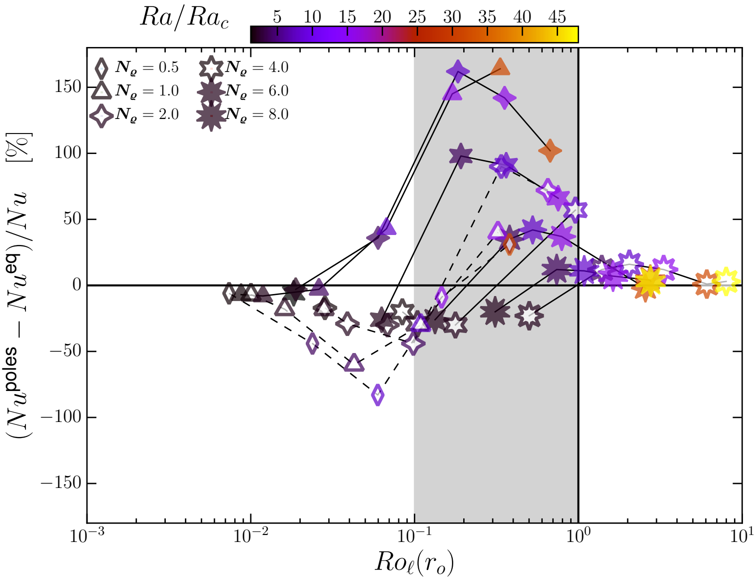


Radial profiles

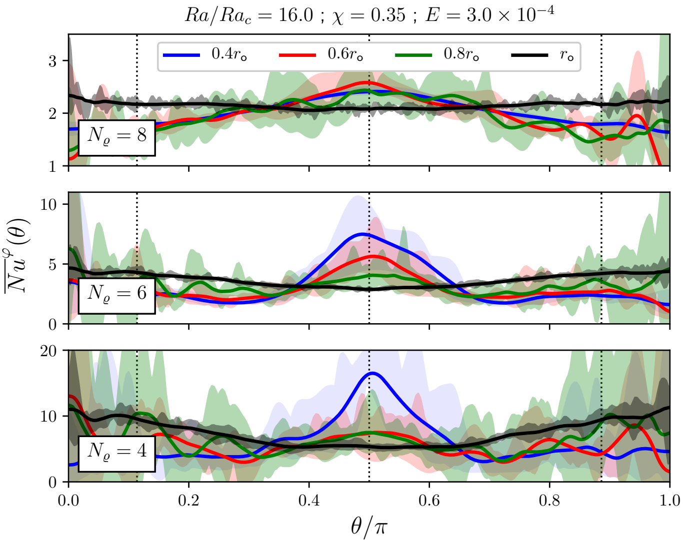


$$Ra/Ra_c = 16,\ \chi = 0.35$$

Uniformisation of the surface heat flux distribution



A screening effect confined to the upper layers



Entropy, velocity magnitude and Nusselt number snapshots

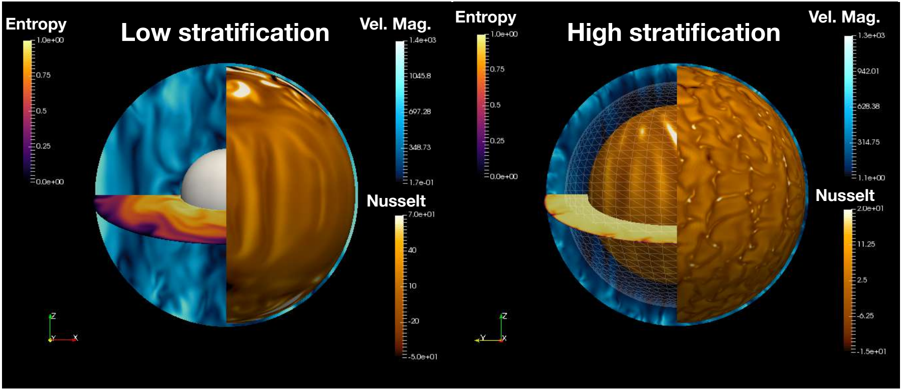
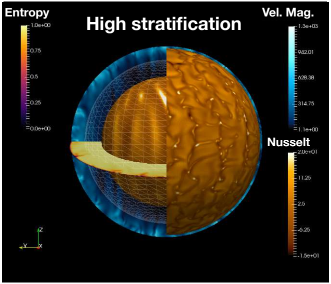


Table of contents

Conclusions

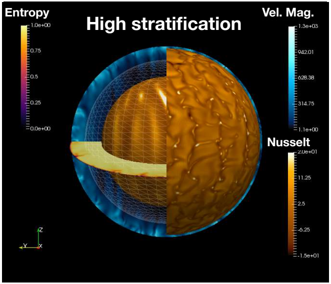
Raynaud et al., “Gravity darkening in late-type stars. I Coriolis effect”,*A&A* 609, A124 (2018)



1. zonal jet can be efficient at impeding the radial heat transfer at low latitudes

Conclusions

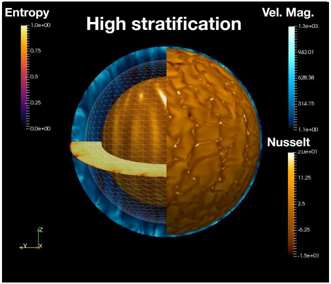
Raynaud et al., “Gravity darkening in late-type stars. I Coriolis effect”,*A&A* 609, A124 (2018)



1. zonal jet can be efficient at impeding the radial heat transfer at low latitudes
2. strong impact of the density stratification

Conclusions

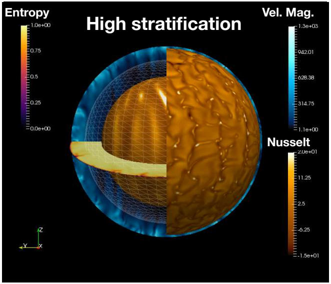
Raynaud et al., "Gravity darkening in late-type stars. I Coriolis effect", *A&A* 609, A124 (2018)



1. zonal jet can be efficient at impeding the radial heat transfer at low latitudes
2. strong impact of the density stratification
3. uniformisation of the heat flux is primarily controlled by the surface value of the local Rossby number

Conclusions

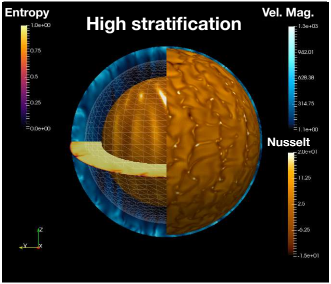
Raynaud et al., "Gravity darkening in late-type stars. I Coriolis effect", *A&A* 609, A124 (2018)



1. zonal jet can be efficient at impeding the radial heat transfer at low latitudes
2. strong impact of the density stratification
3. uniformisation of the heat flux is primarily controlled by the surface value of the local Rossby number
4. partly consistent with the "solar heat flux problem" (weak 0.1% enhancement of the luminosity at polar latitudes)

Conclusions

Raynaud et al., "Gravity darkening in late-type stars. I Coriolis effect",*A&A* 609, A124 (2018)



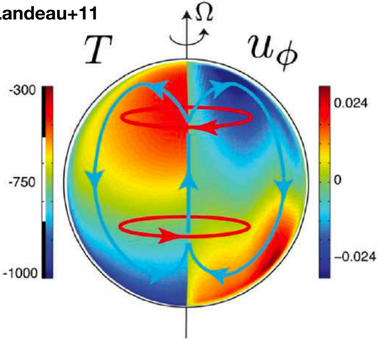
1. zonal jet can be efficient at impeding the radial heat transfer at low latitudes
2. strong impact of the density stratification
3. uniformisation of the heat flux is primarily controlled by the surface value of the local Rossby number
4. partly consistent with the "solar heat flux problem" (weak 0.1% enhancement of the luminosity at polar latitudes)
5. robustness of the results . . . ?

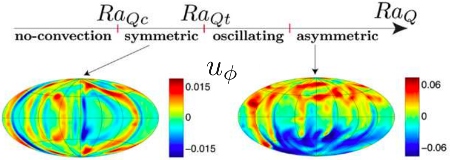
Table of contents

Equatorially asymmetric convection: fixed flux forcing

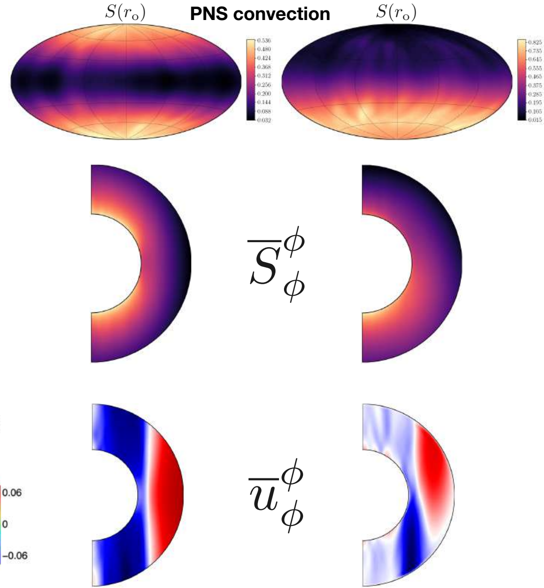
Boussinesq convection

Landeau+11

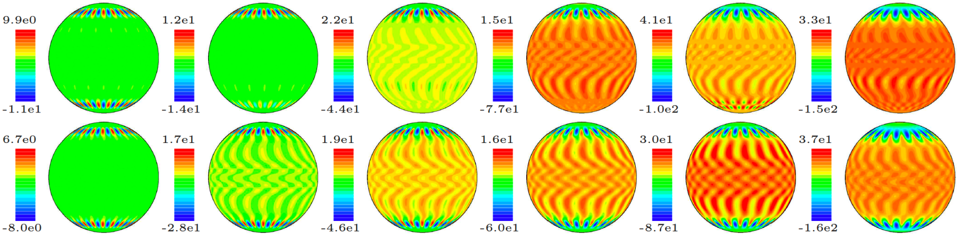




PNS convection



Low Prandtl regime

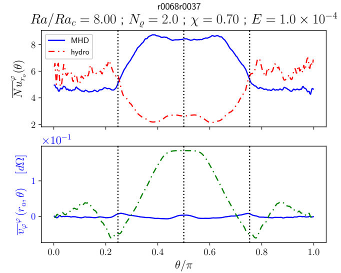


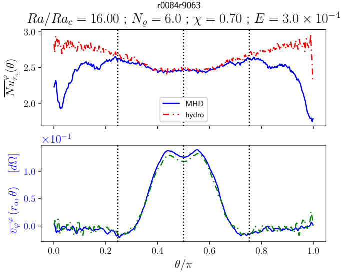
Boussinesq convection in thin shell with

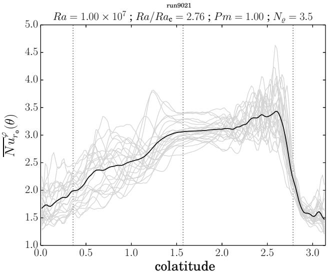
$$\chi = 0.9, \quad Pr = 3 \times 10^{-3}, \quad E \sim 10^{-4}$$

Garcia et al. PRF 4, 074802 (2019)

Dynamo cases (current work)







1. magnetic quenching of differential rotation
2. polar spot (dipole solution)
3. hemispherical dynamo