Abrupt transitions in geophysical flows: the case of superrotation

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November 29, 2019 — CEA Saclay *Extreme Events*

In collaboration with F. Bouchet (ENS Lyon), C.-E. Brehier (Université Lyon), R. Caballero (Stockholm University), T. Lestang (University of Oxford), F. Ragone (ENS Lyon).

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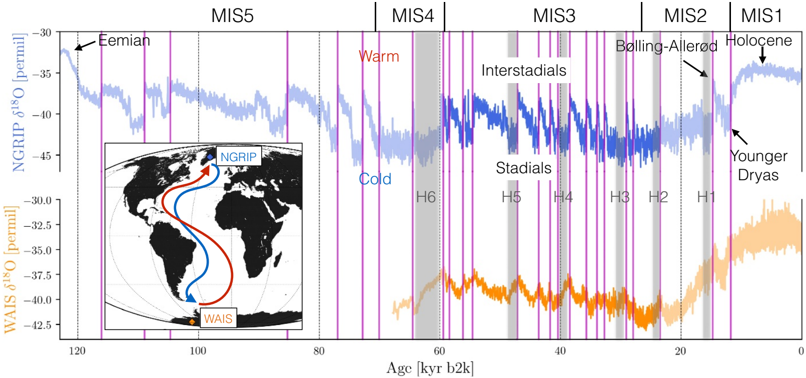
Marie Skłodowska-Curie Actions

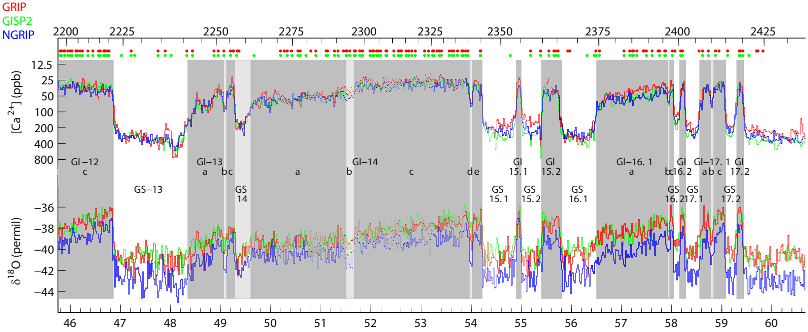
Abrupt events in glacial climate



N. Boers, M. Ghil, and D.-D. Rousseau (2018). *Proc. Natl. Acad. Sci. U.S.A.*

- Dansgaard-Oeschger events: rapid NH warming ($\sim$ 5 K in $\sim$ 10 y)
- Heinrich events: massive iceberg rafts

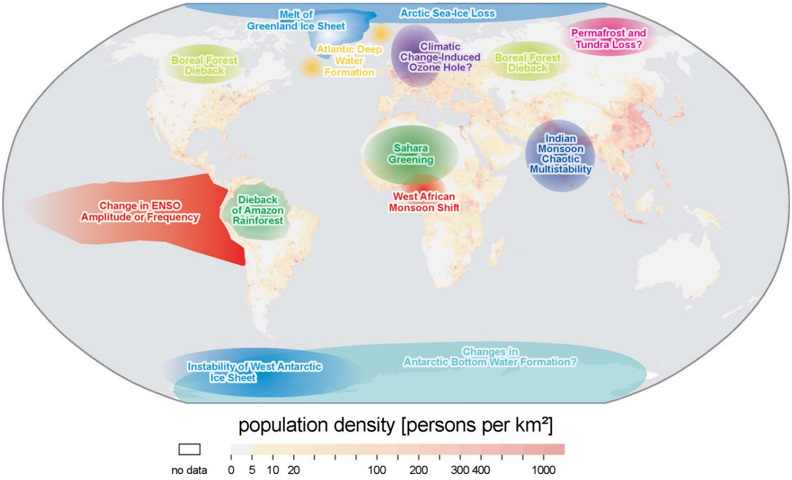
Abrupt events in glacial climate: high-resolution



S. O. Rasmussen et al. (2014). *Quaternary Science Reviews*

- Transitions occur on very fast timescales ($\leq$ 10 years)
- Something happens in the atmosphere

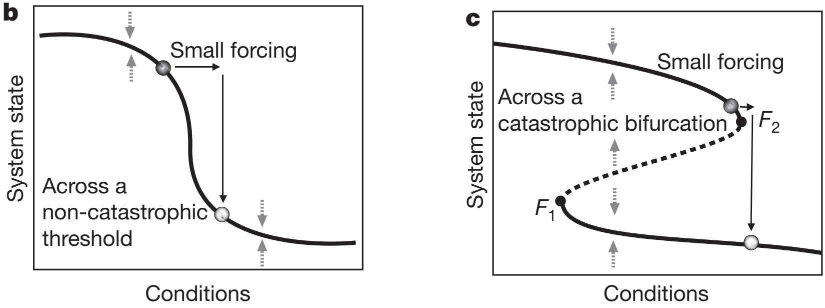
Tipping points



T. Lenton et al. (2008). *Proc. Natl. Acad. Sci. U.S.A.*

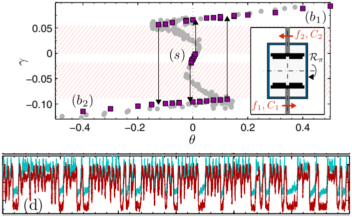
Many potential mechanisms for abrupt transitions have been suggested.

Continuous or discontinuous?



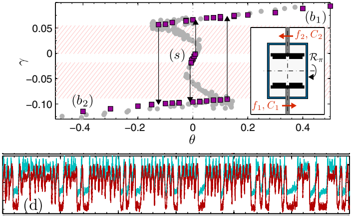
M. Scheffer et al. (2009). *Nature*

Bistability in turbulent flows



B. Saint-Michel, B. Dubrulle, L. Marié, F. Ravelet, and F. Daviaud (2013). *Phys. Rev. Lett.*

Bistability in turbulent flows



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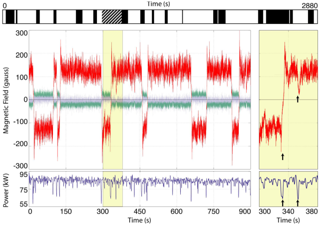
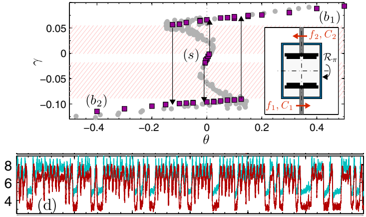


M. Berhanu et al. (2007). *EPL*

Bistability in turbulent flows



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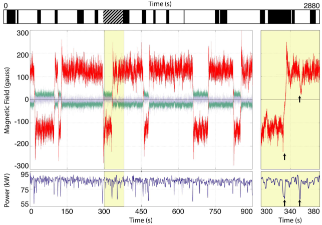

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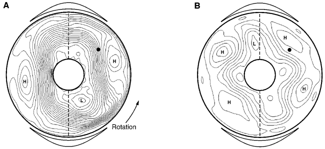

E. R. Weeks et al. (1997). *Science*

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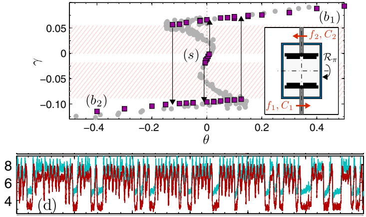

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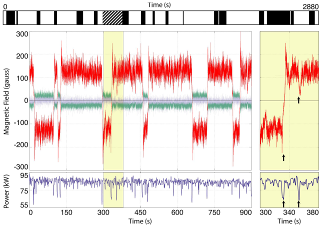

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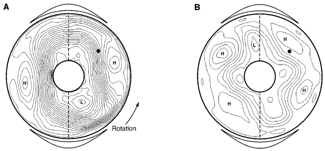

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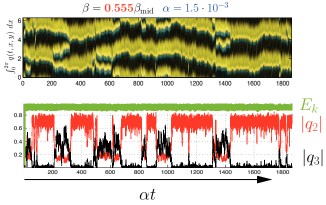


F. Bouchet, J. Rolland, and E. Simonnet (2019). *Phys. Rev. Lett.*

Main questions

General questions

- Does the general circulation admit multiple attractors? Can we compute them?
- What is the probability of transitions between climate attractors?
- Which dynamical mechanisms at play?
- Which aspects are predictible?

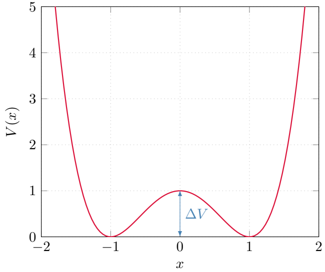
I will focus on a candidate for bistability of the general circulation of the atmosphere: *Superrotation*

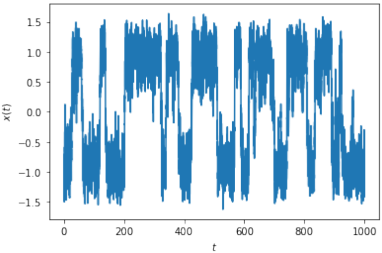
Outline

The Kramers problem[1]

Overdamped Langevin dynamics:

$$dX_t = -V'(X_t)dt + \sqrt{2\epsilon}dW_t, \quad V(x) = (x^2 - 1)^2.$$





[1] H. A. Kramers (1940). *Physica*.

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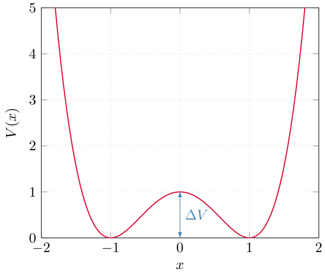

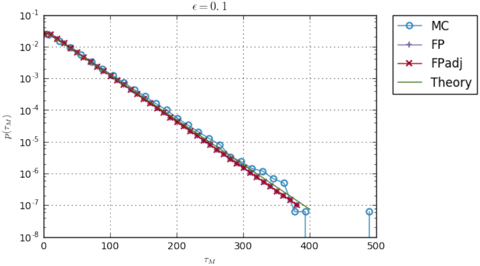

Transition probability

In the weak noise limit ($\epsilon \to 0$), transition times form a Poisson point process with transition rate $\lambda = \tau^{-1}e^{-\Delta V/\epsilon}$.

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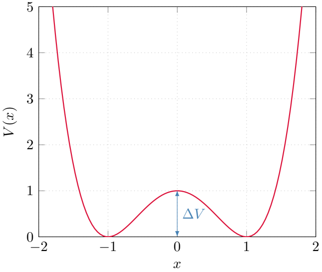

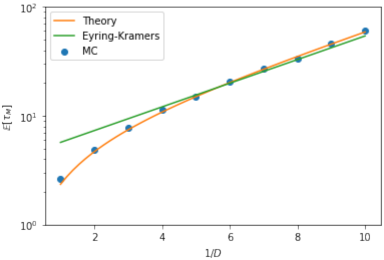

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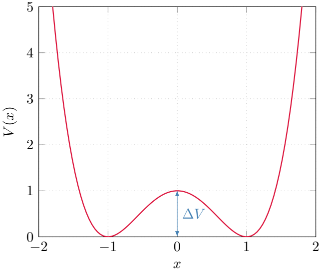


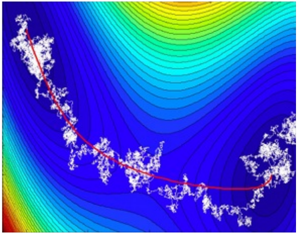


Fig. E. Vanden-Eijnden (Courant)

Instantons

Path integral formalism

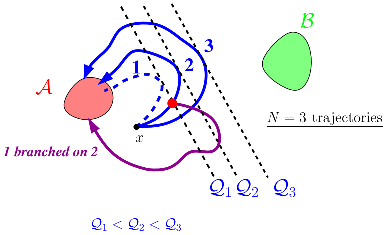
$$\mathbb{E}[\mathcal{O}] = \int \mathcal{D}[x]\mathcal{O}[x]\exp(-\mathcal{A}[x]/\epsilon), \quad \text{Action: } \mathcal{A}[x] = \frac{1}{4}\int dt(\dot{x} + V'(x))^2.$$

Instanton: most probable path: $\min_x\{\mathcal{A}[x] | x(-\infty) = -1, x(+\infty) = 1\}$.

[1] H. A. Kramers (1940). *Physica*.

Rare event algorithms

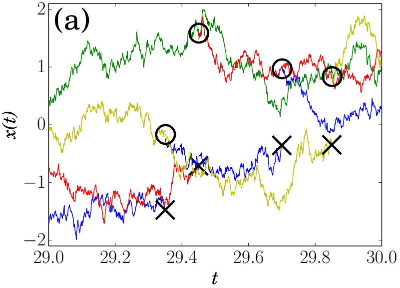
Adaptive Multilevel Splitting



F. Cérou and A. Guyader (2007). *Stoch. Anal. Appl.*

Figure from J. Rolland, F. Bouchet, and E. Simonnet (2016). *J. Stat. Phys.*

Population dynamics



C. Giardina, J. Kurchan, and L. Peliti (2006). *Phys. Rev. Lett.*

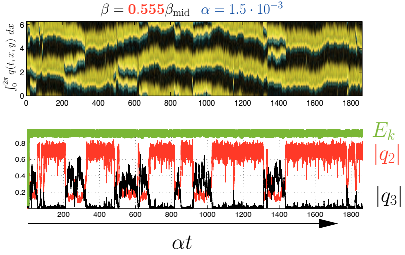
V. Lecomte and J. Tailleur (2007). *J. Stat. Mech.*

Statistics biased in a controlled way to favor the occurrence of the rare event of interest.

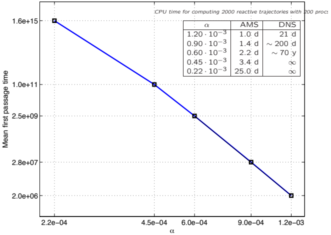
Rare transitions in jet dynamics[2]

Zonal jets in the stochastic barotropic vorticity equation:

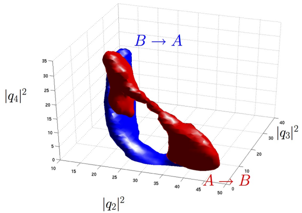
$$\partial_t \omega + \mathbf{u} \cdot \nabla \omega + \beta v = -\alpha \omega + \nu \Delta \omega$$



Arrhenius law:



"Instantons":



Efficient sampling of reactive trajectories with AMS algorithm.

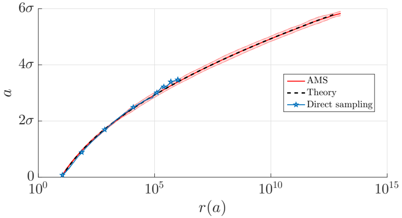
[2] F. Bouchet, J. Rolland, and E. Simonnet (2019). *Phys. Rev. Lett.*

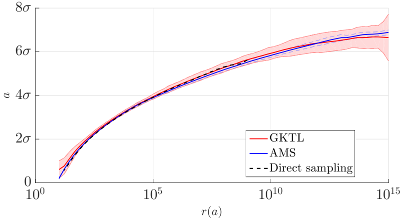
Return times: Ornstein-Uhlenbeck Process[3]

Return time of fluctuations of amplitude a for the Ornstein-Uhlenbeck process:

$$dX_t = -\alpha X_t dt + \sqrt{2\epsilon} dW_t. \qquad \text{(we use } \alpha = 1, \epsilon = 1/2\text{)}$$

- Instantaneous Observable: $A(X_t) = X_t$.
- Time-averaged Observable: $\overline{X}_T(t) = \frac{1}{T}\int_{t-T}^{t} X_s ds, \ t \in [T, T_a]$.





Instantaneous observable. $N = 100$, $T_a = 5\tau_c$, $a_{\max} = 7\sigma$, score function: $\xi(x, t) = x$. Theoretical formula:

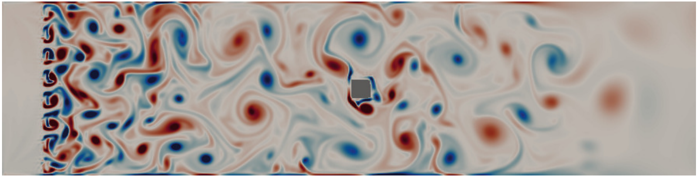
Time-averaged observable ($T = 10\tau_c$). $N = 100$, $T_a = 50$, $a_{\max} = 6.5\sigma_T$, $K = 10$ repetitions, and score function $\xi(t) = \bar{X}_T(t)$.

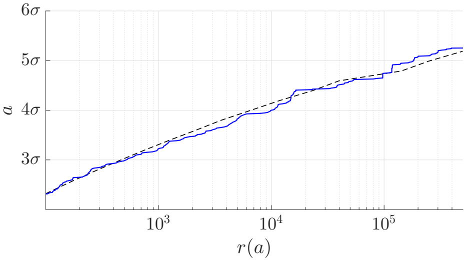
$$\mathbb{E}_s[\tau_a] = \sqrt{\frac{\alpha}{2\pi\epsilon^3}} \int_{-\infty}^{a} dy e^{\frac{\alpha y^2}{2\epsilon}} \left(\int_{-\infty}^{y} dz e^{-\frac{\alpha z^2}{2\epsilon}} \right)^2.$$

[3] T. Lestang, F. Ragone, C.-E. Bréhier, C. Herbert, and F. Bouchet (2018). *J. Stat. Mech.*

Return times: drag force in a turbulent flow[4]

Grid turbulence: 2D channel flow with square obstacle in the middle of the domain, simulated with a Lattice Boltzmann method.



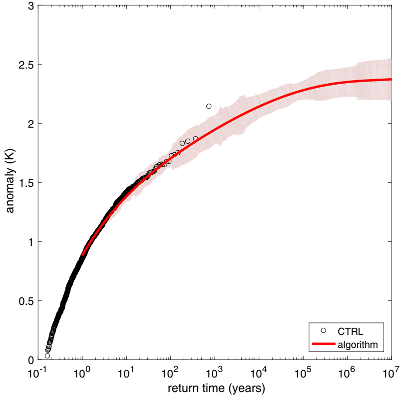


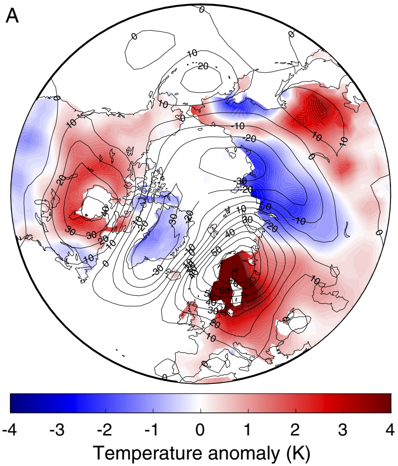
Return time plot for time-averaged drag ($5\tau_c$): GKTL (blue), $N = 128$, $T_a = 10\tau_c$, $K = 10$ (cost $10^4\tau_c$) and direct sampling (dashed, cost $10^6\tau_c$).

[4] T. Lestang, F. Ragone, C.-E. Bréhier, C. Herbert, and F. Bouchet (2018). *J. Stat. Mech.*

Heat Waves[6]

Return time plot for 90-day temperature anomaly over Europe, computed with the PLASIM model [5], using direct sampling (black, 1000 years) and the GKTL algorithm (red), at constant computational cost.





Average surface temperature (colors) and 500 hPa geopotential (contours) anomalies conditioned on the occurrence of heat wave conditions.

[5] K. Fraedrich, H Jansen, E Kirk, U Luksch, and F. Lunkeit (2005). *Meteorol. Z.*

[6] F. Ragone, J. Wouters, and F. Bouchet (2018). *Proc. Natl. Acad. Sci. U.S.A.*.

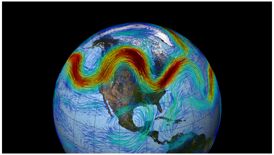
Outline

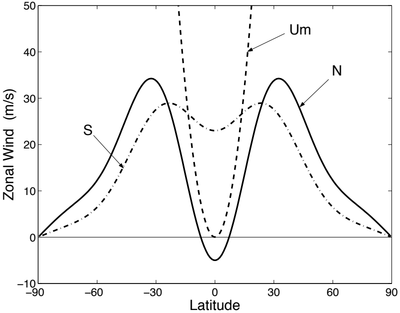
Superrotation: a robust phenomenon

Angular momentum:

$$M = a\cos\phi(\Omega a\cos\phi + u)$$

$$M > \Omega a^2 \iff u > \Omega a \frac{\sin^2\phi}{\cos\phi} \equiv U_m$$

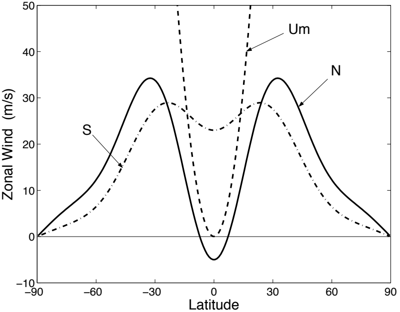




I. M. Held (1999). "Equatorial superrotation in Earth-like atmospheric models". In: *Bernhard Haurwitz Memorial Lecture*

Superrotation: a robust phenomenon

In the solar system[7]



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[7] P. L. Read and S. Lebonnois (2018). *Annu. Rev. Earth Planet. Sci.*

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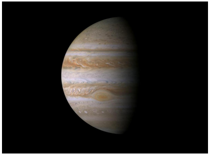

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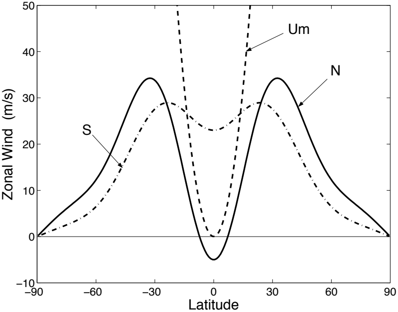

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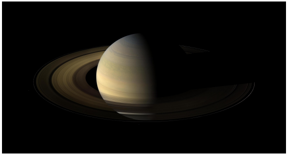

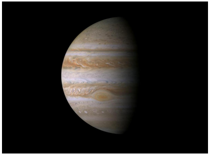

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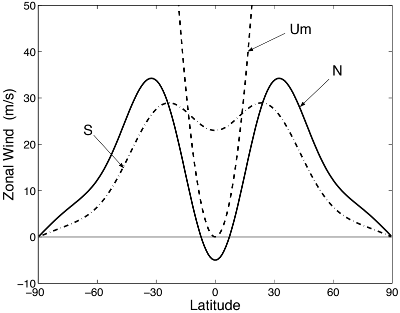

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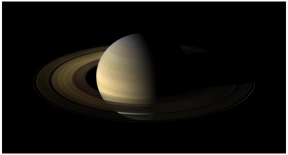

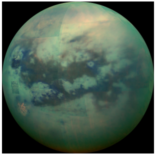

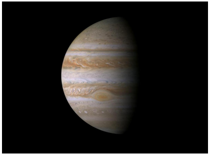

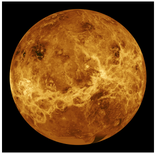

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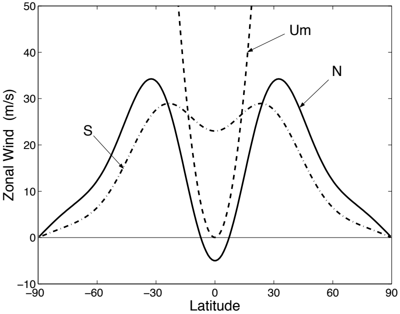

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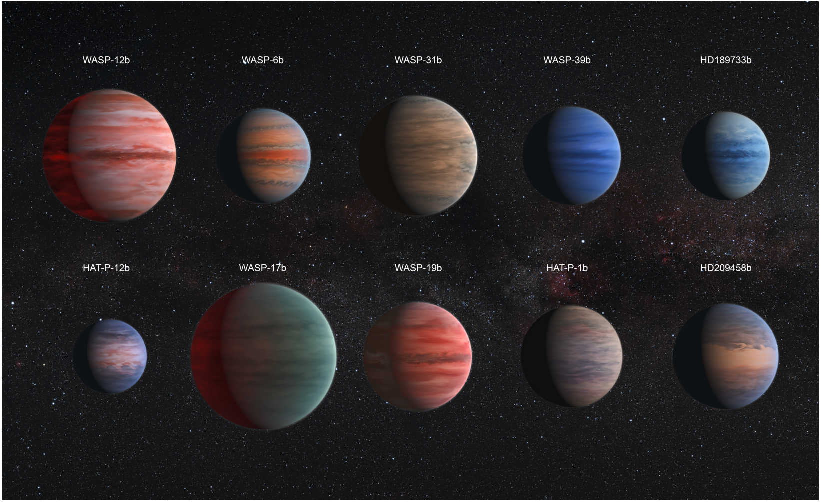

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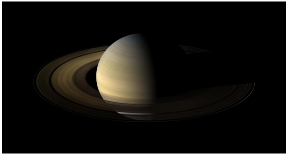
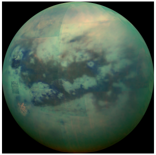
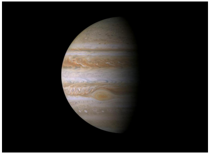
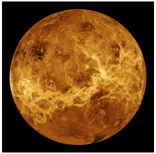
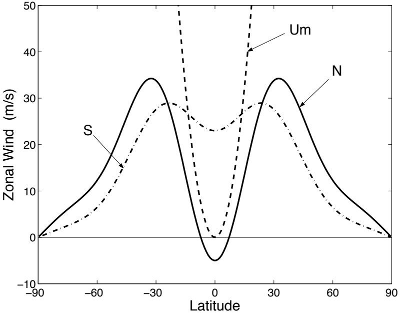




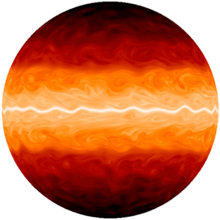




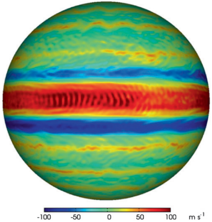


I. M. Held (1999). "Equatorial superrotation in Earth-like atmospheric models". In: *Bernhard Haurwitz Memorial Lecture*

Shallow-water equations[7]



Primitive equations[8]

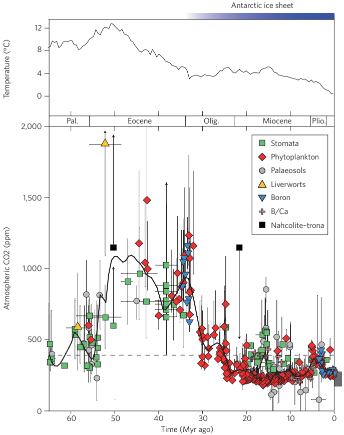


[7] R. K. Scott and L. M. Polvani (2008). *Geophys. Res. Lett.*

[8] T. Schneider and J. Liu (2009). *J. Atmos. Sci.*

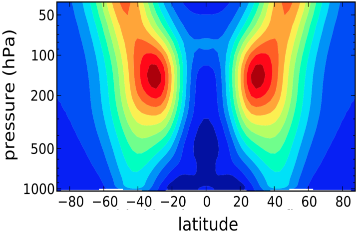
Superrotation in past climates

Was the atmosphere superrotating during the Eocene?

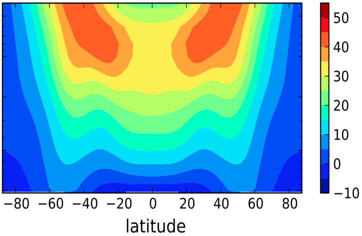


D. J. Beerling and D. L. Royer (2011). *Nature Geoscience*

COLD ($1xCO_2$)



HOT ($32xCO_2$)



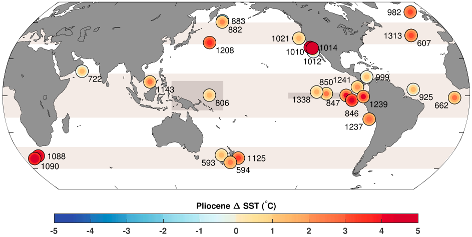
Zonal-mean zonal wind (CAM3).

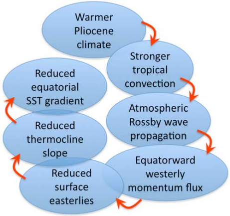
R. Caballero and M. Huber (2010). *Geophys. Res. Lett.*

Superrotation in past climates

Observations indicate a reduced Pacific SST gradient in the Pliocene.

Could superrotation alter El Niño?

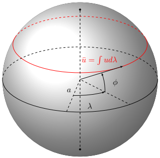




J. E. Tierney, A. M. Haywood, R. Feng, T. Bhattacharya, and B. L. Otto-Bliesner (2019). *Geophys. Res. Lett.*

E. Tziperman and B. Farrell (2009). *Paleoceanography*

Dynamical mechanisms and bistability

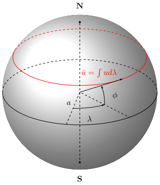


$$\frac{\partial \bar{u}}{\partial t} = 2\Omega \bar{v} \sin\phi - \frac{\bar{v}}{a\cos\phi}\frac{\partial(\bar{u}\cos\phi)}{\partial\phi} - \bar{\omega}\frac{\partial \bar{u}}{\partial p} + F + D \equiv \mathcal{F}[\bar{u}],$$

$$F = -\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\overline{u'v'\cos\phi} - \frac{\partial}{\partial p}\overline{u'\omega'}.$$

- Which dynamical mechanisms maintain the jet?

Dynamical mechanisms and bistability



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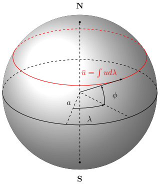
- Which dynamical mechanisms maintain the jet?

- Meridional mean circulation conserves angular momentum

Dynamical mechanisms and bistability



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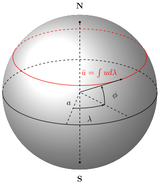
- Which dynamical mechanisms maintain the jet?

- Meridional mean circulation conserves angular momentum
- Dissipation mixes it

Dynamical mechanisms and bistability



$$\frac{\partial \bar{u}}{\partial t} = 2\Omega \bar{v} \sin\phi - \frac{\bar{v}}{a\cos\phi}\frac{\partial(\bar{u}\cos\phi)}{\partial\phi} - \bar{\omega}\frac{\partial \bar{u}}{\partial p} + F + D \equiv \mathcal{F}[\bar{u}],$$

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- Which dynamical mechanisms maintain the jet?

- Meridional mean circulation conserves angular momentum
- Dissipation mixes it
- Eddy momentum flux convergence ($F > 0$) is required!

Momentum convergence by Rossby waves

Localised sources of Rossby waves produce momentum convergence.

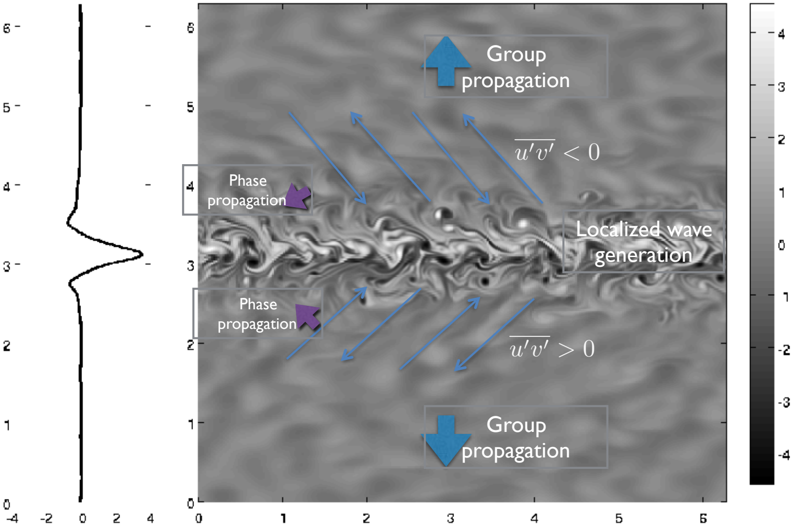


Image: W. Young.

Forced equatorial waves: Matsuno-Gill problem

Linearized shallow-water equations on an equatorial beta plane:

$$\begin{aligned} \partial_t u - \beta y v + g\partial_x h &= 0, \\ \partial_t v + \beta y u + g\partial_y h &= 0, \\ \partial_t h + H\partial_x u + H\partial_y v &= 0. \end{aligned}$$

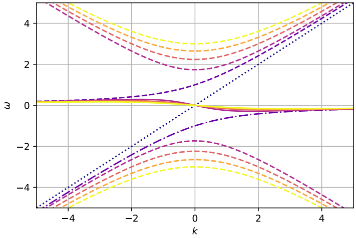
$$\text{or } \dot{X} + LX = 0 \qquad L^\dagger = -L,$$

Forced equatorial waves: Matsuno-Gill problem

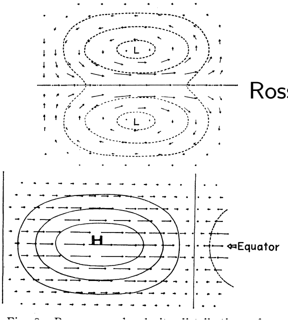
Linearized shallow-water equations on an equatorial beta plane:

$$\partial_t u - \beta y v + g\partial_x h = 0, \qquad \text{or } \dot{X} + LX = 0$$
$$\partial_t v + \beta y u + g\partial_y h = 0, \qquad L^\dagger = -L,$$
$$\partial_t h + H\partial_x u + H\partial_y v = 0.$$

Free modes $LX_{n,\ell} = i\omega_{n,\ell} X_{n,\ell}$



Dispersion relation $\omega_{n,\ell}(k)$.



Rossby $(n = 1)$

Kelvin

T. Matsuno (1966). *J. Meteor. Soc. Japan*; A. E. Gill (1980). *Q. J. R. Meteorol. Soc.*

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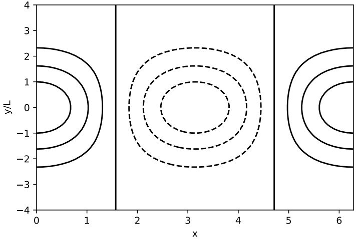
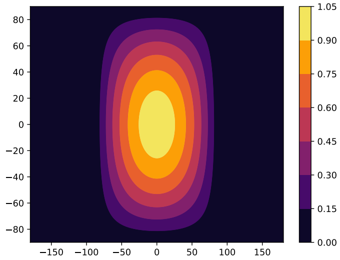
or $\dot{X} + LX = B - DX$

$$L^\dagger = -L,$$

Forcing



$$Q = Q_0 \cos(kx) e^{-y^2/2}.$$



Instellation for a planet in synchronous rotation.

Forced equatorial waves: Matsuno-Gill problem

Linearized shallow-water equations on an equatorial beta plane:

$$
\begin{aligned}
\partial_t u - \beta y v + g\partial_x h &= -\epsilon u, \\
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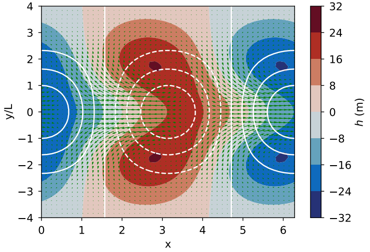
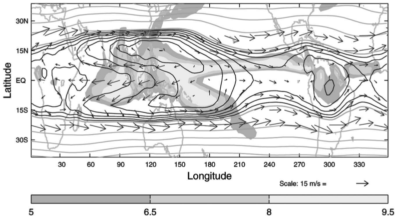
\end{aligned}
\qquad \text{or } \dot{X} + LX = B - DX, \quad L^\dagger = -L,
$$

Matsuno-Gill response: $X = (L + D)^{-1} B$.



Stationary height (colors) and velocity (arrows) perturbations for imposed forcing (white lines).



I. M. Dima, J. M. Wallace, and I Kraucunas (2005). *J. Atmos. Sci.*

Reanalysis (NCEP) 150-hPa annual-mean geopotential height (contours) and wind (arrows).

Forced equatorial waves: Matsuno-Gill problem

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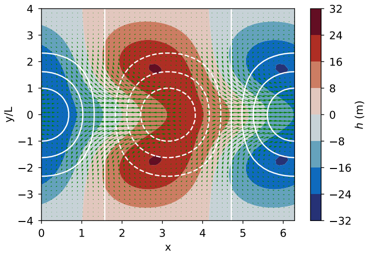
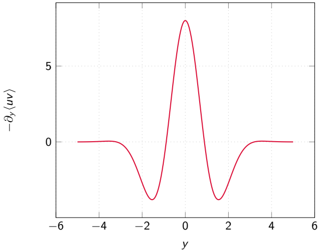


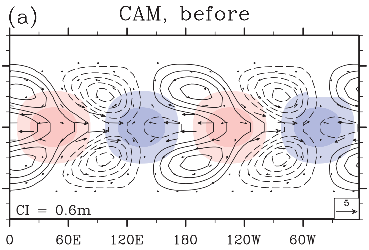
Stationary height (colors) and velocity (arrows) perturbations for imposed forcing (white lines).



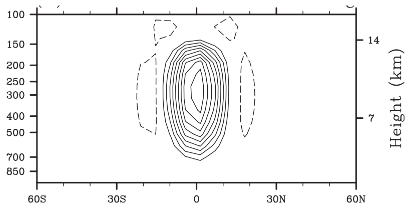
Reynolds stress $F = -\partial_y \overline{uv}$.

Forced equatorial waves: GCM results

General Circulation Model simulations:



200 hPa geopotential height (contours) and wind (arrows).



Zonally averaged zonal wind.

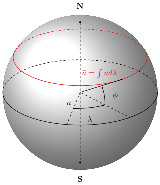
N. P. Arnold, E. Tziperman, and B. Farrell (2012). *J. Atmos. Sci.*

CAM3.1, T42, 26 levels. Held-Suarez forcing[9].

[9] I. M. Held and M. J. Suarez (1994). *Bull. Amer. Meteor. Soc.*

Outline

Dynamical mechanisms for bistability



$$\frac{\partial \bar{u}}{\partial t} = 2\Omega \bar{v} \sin\phi - \frac{\bar{v}}{a\cos\phi}\frac{\partial(\bar{u}\cos\phi)}{\partial\phi} - \bar{\omega}\frac{\partial \bar{u}}{\partial p} + F \equiv \mathcal{F}[\bar{u}],$$

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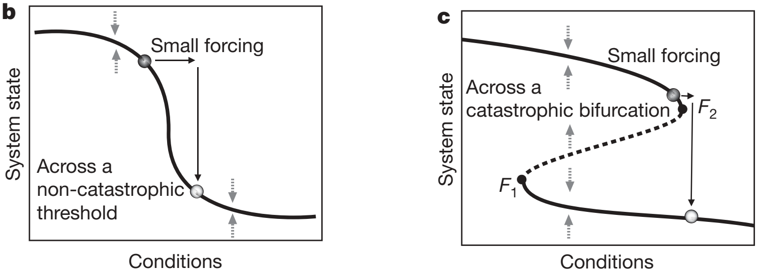
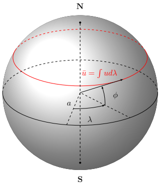
- Which dynamical mechanisms maintain the jet?
- Is the transition smooth or abrupt? Bistability requires a positive feedback: $\frac{\delta \mathcal{F}}{\delta \bar{u}} > 0$.



M. Scheffer et al. (2009). *Nature*

Dynamical mechanisms for bistability



$$\frac{\partial \bar{u}}{\partial t} = 2\Omega \bar{v} \sin\phi - \frac{\bar{v}}{a\cos\phi}\frac{\partial(\bar{u}\cos\phi)}{\partial\phi} - \bar{\omega}\frac{\partial \bar{u}}{\partial p} + F \equiv \mathcal{F}[\bar{u}],$$

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Potential feedback mechanisms

1. Hadley cell feedback
2. Wave-jet resonance

In which range of parameters are those positive feedback robust? Are they compatible? Are they robust to model complexity (will we see them in GCM and actual planets)?

The Hadley cell feedback: theory[10]

1D axisymmetric 1-1/2 layer shallow-water equations:

$$\frac{\partial u}{\partial t} + \frac{v}{a}\frac{\partial u}{\partial \phi} - \frac{uv\tan\phi}{a} = 2\Omega v\sin\phi + F + R - \epsilon u, \qquad R = -\bar{Q}u/h\Theta(\bar{Q}),$$

$$\frac{\partial v}{\partial t} + \frac{v}{a}\frac{\partial v}{\partial \phi} + \frac{u^2\tan\phi}{a} = -2\Omega u\sin\phi - \frac{g^*}{a}\frac{\partial h}{\partial \phi} - \epsilon v,$$

$$\frac{\partial h}{\partial t} + \frac{1}{a\cos\phi}\frac{\partial(hv\cos\phi)}{\partial \phi} = -\frac{h-h_{eq}}{\tau} \equiv \bar{Q}.$$

[10] K. M. Shell and I. M. Held (2004). *J. Atmos. Sci.*

The Hadley cell feedback: theory[10]

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Zonal acceleration budget at equator:

$$F - \epsilon u + \frac{u}{h}\frac{h - h_{eq}}{\tau} = 0.$$

[10] K. M. Shell and I. M. Held (2004). *J. Atmos. Sci.*

The Hadley cell feedback: theory[10]

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Zonal acceleration budget at equator:

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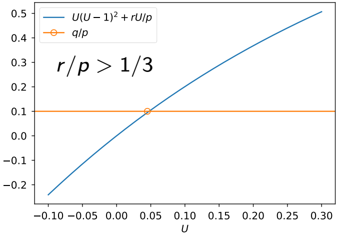
Layer thickness:

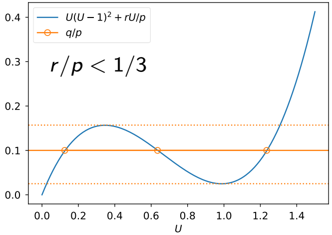
- Geostrophic balance with angular momentum conserving wind in the tropics
- Radiative equilibrium outside

$$h - h_{eq} = -\frac{5}{18g^*}(u_{eq} - u)^2.$$

[10] K. M. Shell and I. M. Held (2004). *J. Atmos. Sci.*

The Hadley cell feedback: theory[10]



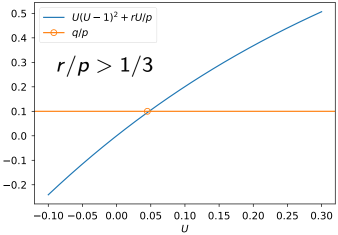


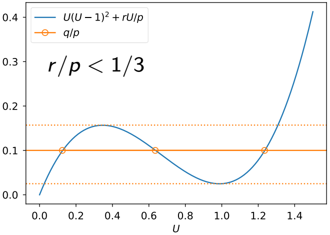
Zonal acceleration budget at equator:

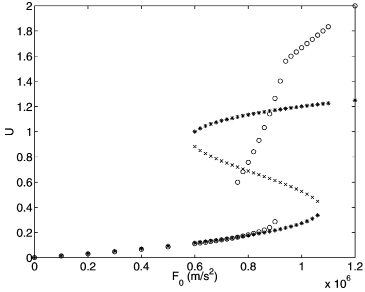
$$pU(U-1)^2 + rU - q = 0, \quad U = u/u_{eq}, \quad p = \frac{5u_{\text{eq}}^2}{18g^\star h_{\text{eq}}}, \quad q = \frac{F\tau}{u_{\text{eq}}}, \quad r = \epsilon\tau.$$

[10] K. M. Shell and I. M. Held (2004). *J. Atmos. Sci.*

The Hadley cell feedback: theory[10]







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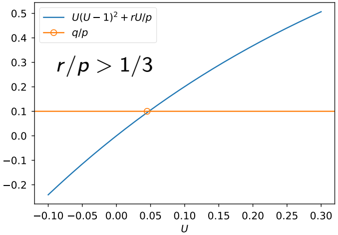
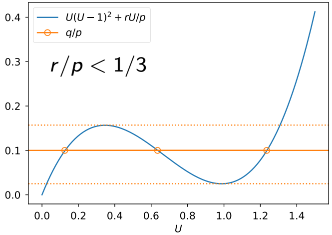
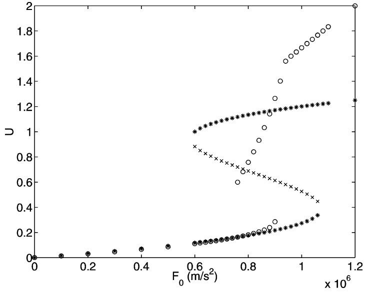




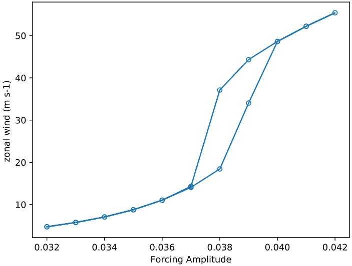


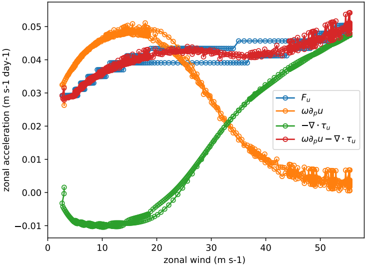
Can we obtain abrupt transitions through the Hadley cell positive feedback in a more complex model?

[10] K. M. Shell and I. M. Held (2004). *J. Atmos. Sci.*

Hadley cell driven bistability: numerical simulations

Axisymmetric primitive equations with prescribed *constant eddy forcing*.



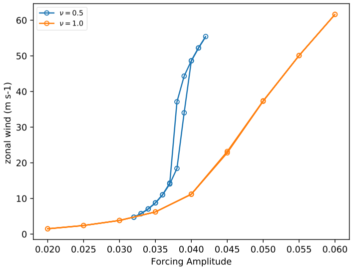


- We observe hysteresis in a 2D model (Shell and Held was 1D).

C. Herbert, R. Caballero, and F. Bouchet (2019). *J. Atmos. Sci.*

Hadley cell driven bistability: numerical simulations

Axisymmetric primitive equations with prescribed *constant eddy forcing*.



- We observe hysteresis in a 2D model (Shell and Held was 1D).
- However, strong sensitivity to vertical resolution and viscosity

C. Herbert, R. Caballero, and F. Bouchet (2019). *J. Atmos. Sci.*

Wave-jet resonance feedback: theory

Linearized shallow-water equations on an equatorial beta plane :

$$\partial_t u - \beta y v + g\partial_x h = -\epsilon u,$$
$$\partial_t v + \beta y u + g\partial_y h = -\epsilon v,$$
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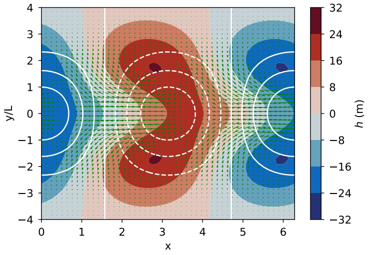

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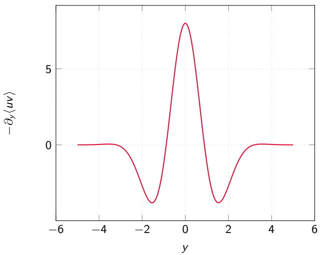


Reynolds stress $F = -\partial_y \overline{uv}$.

Wave-jet resonance feedback: theory

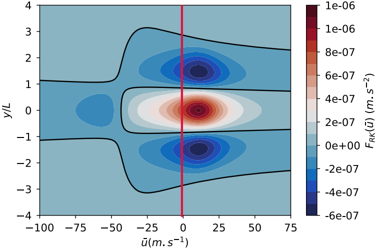
Linearized shallow-water equations on an equatorial beta plane with mean-flow U:

$$\begin{aligned} \partial_t u + U\partial_x u - \beta y v + g\partial_x h &= -\epsilon u, \\ \partial_t v + U\partial_x u + \beta y u + g\partial_y h &= -\epsilon v, \\ \partial_t h + U\partial_x h + H\partial_x u + H\partial_y v &= Q - h/\tau. \end{aligned}$$

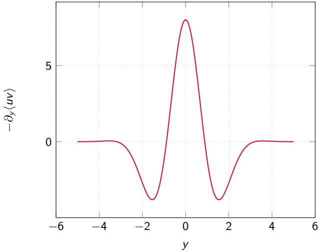
or $\dot{X} + LX + ikUX = B - DX$

$$L^\dagger = -L,$$

Matsuno-Gill response: $X = (L + D + ikU\mathrm{Id})^{-1} B = \sum_{n,\ell} \frac{b_{n,\ell}}{\epsilon + i(\omega_{n,\ell} + k\bar{u})} X_{n,\ell}(y)$.



Reynolds stress $F = -\partial_y \overline{uv}$.



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C. Herbert, R. Caballero, and F. Bouchet (2019). *J. Atmos. Sci.*

Wave-jet resonance feedback: theory

Linearized shallow-water equations on an equatorial beta plane :

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$$\partial_t v + U\partial_x u + \beta y u + g\partial_y h = -\epsilon v, \qquad L^\dagger = -L,$$
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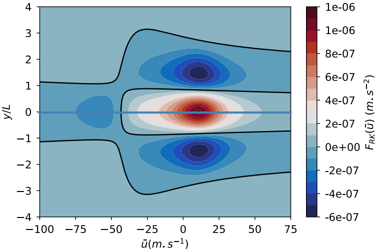

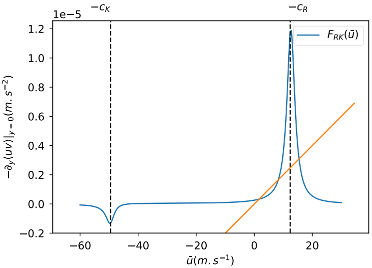

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C. Herbert, R. Caballero, and F. Bouchet (2019). *J. Atmos. Sci.*

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$$\partial_t h + U\partial_x h + H\partial_x u + H\partial_y v = Q - h/\tau.$$

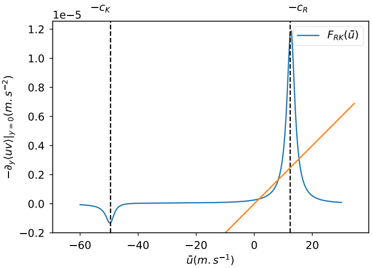
Matsuno-Gill response: $X = (L + D + ikU\mathrm{Id})^{-1} B = \sum_{n,\ell} \frac{b_{n,\ell}}{\epsilon + i(\omega_{n,\ell} + k\bar{u})} X_{n,\ell}(y).$

Reynolds stress at the equator:

$$F = \frac{Q_0^2 \epsilon k^2 (c_K - c_R)(2\bar{u} + c_K - 3c_R)}{12[\epsilon^2 + k^2(\bar{u} + c_R)^2][\epsilon^2 + k^2(\bar{u} + c_K)^2]}$$

Bistability condition: $kc_R/\epsilon \gg 1$.

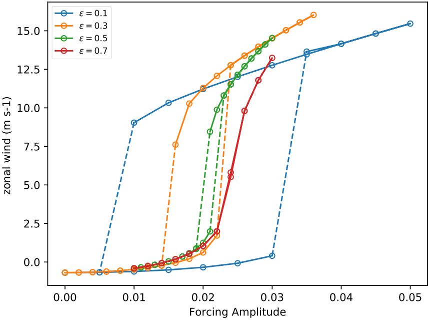
- Small Rossby number
- Large Reynolds number



C. Herbert, R. Caballero, and F. Bouchet (2019). *J. Atmos. Sci.*

Wave-jet resonance driven bistability: numerical simulations

Axisymmetric primitive equations with prescribed *resonant eddy forcing*.
Solved with Climt[11], 91×45 grid points.



Hysteresis experiments for different values of the resonance width ϵ.

C. Herbert, R. Caballero, and F. Bouchet (2019). *J. Atmos. Sci.*

As expected from theory, we observe hysteresis for small ϵ, with a velocity jump of order $\Delta U \approx c_R$.

[11] J. M. Monteiro, J. McGibbon, and R. Caballero (2018). *Geosci. Model Dev.*

Wave-jet resonance driven bistability: numerical simulations

Axisymmetric primitive equations with prescribed *resonant eddy forcing*.
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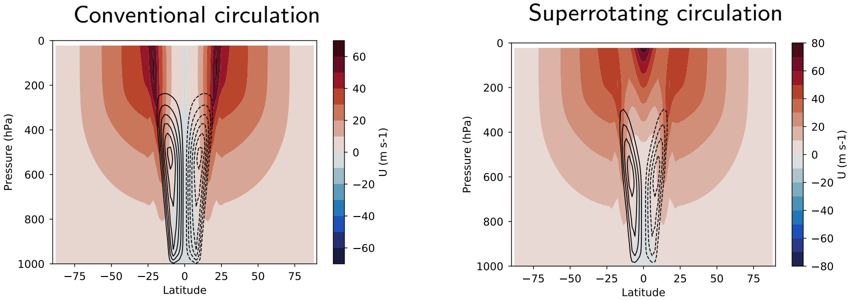

Zonal wind (color) and meridional stream function (contours).

C. Herbert, R. Caballero, and F. Bouchet (2019). *J. Atmos. Sci.*

The Hadley cell does not collapse in the superrotating state.

[11] J. M. Monteiro, J. McGibbon, and R. Caballero (2018). *Geosci. Model Dev.*

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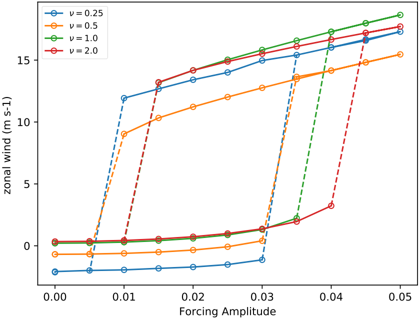

Hysteresis experiments for different values of the eddy viscosity ν.

C. Herbert, R. Caballero, and F. Bouchet (2019). *J. Atmos. Sci.*

Unlike Hadley cell driven bistability, abrupt transitions are robust to changes in viscosity.

[11] J. M. Monteiro, J. McGibbon, and R. Caballero (2018). *Geosci. Model Dev.*

Summary

- Well-defined theoretical framework for noise induced transitions in bistable systems.
- Rare event algorithms allow to study rare transitions in high-dimensional complex systems.
- *We have characterized theoretically the range of parameters for bistability and abrupt transitions to a superrotating state of the atmosphere.*
- Those results have been verified in a 2D axisymmetric primitive equation model.
- The Hadley-cell positive feedback by himself seems not robust and highly dependent on the dissipation.
- The positive feedback of the Rossby wave-jet resonance leads to a broad and robust bistability range.
- Superrotation without collapse of the Hadley cell is possible.

Perspectives

- Can these results be reproduced in a full GCM?
- Do they extend to other eddy momentum flux convergence mechanisms? E.g. Kelvin-Rossby instability[12]

[12] S. Iga and Y. Matsuda (2005). *J. Atmos. Sci.* P. Wang and J. L. Mitchell (2014). *Geophys. Res. Lett.* P. Zurita-Gotor and I. M. Held (2018). *J. Atmos. Sci.*

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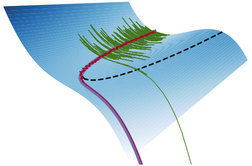
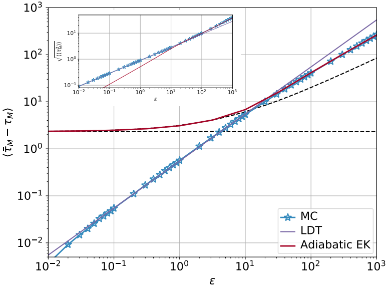
Zurita-Gotor, P. and I. M. Held (2018). *J. Atmos. Sci.* 75, pp. 2299–2316.

Escape in stochastic saddle-node bifurcation[13]

$$dx_t = (x_t^2 + t)dt + \sqrt{2\epsilon}dW_t, \qquad \tau_M = \inf\{t \geq t_0, x_t \geq M\}$$



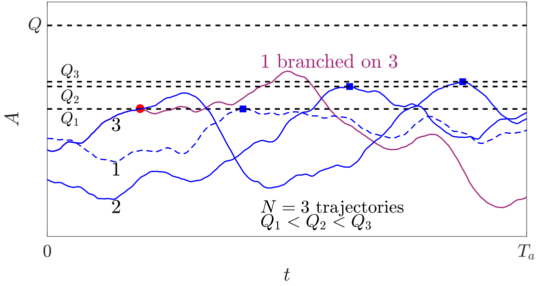


Competition between deterministic and stochastic effects.

[13] C. Herbert and F. Bouchet (2017). *Phys. Rev. E*.

Return times: The TAMS algorithm[14]

Ensemble $\{x_n^{(0)}(t)\}$ of N independent trajectories, duration T_a, weight $w_0 = 1$.



Selection-mutation steps, with *score function* ξ: at iteration $j \geq 1$,

- *Selection.* trajectory $n_j^\star$ with the lowest score:

$$\mathcal{Q}_{n_j^\star}^{(j)} = \min_{1 \leq n \leq N} \mathcal{Q}_n^{(j)}, \text{ with } \mathcal{Q}_n^{(j)} = \sup_{0 \leq t \leq T_a} \xi(t, x_n^{(j-1)}(t)).$$

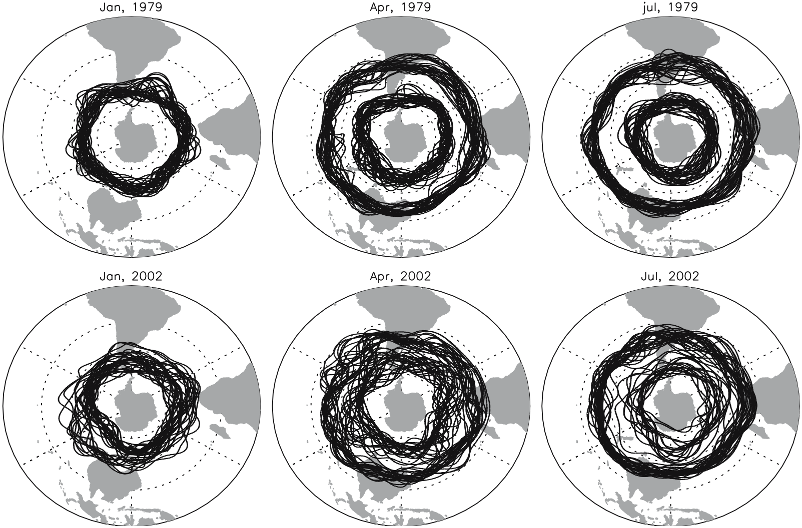
- *Mutation.* Resimulate the trajectory starting from the threshold $\mathcal{Q}_{n_j^\star}^{(j)}$.

  Associate weight $w_j = (1 - 1/N) w_{j-1}$ to the trajectories in the ensemble.

Iterate J times, or until all trajectories reach a fixed threshold $a_{\max}$. Yield $M = N + J$ trajectories with probabilities $p_m = w_m / \sum_m w_m$.

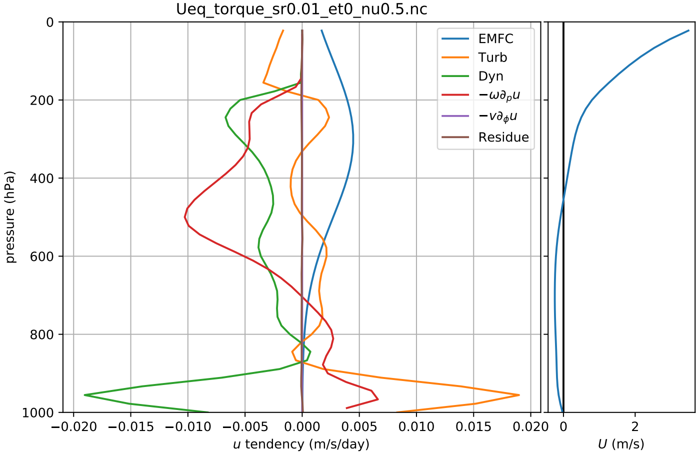
[14] T. Lestang, F. Ragone, C.-E. Bréhier, C. Herbert, and F. Bouchet (2018). *J. Stat. Mech.*

Jet Stream in the Southern Hemisphere[15]



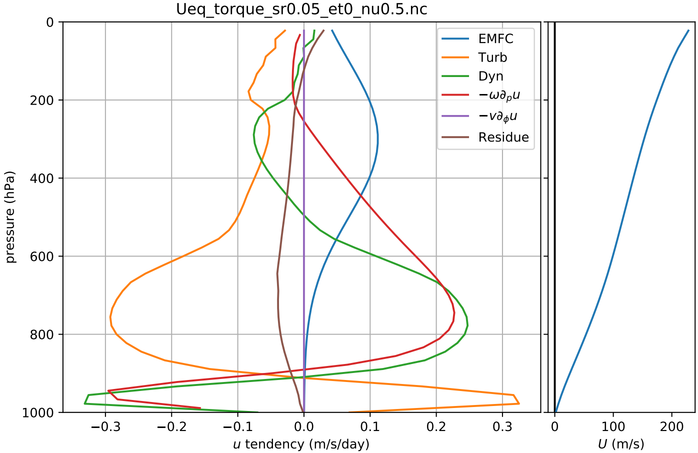
[15] D. Gallego, P. Ribera, R. Garcia-Herrera, E. Hernandez, and L. Gimeno (2005). *Clim. Dyn.*

Emergence of equatorial superrotation with constant forcing



Zonal acceleration budget

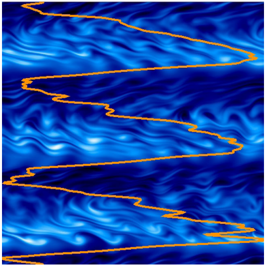
Emergence of equatorial superrotation with constant forcing



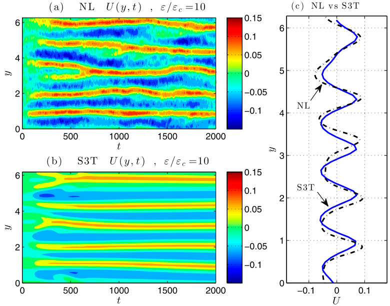
Zonal acceleration budget

The quasi-linear framework

The quasi-linear approximation works well for 2D turbulence[16], barotropic jets[17] or even full 3D GCMs[18].



S. M. Tobias and J. B. Marston (2013). *Phys. Rev. Lett.*



N. C. Constantinou, B. F. Farrell, and P. J. Ioannou (2014). *J. Atmos. Sci.*

[16] J. Laurie, G. Boffetta, G. Falkovich, I. Kolokolov, and V. Lebedev (2014). *Phys. Rev. Lett.* G. Falkovich (2016). *Proc. R. Soc. A*; I. V. Kolokolov and V. Lebedev (2016). *Phys. Rev. E*; A. Frishman (2017). *Phys. Fluids*.

[17] B. F. Farrell and P. J. Ioannou (2003). *J. Atmos. Sci.* J. B. Marston, E Conover, and T. Schneider (2008). *J. Atmos. Sci.* K. Srinivasan and W. R. Young (2012). *J. Atmos. Sci.* F. Bouchet, C. Nardini, and T. Tangarife (2013). *J. Stat. Phys.*

[18] T. Schneider and C. C. Walker (2006). *J. Atmos. Sci.* P. A. O'Gorman and T. Schneider (2007). *Geophys. Res. Lett.* F. Ait-Chaalal, T. Schneider, B. Meyer, and J. B. Marston (2016). *New J. Phys.*

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The quasi-linear approximation works well for 2D turbulence, barotropic jets or even full 3D GCMs.

$$\partial_t \bar{u} - \overline{v(\xi + \beta y)} = -\partial_y \overline{u'v'} - \epsilon \bar{u},$$
$$\partial_t u' + \bar{\mathbf{u}} \cdot \nabla u' + v' \partial_y \bar{u} = \beta y v' - g \partial_x h' - \epsilon u',$$
$$\partial_t v' + \bar{\mathbf{u}} \cdot \nabla v' + v' \partial_y \bar{v} = -\beta y u' - g \partial_y h' - \epsilon v',$$
$$\partial_t h' + \bar{u} \cdot \nabla h' + h' \partial_y \bar{v} + \bar{h} \nabla \cdot \mathbf{u}' + v' \partial_y \bar{h} = Q' - h'/\tau.$$

It remains extremely difficult to compute the mean flow profile and Reynolds stress tensor in general[a].

[a] J. Laurie, G. Boffetta, G. Falkovich, I. Kolokolov, and V. Lebedev (2014). *Phys. Rev. Lett.* E. Woillez and F. Bouchet (2017). *EPL*; A. Frishman and C. Herbert (2018). *Phys. Rev. Lett.* E. Woillez and F. Bouchet (2019). *J. Fluid Mech.*

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$$\begin{aligned}
\partial_t \bar{u} - \overline{v(\xi+\beta y)} &= -\partial_y \overline{u'v'} - \epsilon \bar{u},\\
\partial_t u' + \bar{\mathbf{u}} \cdot \nabla u' + v' \partial_y \bar{u} &= \beta y v' - g \partial_x h' - \epsilon u',\\
\partial_t v' + \bar{\mathbf{u}} \cdot \nabla v' + v' \partial_y \bar{v} &= -\beta y u' - g \partial_y h' - \epsilon v',\\
\partial_t h' + \bar{u} \cdot \nabla h' + h' \partial_y \bar{v} + \bar{h} \nabla \cdot \mathbf{u}' + v' \partial_y \bar{h} &= Q' - h'/\tau.
\end{aligned}$$

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Let us assume there is no meridional shear.

[a] J. Laurie, G. Boffetta, G. Falkovich, I. Kolokolov, and V. Lebedev (2014). *Phys. Rev. Lett.* E. Woillez and F. Bouchet (2017). *EPL*; A. Frishman and C. Herbert (2018). *Phys. Rev. Lett.* E. Woillez and F. Bouchet (2019). *J. Fluid Mech.*