Multi-scale modeling of precipitation extremes

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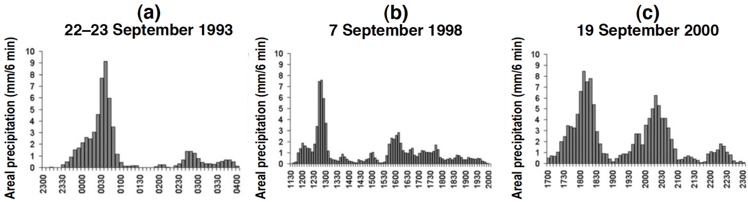
Motivation

- In all talks till now: extremes = extremes of processes considered at a unique temporal scale (eg. hourly, daily, monthly) and a unique spatial scale (e.g. scale of the raing gauges $\approx 100\text{cm}^2$).
- However, rainfall events do have a spatial extension and duration.
- Looking at hourly and rain gauge scales may not be the right scales to assess how extreme an event were.
- E.g. moderate but long-lasting rain: daily extreme, hourly usual. Also localized storms : more frequent (= less extreme) than storms affecting a whole region
- The "extremness" of an event depends on which temporal and spatial scale we look at it.

Illustration

Ramos et al. 2005



50y RP at hourly and very local scale (station)	not extreme at any scale	50y RP at 3h scale and up to 150km^2
Short and very localized event		**Persistent and extended event**

⇒ There is a need for a multi-scale description of the extremness of rain event

= temporal and spatial scales: how extreme it is for a given duration and area.

About the temporal and spatial scales

- Here: temporal scale = duration of the aggregation. $\neq$ Raphael's talk where it was the time.
- Here spatial scale = area of aggregation $\neq$ spatial in Raphael's, Marco's... talks.
- Warning, we will estimate the distribution of extremes for **each spatial aggregation**, and **each temporal aggregation**, independently on how long the rain event lasted, and how extended it was.

Aggregation

- Say we have maps of 1h rainfall over a region, at a given spatial resolution.
- We sum-up all consecutive maps → realizations of 2h-rainfall
- We sum-up all consecutive triplet of maps → realizations of 3h-rainfall
- ...
- For each of the temporally aggregated maps, we aggregate spatially over areas of 1km^2, 10km^2 etc.
- At the end, we obtain (lots of) realizations of duration-areal rainfall.
- With rain gages: interpolation step.

The problem

- $Y_{d,a}(x)$ intensity of rain falling over a region of area a centered on x, during duration d
- $Z_{d,a}(x)$ annual maximum : $Z_{d,a}(x) = \max_i Y_{d,a}^{(i)}(x)$
- Warning: $\max \int_A \neq \int_A \max$
- We want to estimate the distribution of each $Z_{d,a}(x)$.
- In other words, we want to be able to compute quantiles $q_{d,a}(x;p)$ for each d, a, x ($\rightarrow$ return level/return period)
- We expect to have "smooth" quantiles in the geographical space $\{x\}$, but also smooth quantiles in the duration-area-geographical space $\{d, a, x\}$.

Temporal scaling invariance for extreme rainfall

- We momentanely omit the "a" (area).
- How to relate $Z_d(x)$ and $Z_{d'}(x)$? Or how to relate 1h-extreme rainfall $Z_{d_{\text{ref}}}(x)$ to d-hour rainfall $Z_d(x)$.
- Gupta and Waymire, 1990: how to relate Y_d and $Y_{d_{\text{ref}}}(x)$?
  $\Rightarrow$ Concept of simple scaling:

$$Y_d(x) = \left(\frac{d}{d_{\text{ref}}}\right)^{-\nu} Y_{d_{\text{ref}}}(x),$$

  which implies that

$$Z_d(x) = \left(\frac{d}{d_{\text{ref}}}\right)^{-\nu} Z_{d_{\text{ref}}}(x)$$

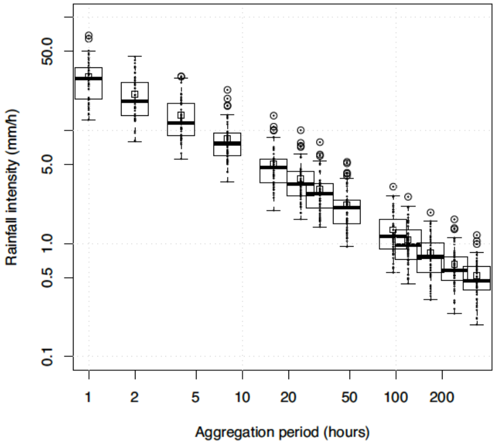
  and so that the IDF (intensity-duration-frequency) curves are such that:

$$q_d(x;p) = \left(\frac{d}{d_{\text{ref}}}\right)^{-\nu} q_{d_{\text{ref}}}(x;p)$$

Temporal scaling invariance for extreme rainfall: illustration

Ceresetti et al. 2011

Annual maxima for station Montpellier, 1920-1972:



Slope $= \hat{\nu}$

Temporal scaling within extreme value framework

- We assume that at hourly (d_{ref}) scale

$$Z_{d_{\text{ref}}}(x) \sim \text{GEV}\{\mu_{d_{\text{ref}}}(x), \sigma_{d_{\text{ref}}}(x), \xi(x)\}$$

- Then using temporal scaling invariance,

$$Z_d(x) \sim \text{GEV}\{\mu_d(x), \sigma_d(x), \xi(x)\}$$

with

$$\begin{aligned} \mu_{d,a}(x) &= \mu_{d_{ref}}(x) \times (d/d_{\text{ref}})^{-\nu(x)} \\ \sigma_{d,a}(x) &= \sigma_{d_{ref}}(x) \times (d/d_{\text{ref}})^{-\nu(x)} \end{aligned}$$

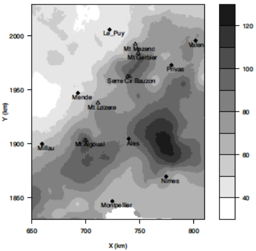
In Cevennes-Vivarais region

- This is used in Ceresetti et al. (2011) for extreme rainfall in Cevennes-Vivarais.
- They estimate the coefficients $\mu_{d_{ref}}(x)$, $\sigma_{d_{ref}}(x)$ and $\xi(x)$ locally + kriging
- Empirical estimates for $\nu(x)$ (slope in the plot of aggregated max) + kriging.

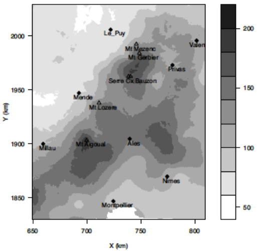
Return level maps

Map of rainfall depth (intensity $\times$ duration) for a return period of 100 years.

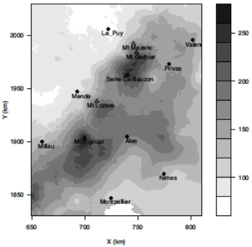
1h



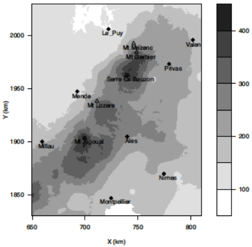
4h



8h

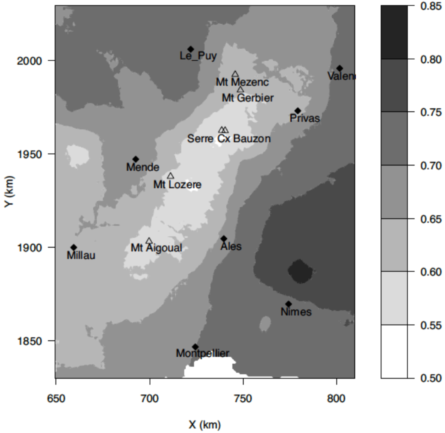


24h



Different duration, different localization of extremes

1h: foothills
24h: mountain ridge.
This is due to the spatial variation of the scaling exponent ν.



Spatio-temporal scaling invariance for extreme rainfall

- Following de Michele et al. (2001), we consider the following decomposition:

$$
\begin{aligned}
Z_{d,a}(x) &= \underbrace{Z_{d,a_{ref}}(x)}_{\substack{\text{rain gauge scale}\\ \text{for duration } d}} \times \underbrace{r_{d,a}}_{\substack{\text{Areal Reduction Factor}\\ \text{(ARF)}}} \\
&= \underbrace{Z_{d_{ref},a_{ref}}(x)}_{\substack{\text{rain gauge scale}\\ \text{hourly}}} \times \left(\frac{d}{d_{ref}}\right)^{-\nu} \times \underbrace{r_{d,a}}_{\substack{\text{Areal Reduction Factor}\\ \text{(ARF)}}} \\
&= Z_{d_{ref},a_{ref}}(x) \times \underbrace{\left(\frac{d}{d_{ref}}\right)^{-\nu} \times \left(1+\omega\frac{a^{\alpha}}{d^{\delta}}\right)^{-\nu/\beta}}_{c_{d,a}(\nu,\omega,\alpha,\delta,\beta)}
\end{aligned}
$$

- Equivalently,

$$q_{d,a}(x;p) = q_{d_{ref},a_{ref}}(x;p) \times c_{d,a}(\nu,\omega,\alpha,\delta,\beta)$$

Scaling invariance within extreme value framework

- We assume that at hourly (d_{ref}) rain gauge scale (a_{ref})

$$Z_{d_{\text{ref}},a_{\text{ref}}}(x) \sim \text{GEV}\{\mu_{d_{\text{ref}},a_{\text{ref}}}(x), \sigma_{d_{\text{ref}},a_{\text{ref}}}(x), \xi(x)\}$$

- Then using scaling invariance,

$$Z_{d,a}(x) \sim \text{GEV}\{\mu_{d,a}(x), \sigma_{d,a}(x), \xi(x)\}$$

with

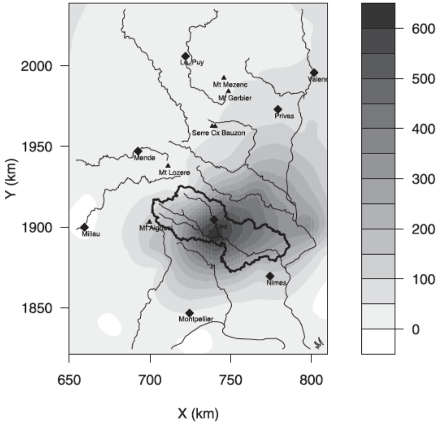
$$\begin{aligned}
\mu_{d,a}(x) &= \mu_{d_{ref},a_{ref}}(x) \times c_{d,a}(\nu, \omega, \alpha, \delta, \beta) \\
\sigma_{d,a}(x) &= \sigma_{d_{ref},a_{ref}}(x) \times c_{d,a}(\nu, \omega, \alpha, \delta, \beta)
\end{aligned}$$

In Cevennes-Vivarais region

- This is used in Ceresetti et al. (2012) for extreme rainfall in Cevennes-Vivarais.
- They estimate the coefficients $\mu_{d_{ref},a_{ref}}(x)$, $\sigma_{d_{ref},a_{ref}}(x)$ and $\xi(x)$ locally + kriging
- Empirical estimates for $\nu, \omega, \alpha, \delta, \beta$ with two regions: flat area and mountainous region.

Outch...

Event of 8 Sept. 2002.
12h accumulation:



According to the fitted model, the 12h-aggregated rainfall has a return period > 2000y for areas $> 10\text{km}^2$.

Going further

"Pure" scaling might be a too strong assumption.

$$Z_{d,a}(x) = Z_{d_{ref},a_{ref}}(x) \times c_{d,a}(\nu, \omega, \alpha, \delta, \beta).$$

$\Rightarrow$ Introduce stochasticity (= uncertainty) in the model within a hierachical framework, e.g.

$$\begin{aligned} \mu_{d_{\text{ref}},a_{\text{ref}}}(.) &= \mu_0 + \varepsilon_\mu(.), \\ \log \sigma_{d_{\text{ref}},a_{\text{ref}}}(.) &= \log \sigma_0 + \varepsilon_\sigma(.), \end{aligned}$$

with $\varepsilon_\mu(.)$ and $\varepsilon_\sigma(.)$ 0-mean GP, and e.g.

$$\nu(.) = \nu_0 + \varepsilon_\nu(.).$$

Plus & Minus

Good points:

- integrated approach (all in one step, no kriging)
- gives model uncertainties
- 'lighter' assumptions of scaling.

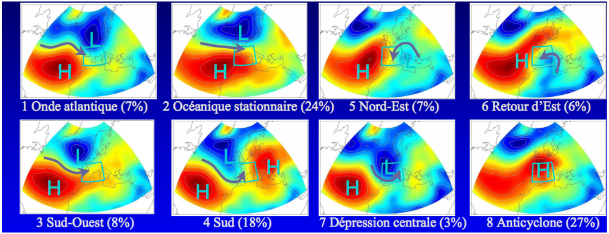
Bad points:

- no physics $\rightarrow$ brings very few in terms of understanding.
- the modeling assumes independent extremes $Z_{d,a}(x)$.

Adding physics: using weather types?

Idea: **Conditioning on weather types**

Cf Emmanuel Paquet's talk.



Ok at daily scale but might not be relevant at short-time scale.

Finding weather types: aleady not easy in the usual daily case, even more difficult here: $d + a$...

Adding physics: covariates?

Idea: **To link the model parameters to some covariates**, e.g.

$$\mu_{d_{\text{ref}},a_{\text{ref}}}(.) = f_\mu(\text{covariates}) + \varepsilon_\mu(.)$$
$$\log \nu(.) = f_\nu(\text{covariates}) + \varepsilon_\nu(.)$$

$\rightarrow$ Which covariates?

Covariates for the Mediterranean region (Kallache et al. 2011, Tramblay et al. 2011, 2012, 2013, ...): sea level pressure, principal components of fields of sea level pressure, geopotential height at 850 hPa, principal components of fields of geopotential height at 850 hPa., maximum temperature (tmax), wind velocity, wind direction at 10 hPa, the difference of wind direction at 10 hPa and 850 hPa, the difference of wind velocity at 10 hPa and 850 hPa, the difference of vertical wind intensity at 100 hPa and 850 hPa, wind velocity, wind direction, vertical wind intensity at 850 hPa and at 100 hPa., time averaged large-scale precipitation, humidity flux, ...

Many possibilities... how to select them?

Variable selection = open problem in EVA in general, even more difficult here:

$x+d+a$...

Modelling dependence?

There is obviously a dependence on x, but also on d and a.

- We could consider $Z(.) = \{Z_{d,a}(x), (d, A, x) \in S\}$ as a max-stable process on S.
- For this we would need to define a distance on S.
- However, what is the distance between $(12\text{h}, 10\text{km}^2, x_1)$ and $(24\text{h}, 100\text{km}^2, x_2)$???
- This issue of distance actually also applies on $\{x\}$ itself: is the Euclidean distance relevant? How to consider a more 'climatic' distance (Cooley et al. 2007, Blanchet and Davison 2012)?

  $\Rightarrow$ also a question of variable selection?

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Is this worth just to get the quantiles?