Conditional Modelling of Extreme Wind Gusts by Bivariate Brown-Resnick Processes

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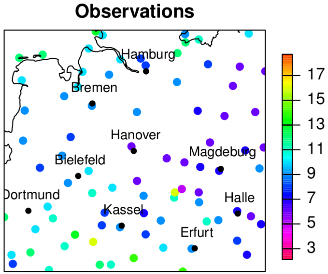
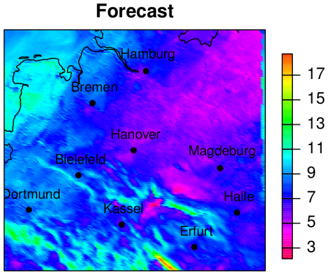
Extreme Wind Gusts

- wind gusts are strongly varying in space
- high uncertainty in forecasts, particularly for extreme wind gusts





Extreme Wind Gusts

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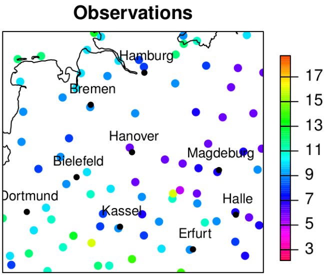
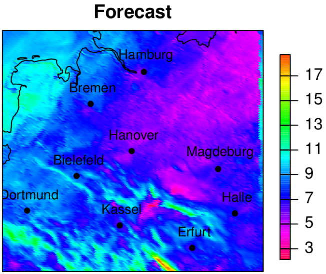
Goal:

Model for the (observed) wind gusts V_{max}^{obs} conditional on the forecast V_{max}^{pred}





Outline

1. General Ideas
2. Brown-Resnick Processes
3. Application to Data

Step 1: standardize distribution at each station

- model $V_{\text{max}}^{\text{obs}}$ and $V_{\text{max}}^{\text{pred}}$ by GEV distribution

$$\mathbb{P}(V \leq x) = \exp\left(- \left(1 + \xi \frac{x - \mu}{\sigma}\right)^{-1/\xi} \right)$$

for $1 + \xi \frac{x-\mu}{\sigma} > 0$

- transform to standard Gumbel margins

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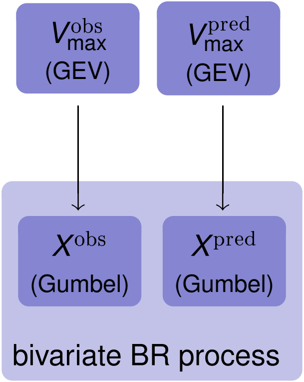
Step 2: bivariate Brown-Resnick model for standardized observation and forecast

$\rightsquigarrow$ flexible modelling of

- dependence in space
- dependence between observations and forecast



Marginal Model

wind speed $V(l, d)$ at location l and day d:

$$V(l, d) =_d s(l, d) V_0 + m(l, d)$$

V_0 : standardized distribution

m : mean

s : std. deviation

(m and s:) depend on general weather situation

Marginal Model

wind speed $V(l, d)$ at location l and day d:

$$V(l,d) =_d s(l,d) V_0 + m(l,d)$$

maximal wind speed $V_{\max}(l, d)$ at location l and day d:

$$\mathbb{P}\left(\frac{V_{\max}(l,d) - m(l,d)}{s(l,d)} \leq x\right) \approx \exp\left(-\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-1/\xi}\right)$$

for $1 + \xi\frac{x-\mu}{\sigma} > 0$

GEV parameters:

- ξ constant in space in time
- error model allows μ, σ to vary spatially

Marginal Model (cont'd)

Observations

$$\frac{V_{\max}^{\mathrm{obs}}(l,d) - m(l,d)}{s(l,d)} \sim GEV(\xi^{\mathrm{obs}}, \mu^{\mathrm{obs}}, \sigma^{\mathrm{obs}})$$

Forecast

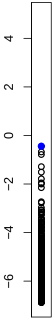
$$\frac{V_{\max}^{\mathrm{pred}}(l,d) - m(l,d)}{s(l,d)} \sim GEV(\xi^{\mathrm{pred}}, \mu^{\mathrm{pred}}, \sigma^{\mathrm{pred}})$$

Parameter Estimation:

- "weather parameters" (m, s): estimated via mean wind forecast
- GEV parameters (ξ, μ, σ): fitted to wind gust data

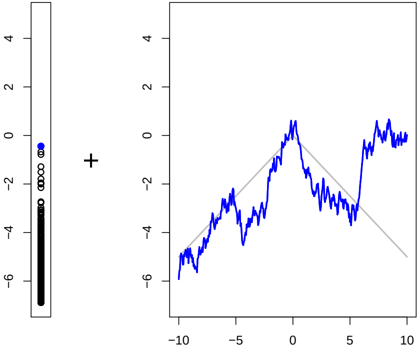
Brown-Resnick Processes (Brown and Resnick 1977)

- $\{U_k\}_{k\in\mathbb{N}}$: Poisson point process w. intensity $e^{-u}\,\mathrm{d}u$ (magnitudes)



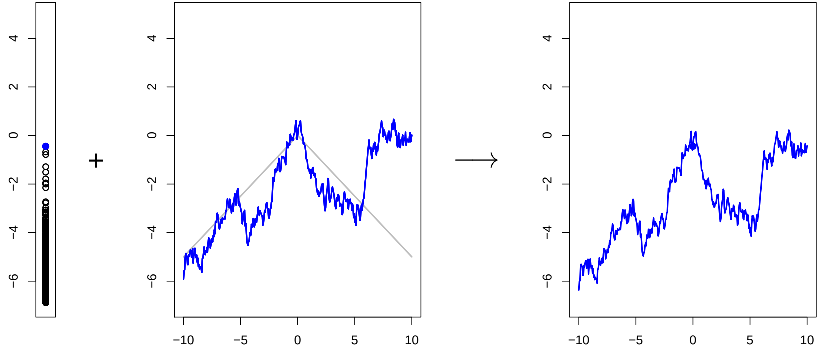
Brown-Resnick Processes (Brown and Resnick 1977)

- $\{U_k\}_{k\in\mathbb{N}}$: Poisson point process w. intensity $e^{-u}\,\mathrm{d}u$ (magnitudes)
- $\{W_k(\cdot)\}_{k\in\mathbb{N}}$: i.i.d. standard Brownian motions
  $W_k(\cdot) - |\cdot|/2$: Brownian motions with trend (spatial profile)



Brown-Resnick Processes (Brown and Resnick 1977)

$$X(t) = \max_{k \in \mathbb{N}} \left(U_k + W_k(t) - \frac{|t|}{2} \right), \quad t \in \mathbb{R}$$



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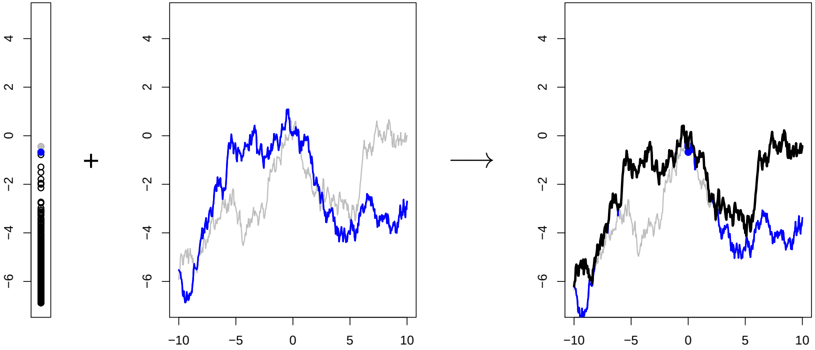

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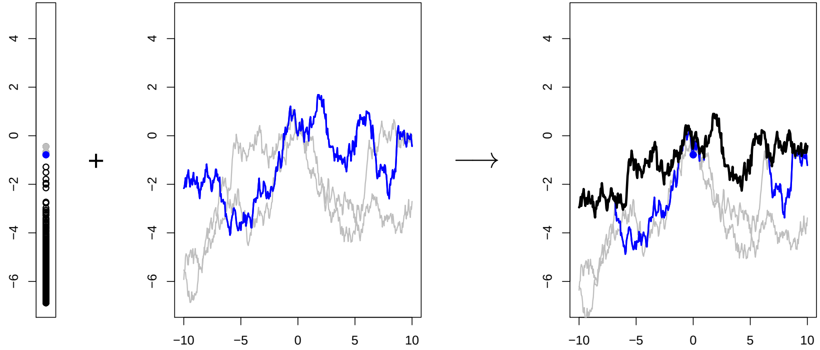

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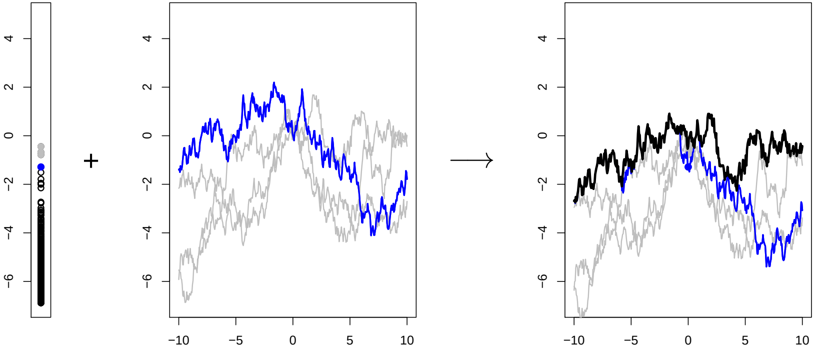

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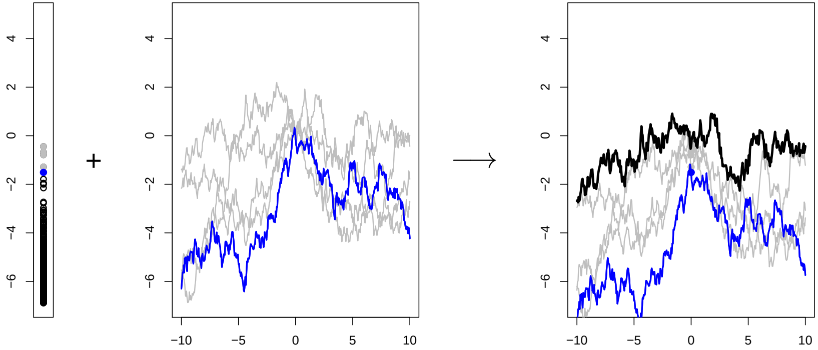

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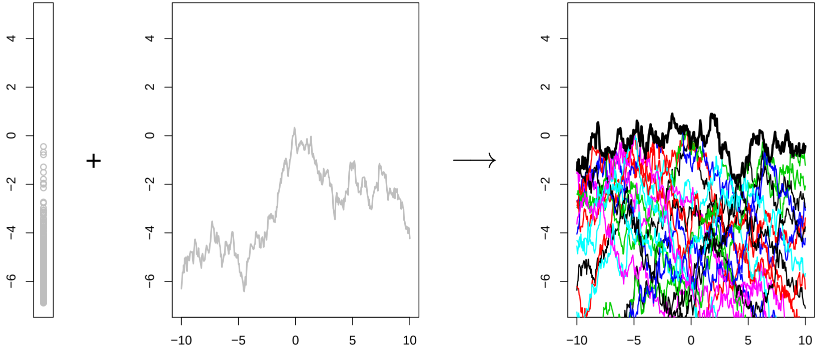

X is max-stable and stationary!

Generalization

(cf. Kabluchko, Schlather & de Haan, 2009, Stucki & Molchanov, 2013)

- $\{U_k\}_{k\in\mathbb{N}}$: Poisson point process with intensity $e^{-u}\,\mathrm{d}u$
- $W(\cdot) = \left(W^{(1)}(\cdot), W^{(2)}(\cdot)\right)$: centered Gaussian process s.t.

pseudo-variogram

$$\gamma(s,t) = (\mathrm{Var}(W^{(i)}(s) - W^{(j)}(t)))_{i,j=1,2}$$

depends on $s - t \in \mathbb{R}^d$ only

- $\{W_k(\cdot)\}_{k\in\mathbb{N}}$: independent copies of W

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$$X^{(i)}(t) = \max_{k\in\mathbb{N}} \left(U_k + W_k^{(i)}(t) - \mathrm{Var}(W_k^{(i)}(t))/2 \right), \quad t \in \mathbb{R}^d,\ i = 1,2,$$

Then,

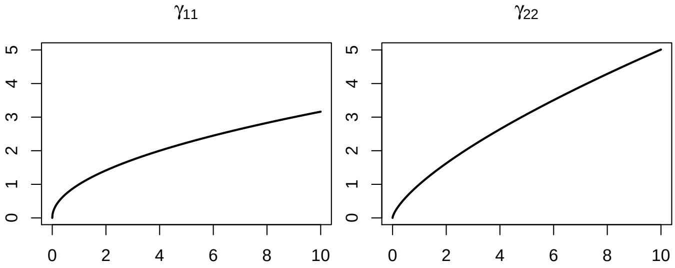
- X is max-stable and stationary (as bivariate process)
- law of X depends on γ only

What does a pseudo-variogram look like?

$$\gamma(s,t) = (\mathrm{Var}(W^{(i)}(s) - W^{(j)}(t)))_{1\leq i,j\leq 2}$$

Question: Can a pseudo-variogram have the form

$$\gamma(t+h,t) = \begin{pmatrix} \|h\|^{\alpha} & ? \\ ? & \|h\|^{\beta} \end{pmatrix}, \quad 0 < \alpha \neq \beta \leq 2?$$



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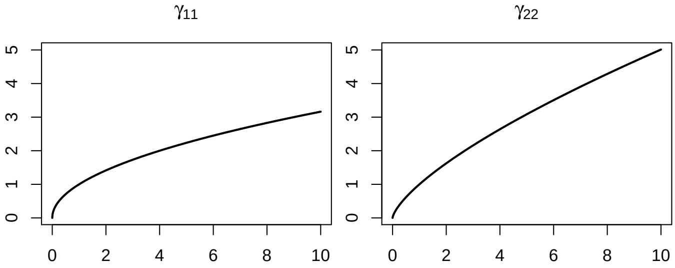


Answer: No!

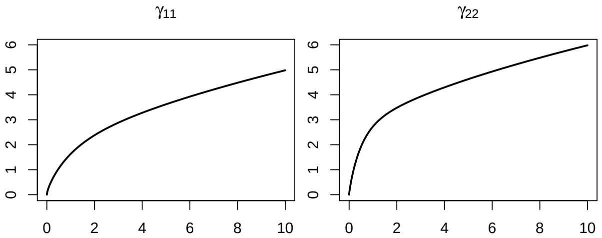
What does a pseudo-variogram look like? (cont'd)

Theorem

Let $\gamma(s,t)$ a pseudo-variogram that depends on $s-t$ only.
Then, γ is of the form

$$\gamma(t+h,t) = \begin{pmatrix} \gamma^*(h) & \gamma^*(h) \\ \gamma^*(h) & \gamma^*(h) \end{pmatrix} + \begin{pmatrix} f_{11}(h) & f_{12}(h) \\ f_{21}(h) & f_{22}(h) \end{pmatrix}, \qquad t,h \in \mathbb{R}^d,$$

for some univariate variogram γ^* and bounded functions $(f_{ij}(\cdot))_{1\leq i,j\leq 2}$.



Construction Principle

- $U(\cdot)$: univariate Gaussian process with stationary increments and variogram γ^*
- $V(\cdot) = (V^{(1)}(\cdot), V^{(2)}(\cdot))$: bivariate stationary Gaussian process with covariance fctn. $C(h) = \begin{pmatrix} C_{11}(h) & C_{12}(h) \\ C_{21}(h) & C_{22}(h) \end{pmatrix}$

$W(\cdot) = (U(\cdot) + V^{(1)}(\cdot), U(\cdot) + V^{(2)}(\cdot))$ has pseudo-variogram

$$\gamma(h) = \left(\gamma^*(h) + \frac{1}{2}C_{ii}(0) + \frac{1}{2}C_{jj}(0) - C_{ij}(h)\right)_{i,j=1,2}.$$

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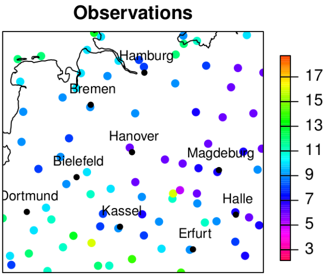
Example:

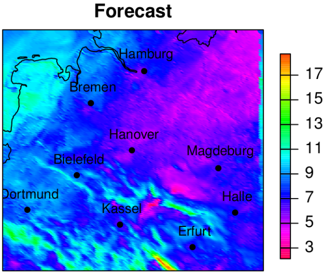
γ^*: $\gamma^*(h) = \frac{\|h\|^2}{(1 + \|h\|^2)^\beta}, \quad \beta \in (0,1)$

C: bivariate Matérn model (Gneiting et al., 2010)

The Data

- observed maximal wind speed at 110 DWD stations in Northern Germany on 360 days (03/2011 - 02/2012) $\rightsquigarrow V_{\max}^{\mathrm{obs}}$
- predictions from COSMO-DE EPS
  - for the maximal wind speed $\rightsquigarrow V_{\max}^{\mathrm{pred}}$
  - for the mean wind speed $\rightsquigarrow$ covariates for GEV parameters





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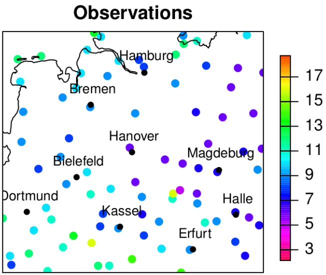
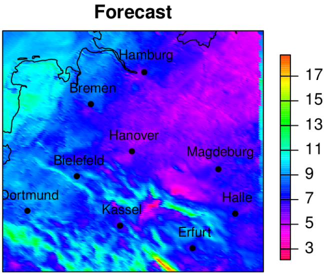
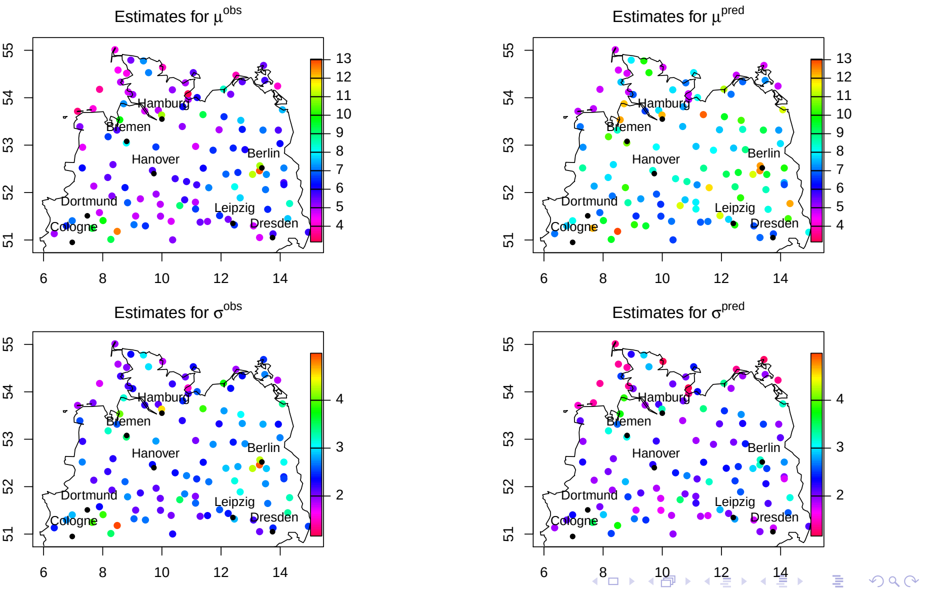




Model Fitting:

- GEV parameters: maximum likelihood
- pseudo-variogram for BR process: via extremal coefficient function

GEV parameters

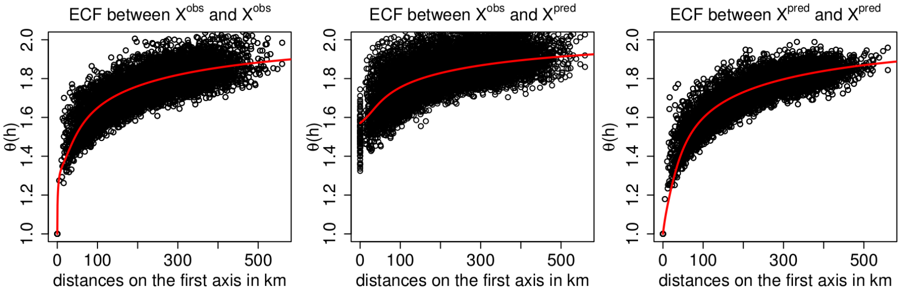


Extremal Coefficient Function

(cf. Schlather and Tawn 2003, Cooley, Naveau, Poncet 2009)

ECF between X^{obs} and X^{obs}

$$\mathbb{P}(X^{\text{obs}}(h) \leq x, X^{\text{obs}}(0) \leq x) = \mathbb{P}(X^{\text{obs}}(0) \leq x)^{\theta(h)}$$



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ECF between X^{obs} and X^{pred}

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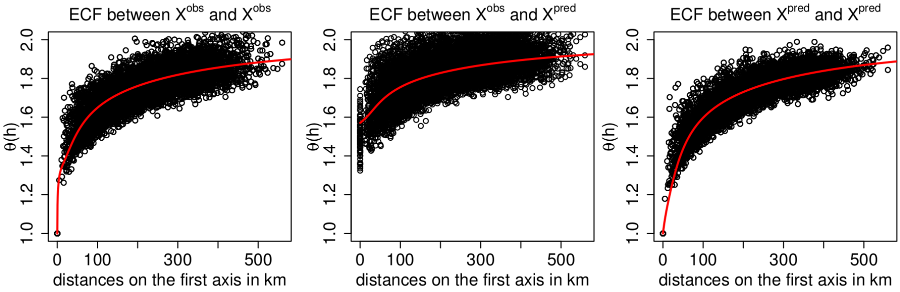

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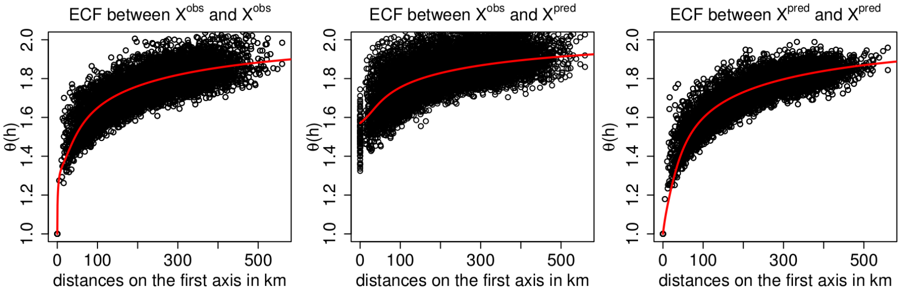

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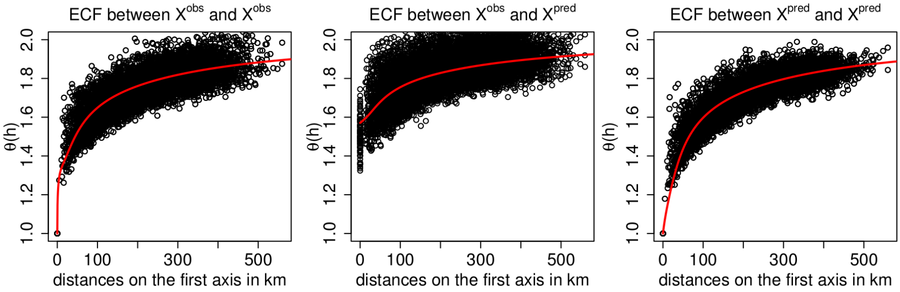
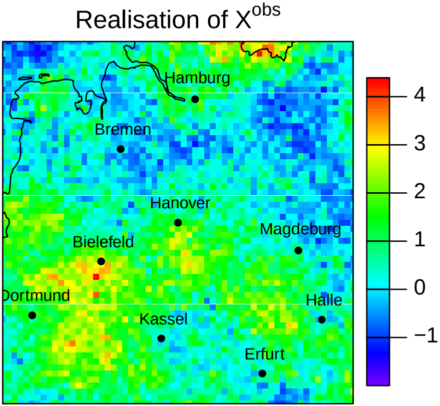
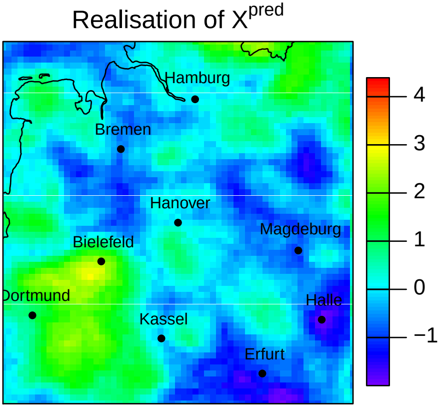


Fit ECF of bivariate BR process to estimated ECF!

Unconditional simulation of the Brown-Resnick process:





Summary:

- bivariate Brown-Resnick processes model
  - dependence in space
  - dependence between forecast and observation
- flexible modelling in spite of restricted class of pseudo-variograms

Outlook/Open Questions:

- simulation of observations conditional on forecast
  $\rightsquigarrow$ conditional simulation of bivariate Brown-Resnick processes
- downscaling (how to choose GEV parameters?)

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