

1 **Multivariate**

2 **– inter-variable, spatial and temporal –**

3 **bias correction**

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ABSTRACT

6
7 Statistical methods to bias correct global or regional climate model output are now common
8 to get data closer to observations in distribution. However, most bias correction (BC) meth-
9 ods work for one variable and one location at a time and basically reproduce the temporal
10 structure of the models. The inter-variable, spatial and temporal dependencies of the cor-
11 rected data are usually poor compared to observations. Here, we propose a novel method for
12 multivariate BC. The empirical copula - bias correction (EC-BC) combines a 1-dimensional
13 BC with a shuffling technique that restores an empirical multidimensional copula. Several
14 BC methods are investigated and compared to high-resolution reference data over the French
15 Mediterranean basin, notably: (i) a 1d-BC method applied independently to precipitation
16 and temperature fields, (ii) a recent conditional correction approach developed for producing
17 correct two-dimensional inter-variable structures, (iii) the EC-BC method.

18 Assessments are realized in terms of inter-variable, spatial and temporal dependencies and
19 an objective evaluation using the integrated quadratic distance (IQD) is presented. As ex-
20 pected, the 1d methods cannot produce correct multidimensional properties. The conditional
21 technique appears efficient for inter-variable properties but not for spatial and temporal de-
22 pendencies. EC-BC provides realistic dependencies in all respects, inter-variable, spatial and
23 temporal. The IQD results are clearly in favour of EC-BC. As many BC methods, EC-BC
24 relies on a stationarity assumption and is only able to reproduce patterns inherited from his-
25 torical data. However, due to its easiness of coding, its speed of application and the quality
26 of its results, the EC-BC method is a very good candidate for all needs in multivariate bias
27 correction.

1. Introduction

The use of simulations from climate or meteorological models at large or regional scales is now common in many impact studies, such as hydrological, environmental, or economic studies among others, or more generally in studies on consequences of climate change and adaptation. Although those simulations provide much useful information, they are in general not directly comparable to observations: for example, many observations are point measurements, whereas simulated data represent volume integrated dynamical variables. Moreover, simulated data are associated with potential biases in the sense their statistical distribution differs from the distribution of the observations. This is partly due to the fact that Global Climate Models (GCMs) have too low a spatial resolution to be employed directly in most of the impact models (e.g., Meehl 2007; Christensen et al. 2008). Regional Climate Models (RCMs) reduce some of the biases but not those unrelated to spatial resolution (Maraun 2013; White and Toumi 2013). Statistical bias correction methods – correcting the distribution (e.g., the cumulative distribution function) – are then commonly applied to transform the simulated data into new data with no, or at least fewer, statistical biases with respect to reference, generally observed, time series. In general, there is no clear distinction between a change of support problem (i.e. down- or upscaling), and bias correction.

In all the following, capital letters – e.g., X – represent random variables, while small letters – e.g., x – are used for realizations or values of a random variable. The most employed bias correction (BC) methods are based on quantile-association. The most famous is certainly the so-called quantile-mapping approach (Panofsky and Brier 1958; Haddad and Rosenfeld 1997; Wood et al. 2004; Déqué 2007; Piani et al. 2010; Gudmundsson et al. 2012), trying to

50 map a modeled value x (with a cumulative distribution function – CDF – F_X) to an observed
51 value y (with a CDF F_Y) through a function f , such that their distributions are equivalent
52 (Piani et al. 2010b):

$$y = h(x) \quad \text{such that} \quad F_Y(y) = F_X(x). \quad (1)$$

53 This mapping function h can be derived from distributions, regression-like transformations,
54 in both cases either parametric or non-parametric (see, e.g., Gudmundsson et al. 2012, for
55 some details). A very popular distribution-derived non-parametric approach (e.g., Déqué
56 2007) directly uses the constraint $F_X(x) = F_Y(y)$ to derive the corrected value y from the
57 modeled value x through the so-called "Empirical Quantile Mapping" (EQM):

$$y = F_Y^{-1}(F_X(x)) \quad (2)$$

58 where F^{-1} is the inverse function of the CDF F , both modeled non-parametrically.

59 One major issue of such quantile mapping methods and their variants (e.g., Michelangeli
60 et al. 2009) is that they are essentially univariate: they work only for one variable at a time,
61 one location at a time and basically reproduce the temporal structure of the climate models.
62 Hence, although the resulting marginal (i.e., one-dimensional) statistical distributions of the
63 corrected data are improved, those one-dimensional techniques suffer from various limita-
64 tions. Among the latter, one major limitation for many impact studies is that, because they
65 are applied to one location at a time, the spatial and temporal structures of the corrected
66 time series are misrepresented (Colette et al. 2012; Maraun 2013) and basically correspond
67 to the structures of the model to be corrected. This leads to potentially significant inade-
68 quacies when used as forcing, for example in a hydrological model where spatialization and
69 chronology of the input rainfall are of importance.

70 Moreover, as most of the BC methods correct one variable at a time – e.g., temperature
71 is corrected separately and independently from precipitation –, the corrected variables can
72 be inconsistent between each other and then generate unrealistic situations (e.g., Chen et al.
73 2011; Muerth et al. 2013).

74 Such (spatial, temporal and inter-variable) issues also appear when BC is applied to
75 de-bias GCM outputs prior to downscaling with regional climate models. Although Colette
76 et al. (2012) and White and Toumi (2013) showed that such a “prior” correction of the
77 large-scale inputs for RCMs with a quantile-association based method clearly improves the
78 quality of the RCM simulations, White and Toumi (2013) found that it can nevertheless
79 produce undesirable results in the RCM simulations.

80 Recently, efforts have been made to improve or create BC models that solve (some of)
81 those issues. Piani and Haerter (2012) developed a BC methodology to bypass the problem
82 of physical consistency between two variables (e.g., temperature and precipitation) to be
83 corrected. Their approach consists in applying a univariate BC to the time series of one
84 variable (e.g., precipitation) conditionally on the bias corrected values of the time series
85 for the other variable (e.g., temperature). Their results show the clear improvement of the
86 temperature-precipitation dependence representation with respect to the traditional separate
87 univariate temperature and precipitation bias corrections.

88 Furthermore, to overcome the lack of realistic spatial variability and temporal persistence
89 in precipitation and temperature fields simulated by a numerical weather prediction (NWP)
90 model, Clark et al. (2004) presented a method for reordering NWP outputs to recover the
91 space-time variability. In this approach, each time series is ranked and matched with ob-
92 servation data. The element of the time series are then shuffled to match the original order

93 of the historical dataset. Based on this shuffling technique, Clark et al. (2004) correctly
94 reconstructed the space-time variability of forecasted precipitation and temperature fields.
95 This technique has seen great success in hydrological applications, e.g. for flood forecasts
96 (Voisin et al. 2010, 2011), to construct ensemble forecasts from single-value forecasts of pre-
97 cipitation and temperature (Schaake et al. 2007), or for ensemble post-processing (Verkade
98 et al. 2013; Robertson et al. 2013). The ensemble copula coupling (ECC) is an adaptation
99 thereof to multivariate ensemble postprocessing (Scheffzik et al. 2013a; Möller et al. 2012;
100 Schuhen et al. 2012; Thorarinsdottir et al. 2013b). Related methods are also described in
101 Johnson and Bowler (2009), Pinson (2012) and Roulin and Vannitsem (2012). A recent ar-
102 ticle by Wilks (2014) compares the Schaake shuffle and the ECC in the context of ensemble
103 post-processing. To the best of our knowledge, the shuffle technique has not yet been applied
104 for the purpose of multivariate bias correction or downscaling of climate simulations.

105 The main objective of this article is to promote a technique that is readily available and
106 easy to apply. This technique will be referred to as the “empirical copula - bias correction”
107 (EC-BC) approach and combines a univariate BC method with the shuffling technique pre-
108 sented by Clark et al. (2004). We further provide an intercomparison of this method with a
109 one-dimensional BC method and the conditional approach of Piani and Haerter (2012).

110 This article is organized as follows: In the next section, the data to be corrected and
111 the reference data are first presented, as well as the experimental cross-validation setup. In
112 section 3, a short description of the 1d-bias correction method used as a benchmark in this
113 study is provided. Then, theoretical and technical details are given concerning the bivariate
114 and multivariate bias correction methods compared in this article in Section 4, namely: the
115 “conditional” technique, the shuffling-based method and the EC-BC approach. Section 5

116 contains the results of the intercomparison in terms of inter-variable, spatial and temporal
117 analyses. Finally, general conclusions are given in section 6 as well as a discussion concerning
118 the underlying assumptions and some potential adaptations of the various approaches.

119 **2. Reference and model data**

120 In this article, the reference data are daily temperature and precipitation time series
121 from the SAFRAN reanalysis data (Quintana-Segui et al. 2008) over the south-west region
122 of France $[2^{\circ}E, 7.5^{\circ}E] \times [42^{\circ}N, 45^{\circ}N]$ corresponding to 1506 continental grid-cells with an
123 approximate 8×8 km spatial resolution. Fig. 1(a) displays the map of France with the region
124 of interest in a box, as well as the mean cumulated annual precipitation (fig. 1(b)) and the
125 mean daily temperature (fig. 1(c)). The SAFRAN dataset allows one to avoid gaps in the
126 time series. It has been employed as a reference for evaluation of different statistical or
127 dynamical downscaling approaches in various studies (e.g., Lavaysse et al. 2012; Vrac et al.
128 2012). A detailed description of SAFRAN, its validation and its application over France is
129 given by Quintana-Segui et al. (2008).

130 Model data to be corrected are the ERA-Interim (hereafter ERA-I) daily reanalysis tem-
131 perature and precipitation data with a 0.75° by 0.75° spatial resolution. Using an improved
132 atmospheric model and assimilation system from those used in ERA-40 (Simmons and Gib-
133 son 2000), ERA-Interim represents a third generation reanalysis system (Dee et al. 2011).
134 ERA-Interim reanalyses are now widely employed (e.g., Vautard and 25 authors 2013) and
135 serve as meteorological forcing of the downscaling models involved in the “COordinated

136 Regional Downscaling EXperiment” (CORDEX) initiative¹.

137 For both model and reference datasets, data have been extracted from Jan., 1, 1980 to
138 Dec., 31, 2009. Then, each ERA-I grid-cell has been Co-located with the SAFRAN grid-cell
139 the closest to its center. Hence, each ERA-I grid-cell time series to be corrected has a unique
140 reference SAFRAN grid-cell.

141 Moreover, in the following, all bias correction methods are applied separately to two
142 periods of the year : from October 15th to April 14th (hereafter referred to as ”winter”) and
143 from April 15th to October 14th (hereafter referred to as ”summer”).

144 The calibration of the following BC methods is performed over the period 1980-1994 and
145 all evaluations are performed over the period 1995-2009.

146 3. Univariate bias correction

147 A variant of EQM has been recently developed by Michelangeli et al. (2009) and applied
148 in many climate-related studies (e.g., Oettli et al. 2011; Colette et al. 2012; Tisseuil et al.
149 2012; Vrac et al. 2012; Vigaud et al. 2013, among others). This variant first estimates
150 the distributions F_{Yp} and F_{Xp} for the random variables Y and X over the *projection* time
151 period (either future, or simply evaluation time period) before applying a distribution-derived
152 quantile mapping as defined in Eq. (2) in replacing X and Y by Xp and Yp respectively.
153 If F_{Xp} can be directly modeled – parametrically or not – from the data to be corrected in
154 the projection period, the modeling of F_{Yp} is based on the assumption that a mathematical

¹<http://wcrp-cordex.ipsl.jussieu.fr/>

155 transformation T allows to go from F_X to F_Y in the calibration period:

$$T(F_X(z)) = F_Y(z) \tag{3}$$

156 for any z in the domain of X and Y ; and that T is still valid in the projection period, i.e.:

$$T(F_{X_p}(z)) = F_{Y_p}(z). \tag{4}$$

157 Replacing z by $F_X^{-1}(u)$ in (3), where u is any probability in $[0, 1]$, we obtain

$$T(u) = F_Y(F_X^{-1}(u)), \tag{5}$$

158 corresponding to a simple definition for T . Inserting (5) in (4) leads to a modeling of F_{Y_p} :

$$F_{Y_p}(z) = F_Y(F_X^{-1}(F_{X_p}(z))). \tag{6}$$

159 Once F_{X_p} and then F_{Y_p} are modeled, a distribution-based quantile-mapping is applied as in
160 (2). Hence, this so-called "Cumulative Distribution Function - transform" (CDFt) approach
161 – as named by Michelangeli et al. (2009) – includes the information about the distributions
162 over the projection time period in the quantile-mapping technique. Some more details about
163 CDFt can be found in Vrac et al. (2012).

164 In the following, only the CDFt univariate bias correction approach will be applied.
165 Indeed, preliminary analyses showed that EQM and CDFt display equivalent results in the
166 context of the present study. Although this has not been tested, we strongly expect other
167 univariate bias correction techniques (parametric or not, distribution-based or not) to behave
168 relatively similarly. Hence, the univariate BC method CDFt is first applied independently to
169 precipitation (PR) and 2m-temperature (T2) from ERA-I. This will provide the benchmark
170 bias corrected ERA-I dataset to which some bivariate or multivariate correction procedures
171 will be compared.

172 4. Bivariate / Multivariate bias correction

173 *a. A short reminder on statistical dependence and copulas*

174 The notion of (spatial, temporal or inter-variable) dependence structure is in close re-
175 lationship with the so-called copula functions (e.g., Nelsen 2006). An introduction of the
176 copula approach for climate research is given, e.g., in Schoelzel and Friederichs (2008). The
177 basis of the copula approach is Sklar’s theorem (Sklar 1959) which states that every multi-
178 variate or joint CDF can be expressed by the marginal CDFs of the univariate components of
179 the multivariate random variable and the copula. The copula is a joint CDF that describes
180 the statistical dependence of the transformed random variables $U_j = F_{X_j}(X_j)$, where X_j
181 is the j -th component of the multivariate random variable $\mathbf{X} = (X_1, \dots, X_d)^T$ and F_{X_j} the
182 respective marginal CDF. Sklar’s theorem states that every joint CDF $F_{\mathbf{X}}$ can be expressed
183 as

$$F_{\mathbf{X}} = C_{\mathbf{X}}(F_{X_1}, \dots, F_{X_d}), \quad (7)$$

184 where $C_{\mathbf{X}}$ is the copula of \mathbf{X} . Both bivariate and multivariate BC methods presented next are
185 designed to restore the dependence structure and therefore the underlying copula function.

186 *b. The bivariate “Conditional” approach*

187 Piani and Haerter (2012) developed a bivariate BC method whose the main idea is to
188 apply a univariate BC to precipitation time series conditionally on the bias corrected values
189 of temperature classified into binned temperature values. This “conditional” approach works
190 in three steps: First, a standard 1d-BC method is applied separately to model temperature.

191 Then, the (temperature, precipitation) pairs are grouped into temperature quantile bins.
192 Finally, a standard 1d-BC method is applied for precipitation within each temperature bin.
193 They concluded that this approach improved the 2d temperature-precipitation copula and
194 that even a relatively small number of temperature bins allows to significantly improve the
195 dependence structure (i.e., the copula) between the two physical variables. Technical details
196 can be found in Piani and Haerter (2012). In the following, this conditional approach is
197 applied both ways to our data: to bias correct precipitation time series conditionally on the
198 bias corrected values of temperature, and to bias correct temperature time series condition-
199 ally on the bias corrected values of precipitation. For precipitation given temperature, five
200 quantile bins have been used. Higher numbers of bins have also been tested but the quality
201 of the results did not change significantly (not shown). For temperature given precipitation,
202 only three quantile bins have been used (with the first interval bin including all zeros) to
203 avoid the size of the bins to be too much different due to a larger number of dry days. Note
204 that this 2D approach is relatively independent of the 1d-BC method since the conditional
205 correction can be performed with most of the classical 1d-BC techniques. This is a very
206 interesting feature that makes the procedure flexible.

207 However, this conditional approach reproduces only the 2D inter-variable dependences:
208 we may want to correct the spatial and or temporal structures as well. Then, other techniques
209 have to be employed.

210 *c. The “Schaake Shuffle” method*

211 Clark et al. (2004) highlighted another shuffling technique – sometimes named as the

212 “Schaake shuffle” after Dr. J. Schaake (National Weather Service Office of Hydrologic De-
 213 velopment) – in the context of correcting forecasts from NWP models. This method was
 214 adapted by Schefzik et al. (2013b) and by Möller et al. (2012) in the context of ensemble
 215 postprocessing. Here, the Schaake approach is adapted and presented in the context of bias
 216 correction of time series generated by (global or regional) climate models – potentially previ-
 217 ously dynamically or statistically downscaled – whose spatial, temporal and/or inter-variable
 218 properties have to be corrected.

219 The “Schaake shuffle” as illustrated in Table 1 is very simple to implement. Assume we
 220 have a reference sample of length 4 for the variable Z . The reference sample has a certain
 221 rank structure given by the rank k of an element in the sample with respect to the other
 222 data in the sample. When new samples arrive – e.g., from model output or from 1d-bias
 223 corrected data –, the main idea is to reorder the new samples such that their rank structure
 224 is identical to that of the reference sample. Let’s take the example of the variable Z with
 225 reference sample $Z_R = (0.3, 0.5, 0.9, 0.8)$ and prediction sample (i.e., the sample data to be
 226 corrected) $Z_P = (0.7, 0.5, 0.2, 0.9)$. The associated ranks of Z_R are $k(Z_R) = (k(0.3) = 1,$
 227 $k(0.5) = 2, k(0.9) = 4, k(0.8) = 3)$ – noted as $k(Z_R) = (1, 2, 4, 3)$ – and those of Z_P are
 228 $k(Z_P) = (k(0.7) = 3, k(0.5) = 2, k(0.2) = 1, k(0.9) = 4)$ – noted as $k(Z_P) = (3, 2, 1, 4)$. The
 229 shuffling procedure consists in reordering the elements of Z_P into a new sample $Z_{shuffled}$ such
 230 that the rank of this new sample is identical to the rank of the training sample: $k(Z_{shuffled}) =$
 231 $k(Z_R) = (1, 2, 4, 3)$. Hence, based on the present example, the first element of $Z_{shuffled}$ must
 232 be the element of Z_P with rank 1, that is 0.2; the second element of $Z_{shuffled}$ must be the
 233 element of Z_P with rank 2, that is 0.5; the third element of $Z_{shuffled}$ must be the element of Z_P
 234 with rank 4, that is 0.9; and the fourth element of $Z_{shuffled}$ must be the element of Z_P with rank

235 3, that is 0.7. Therefore, $Z_{shuffled} = (0.2, 0.5, 0.9, 0.7)$ and satisfies $k(Z_{shuffled}) = k(Z_R)$.
236 See Clark et al. (2004) for a more technical and mathematical formulation of the shuffling
237 procedure. Note that Z_R represents the dataset from which the dependence structure is
238 “learned”. In our case it represents one time series in the SAFRAN reference dataset during
239 the training period. Z_P represents the prediction, which in our study is the corresponding
240 ERA-I time series, either bias corrected or not. The main difference between the shuffling
241 methods mentioned in the introduction, namely the Schaake shuffle and the ECC, is the
242 dataset that determines the dependence structure (i.e. the ranks).

243 In the present work, for practical reasons, the rank associated with exact same values
244 (such as zeros for precipitation) is supposed to be increasing with time. In other words,
245 if $z_{t_1} = z_{t_2} = 0$ are precipitation values at time t_1 and t_2 respectively, with $t_1 < t_2$, then
246 $rank(z_{t_1}) < rank(z_{t_2})$. In the context of a 3-dimensional data matrix (say, n time steps,
247 s gridcells or stations, p physical variables), the Schaake method is applied separately to
248 the n -component vector resulting from each combination “one gridcell \times one variable” (i.e.,
249 it is applied $s \times p$ times). The remarkable effect is that simply by reordering the data
250 independently in time, not only the temporal, but also intervariable and spatial dependencies
251 are restored. How powerful the “Schaake shuffle” is will be shown in section 5.

252 Why is Sklar’s theorem (Eq. 7) of relevance for the shuffling method presented here? An
253 important property of the transformed random variables U_j is that if Z_j has the CDF F_{Z_j} then
254 $U_j = F_{Z_j}(Z_j) \sim Unif(0, 1)$, i.e. U_j are uniformly distributed on the interval $[0, 1]$. Lets now
255 assume we have a sample $z_j^{(i)}, i = 1, \dots, N$ of Z_j without knowing F_{Z_j} , then $u_j^{(i)} = F_{Z_j}(z_j^{(i)})$
256 is generally estimated as the rank $k_j^{(i)}$ of $z_j^{(i)}$ with respect to the sample $z_j^{(i)}, i = 1, \dots, N$
257 divided by $N + 1$, i.e. $\hat{u}_j^{(i)} = k_j^{(i)} / (N + 1)$. Hence, re-shuffling of the multivariate data with

258 respect to their ranks $k_j^{(i)}$ has the potential to restore (parts of) the dependence structure,
259 namely the copula $C_{\mathbf{Z}}$. It is the same reason why rank correlation is an adequate measure to
260 assess dependence between random variables. An important consequence of Sklar’s theorem
261 (Eq. 7) is that the BC of the marginals and the restoration of the dependence structure
262 can be performed independently, at least as long as the BC of the marginals does not affect
263 the ranks of the data (this is generally given since transfer functions are usually monotonic
264 functions). In the following, the application of the Schaake shuffling technique to previously
265 1d-bias corrected time series will be referred to as “Empirical Copula - Bias Correction”
266 (EC-BC).

267 *d. Raw and shuffled ERA-I reanalyses*

268 For comparison purposes, the “raw” ERA-I data (i.e., without any correction) as well
269 as the Schaake shuffling technique are directly applied to ERA-I without any preliminary
270 univariate bias correction are also evaluated. Hence, in the “Results” section, the following
271 BC methods are intercompared:

- 272 • the **independent univariate bias corrections** (CDFt) of the ERA-I reanalyses of
273 precipitation and temperature;
- 274 • CDFt on ERA-I followed by the Schaake shuffle method, i.e., **the EC-BC approach**;
- 275 • the **conditional approach** based on CDFt on ERA-I (with precipitation corrected
276 conditionally on temperature and the other way around);
- 277 • the **raw ERA-I data** (i.e., without any correction);

278 • and the **Schaake shuffling technique directly applied to ERA-I** without any
279 preliminary univariate bias correction.

280 5. Results

281 The various BC methods are evaluated according to three different angles: How do the
282 corrected data reproduce the inter-variable statistical properties? How do they reproduce
283 the spatial properties? How do they reproduce the temporal properties? In the following,
284 due to the large number of figures available, only winter evaluations are shown. However,
285 summer plots are fairly equivalent or provide equivalent conclusions and are provided as
286 auxiliary material.

287 *a. Inter-variable correlations*

288 For many impact models (e.g., hydrology, agriculture), the correlation between variables
289 – here precipitation and temperature – is an important feature that must be accurately mod-
290 eled by the meteorological input data. Hence, fig. 2 shows maps of inter-variable Spearman
291 correlation coefficients between PR and T in winter over the evaluation period for the various
292 BC models as well as for the SAFRAN dataset. While the Pearson correlation coefficient
293 is the most widely used, the Spearman correlation is employed here. Indeed, the Pearson
294 coefficient measures the strength of the linear relationship between normally distributed
295 variables. However, precipitation is not normally distributed and, besides, the relationship
296 between temperature and precipitation is not supposed to be linear. Hence, in that context,

297 it is more appropriate to use the Spearman correlation that does not require a linear rela-
298 tionship, neither the variables to be normally distributed (e.g., Hauke and Kossowski 2011).
299 In fig. 2, only correlations that are statistically equivalent to the SAFRAN correlation (i.e.,
300 not significantly different at 95%) are shown in those plots. A bootstrap technique (Efron
301 and Tibshirani 1993) with block-replacement of 10-day blocks has been applied to deter-
302 mine if the correlations were significantly different or not at 95%. The procedure was the
303 following for each grid-cell: (i) Take the N daily observations in the verification period; (ii)
304 Generate 1000 times N -day long bootstrapped samples with replacement (i.e., each sample
305 is constituted of $(N/10)$ 10-day blocks); (iii) Compute the 2.5% and 97.5% percentiles from
306 the 1000 correlations as the 95% uncertainty interval: if the correlations of the BC data are
307 outside this range, they are considered as significantly different. The length of the blocks (10
308 days) has been chosen to account for temporal correlations, i.e., that the effective number
309 of degrees of freedom in the daily time series is significantly smaller than N . In each panel
310 of fig. 2, the percentage of grid-points with correlation significantly different (%GPCSD)
311 from that of SAFRAN is also indicated. As expected, ERA-I correlations appear clearly as
312 inappropriate (%GPCSD is more than 66%). This is true also for the correlations from the
313 univariate BC method that roughly reproduce the ERA-I pattern (%GPCSD \simeq 61%). Inter-
314 estingly, the “conditional” approach does not give the same correlations when applied to
315 correct temperature given the precipitation (fig. 2(d)) or to correct precipitation given the
316 temperature (fig. 2(e)): the former provides much better correlations in the present setting
317 (about 30% vs. 75% for the %GPCSD). One explanation is that while a given PR interval
318 provides useful constraints on the possible range of associated temperatures, the opposite
319 is not true: temperature is not a good predictor of precipitation that remains relatively

320 highly variable even for a given small interval of temperatures. The EC-BC method gener-
321 ates equivalently good results in terms of inter-variable dependence and provides satisfactory
322 correlations (%GPCSD \simeq 27%). This is true also when the Schaake shuffle is applied directly
323 to ERA-I reanalyses (%GPCSD \simeq 38%). It is interesting to note that some "not-significantly
324 equivalent correlations" regions are different from one model to another. Some additional
325 analyses and experiments (not shown) illustrate that the EC-BC method is not sensitive
326 to the choice of the univariate BC method (CFT or EQM) as preliminary step. This is
327 not exactly the case for the conditional approach where some differences appear between
328 "Cond. CFT" and "Cond. EQM" (not shown) and one must be cautious to this point when
329 applying the bivariate conditional approach.

330 *b. Spatial correlations*

331 The statistical spatial properties are also very important in many impact studies. A very
332 common way to investigate spatially coherent variability is a principal component analysis
333 (PCA). It has first to be noted, that the dominant empirical orthogonal function (EOF)
334 for both temperature and precipitation represents almost constant changes over the entire
335 region. This is due to the small spatial extent of the region, where day to day weather vari-
336 ability is large and affects the whole domain in a very similar way. We thus first investigate
337 the variability of the area-mean temperature and precipitation times series, which is then
338 removed from the data for the PCA.

339 Figure 3 represents bivariate histograms of area-mean 2m temperatures. We here consider
340 the complete verification period taking summer and winter data together. In order to also

341 show equivalent figures for the reference data, we generated a perturbed series of observed
342 area-mean temperatures by randomly changing the order of the years while preserving the
343 order of the day in the year. The reference data reveal a distinct seasonal cycle with an
344 amplitude of more than 15 K. The seasonal cycle seems well reproduced in the bias corrected
345 temperature, whereas it is largely underestimated in ERA-Interim (Fig. 3 (b) and (f)). Thus,
346 univariate BC is helpful to correct the amplitude of the seasonal cycle. EC-BC or conditional
347 BC seem not to significantly improve the distribution of the area-mean values.

348 Since the distribution of precipitation is highly skewed, we set the zero precipitation
349 values to a small value different from zero (0.00033) and work in the following on the loga-
350 rithm of precipitation. For area-mean precipitation (Fig. 4) no obvious seasonal cycle exists.
351 The effect of the Schaake shuffle (e.g., comparing Figs. 4 (b) and (f)) seems to concentrate
352 the area-mean precipitation values, presumably because the Schaake shuffle increases the
353 spatial variability of ERA-Interim precipitation (i.e., induces more small scale structures by
354 shuffling). The conditional BC seems to shift the modus of the precipitation values to lower
355 values. None of the area-mean precipitation series seems superior from this analysis.

356 For the PCA we now removed the area-mean from the data at each time step. We
357 concentrate on winter data, but the results are similar for summer. Figs. 5 and 6 show
358 the eigenvalues and explained variance fractions of the leading EOF for temperature and
359 log-precipitation, respectively. Zero precipitation values were again set to a small value of
360 0.00033. Note that a principal component analysis for the still non-Gaussian log-precipitation
361 fields should be interpreted with caution. We think, however, that in our case it is a valuable
362 tool to compare spatially coherent modes of variability.

363 The eigenvalue spectra for temperature in Fig. 5 show, that the total variance, i.e. the

364 sum of the eigenvalues, is generally largest for the reference data, and smallest for ERA-
365 Interim, either shuffled or not. Thus, one important effect of BC is to correct for total
366 variance of the data. The conditional BC approach has a realistic variance spectrum, whereas
367 the EC-BC provides an eigenvalue spectrum very close to that of the reference data. The
368 explained variance spectra in Fig. 5 in turn give an indication of the relative importance of
369 the leading EOF. A flat spectrum indicates weak coherence in the spatial patterns, whereas
370 a steep spectrum generally indicates the presence of large-scale coherent structures. Since
371 independent BC inherits the spatial dependence of ERA-Interim, they both have a very
372 dominant first EOF. The explained variance spectra for the conditional and the EC-BC
373 approaches are very realistic.

374 Similar results are obtained for precipitation (Fig. 6). The total variance of ERA-Interim
375 and the shuffled ERA-Interim data is much too small, whereas BC has a very positive effect
376 even for the independent BC. The conditional BC seems to underestimate the variance of
377 the first EOF. The explained variance spectra show only small differences. Precipitation
378 generally has much more small-scale variability which is reflected in the small explained
379 variance fraction of the leading EOF. ERA-Interim and independently bias-corrected ERA-
380 Interim exhibit slightly larger scale dominant patterns.

381 The differences become even more evident in the structure of the leading EOFs. The
382 leading EOF for temperature (Fig. 7) in the reference data represents a dipole pattern with
383 higher than normal temperatures near the Mediterranean coast and colder temperatures
384 in the northern and north-eastern part of the region. All BC methods except those that
385 apply the Schaake shuffle, reproduce the checked pattern imposed by the ERA-Interim grid
386 structure and an east-west dipole. The conditional approach only slightly modifies the

387 large-scale pattern. This effect also pervades higher order EOF (not shown). In contrast,
388 the EC-BC has a very realistic leading EOF, and albeit with a smaller amplitude, the first
389 EOF is also well reproduced in the shuffled ERA-I dataset.

390 For log-precipitation (Fig. 8) the results are similar. Again, the leading EOF of the
391 EC-BC data set is very close to that of the reference data. The conditional BC introduces
392 some noise, but besides this its first EOF is very close to the first EOF of ERA-I. Note
393 that the conditional approach has been applied here to model temperature conditionally
394 on precipitation (Figs. 5 and 7) or the other way around (Figs. 6 and 8), i.e., in an inter-
395 variable context and not a spatial one. One can expect this conditional technique to work
396 better if applied in a spatial one, e.g., if the station i is modeled according to the station j .
397 Nevertheless, one could get as many references as stations j . Hence, the correction is then
398 not unique and therefore may be quite complicated to interpret. Besides, the combinatory
399 of BC to be applied can quickly increase and make the practical implementation intractable.

400 Globally, EC-BC shows the most satisfying spatial variance pattern, whenever designed
401 with CDFt or EQM (not shown for EQM). The results are also satisfactory – to a lesser
402 extent – for the Schaake shuffle directly applied to the raw ERA-I data. In order to assess
403 the similarity of spatial variance patterns more objectively, we performed a reduction of
404 spatial degrees of freedom. To this end, we calculate the EOF of the reference data and
405 project all data onto the leading 10 EOF of the reference data. We thus obtain 10 times
406 series (i.e. expansion coefficients) for each dataset. The analysis is now performed within
407 the 10 dimensional subspace spanned by the 10 leading EOF.

408 We first examine the covariance matrices of the reduced data sets for 2m temperature
409 (Fig. 9). By construction, the expansion coefficients of the reference data show a diagonal

410 covariance matrix. The covariances between the expansion coefficients are zero since the
 411 eigenvectors of the covariance matrix are statistically orthogonal. This is not anymore the
 412 case for the other datasets. Here, the covariances between the expansion coefficients are
 413 generally non-zero. The degree to which the off-diagonal are different from zero indicates
 414 how different the respective variation patterns are. EC-BC seems to project very well on the
 415 EOF of the reference data, all other methods show substantial differences. For precipitation
 416 (Fig. 10) results are similar. The similarity of the covariance matrix obtained from EC-BC
 417 with that of the reference data is again striking.

418 We finally want to quantify the quality of each of the approaches by using a distance
 419 function between the empirical (multivariate) distribution of the reference data and each
 420 of the BC methods. As distance measure we use the integrated quadratic distance (IQD),
 421 which is a proper divergence function Thorarinsdottir et al. (2013a). It measures the distance
 422 between two distribution functions. The IQD between two distribution functions F and G
 423 is defined as the integral

$$d(F, G) = \int_{\Omega} (F(\omega) - G(\omega))^2 d\omega, \quad (8)$$

424 where Ω represents the sample space. The IQD is closely related to the energy score used
 425 in forecast verification (Gneiting and Raftery 2007). It may be empirically estimated using
 426 the equivalent formulation

$$d(F, G) = E\|\mathbf{X} - \mathbf{Y}\|^2 - \frac{1}{2}E\|\mathbf{X} - \mathbf{X}'\|^2 - \frac{1}{2}E\|\mathbf{Y} - \mathbf{Y}'\|^2, \quad (9)$$

427 where \mathbf{X} and \mathbf{X}' , and \mathbf{Y} and \mathbf{Y}' represent independent draws from multivariate distribution
 428 functions F and G , respectively. The vector norm used here is the Euclidian norm.

429 In our application, \mathbf{X} and \mathbf{Y} are the expansion coefficients of the reference and the BC

430 data, respectively. To get independent random realizations of the differences we randomly
431 draw 50.000 vectors with replacement for \mathbf{X} , \mathbf{X}' , \mathbf{Y} and \mathbf{Y}' out of the available data sets of
432 length 2.734 winter days, respectively, and calculate the IQD using (9). In order to assess
433 the uncertainty of the IQD, we additionally apply a bootstrap method with replacement
434 (Efron and Tibshirani 1993). Repeating this 200 times provides estimates of the uncertainty
435 of the IQD. Fig. 11 shows the IQD estimates together with the 95% bootstrap sampling
436 uncertainty. The IQD in Fig. 11 is evaluated hierarchically, first in the subspace of the
437 leading, then the first 2 leading upto the first 10 leading EOF of the reference data.

438 The IQD quantitatively confirms the superiority of EC-BC. For 2m temperature (Fig. 11
439 (a)) the IQD for the EC-BC data varies closely above zero throughout the hierarchy. It only
440 slightly increases with a higher dimensionality. There is a rather clear ranking between the
441 different approaches, with EC-BC performing best, conditional BC second best when using
442 more than 2 EOF, followed by ERA-Interim with Schaake shuffle, and independent BC. The
443 raw ERA-Interim data have the largest IQD, so any approach provides improvements in
444 terms of IQD. For temperature, large improvements of the spatial covariances are obtained
445 solely by the Schaake shuffle. Its effect on the IQD is stronger than that of the independent
446 BC of the marginals. In comparison to independent BC, the conditional BC only slightly
447 improves the spatial covariances.

448 For precipitation (Fig. 11 (b)) again EC-BC is clearly superior, but the ranking is not the
449 same as for temperature and less distinct. EC-BC performs best, followed by independent
450 BC. Interestingly, the Schaake shuffle applied without BC seems to degrade the IQD. The
451 most important correction here is the BC of the marginals, whereas the correction for the
452 dependence structure is less important for precipitation. The conditional approach seems to

453 work less well for precipitation under this respect.

454 *c. Temporal correlations*

455 We finally investigate the temporal structure of the time series, which used as input in
456 impact models may also have great consequences. Its accurate modeling may then be crucial.

457 To this end, n -day lag autocorrelations have been studied for n between 1 and 5. Fig-
458 ures 12 and 13 display the lag-1 auto-correlations for the different BC models in winter for
459 temperature and precipitation respectively.

460 For temperature, the conditional approach (Fig. 12(d)) clearly underestimates lag-1 auto-
461 correlations, while, in that temporal context, the independent BC (Fig. 12(c)) gives rel-
462 atively consistent results, although strongly imperfect due to the structure in “squares”
463 already present in non-corrected ERA-I auto-correlations (Fig. 12(b)). The shuffling pro-
464 cedure provides the best temporal dependencies either applied to CDFt results (i.e., the
465 EC-BC approach, Fig. 12(e)) or directly to ERA-I data (Fig. 12(f)). Most of the conclu-
466 sions from lag-1 temperature auto-correlation are still valid for lag-5 auto-correlations (not
467 shown): The results of the shuffling procedure (on 1d-BC or non-corrected data) are still
468 very close to the reference, while the conditional approach provides too low correlations and
469 ERA-I data continue to have too high correlations. However, the independent BC method
470 is not as consistent as for lag-1 results, with too low lag- n auto-correlations for $n \geq 2$.

471

472 In terms of precipitation, (Fig. 13), contrary to temperature, lag-1 correlations from in-
473 dependent BC (Fig. 13(c)) are not acceptable, showing a pronounced underestimation. On

474 the opposite, the direct shuffling of ERA-I (Fig. 13(f)) globally overestimates the 1-day auto-
475 correlation, especially on the North-East part of the domain. This was already true (with a
476 smaller magnitude) for uncorrected ERA-I (Fig. 13(b)). The EC-BC approach (Fig. 13(e))
477 provides the precipitation lag-1 auto-correlation structures and intensities the closest to
478 those of the SAFRAN dataset (Fig. 13(a)), while the conditional approach (Fig. 13(d)) gives
479 correct auto-correlation magnitudes but with relatively inappropriate structures. For lags
480 longer than one day, the precipitation auto-correlation drops very quickly close to zero for
481 observations and almost all models (not shown), except for the direct shuffling of ERA-I
482 data that continues to provide very high (unobserved and unrealistic) auto-correlations of
483 about 0.8 – at least until a 5-day lag – for the north-east region. This is somehow surprising
484 since the “raw” ERA-I data (i.e., without any bias correction) do not show such a strong
485 feature although with a very slight overestimation of the auto-correlation for this region.

486

487 Moreover, to describe more specifically the rainfall occurrence temporal structure ob-
488 tained from the BC methods, the maps of the probability of a dry day given that the
489 previous day was wet – i.e., $Proba(dry|wet)$ noted as Pdw –, as well as the opposite – i.e.,
490 $Proba(wet|dry)$ noted as Pwd – have been computed and are displayed in Figs. 14 and 15
491 respectively. For the maps of $Proba(dry|wet)$, it is quickly seen that ERA-I (Fig. 14(b))
492 and the conditional approach (Fig. 14(d)) globally overestimate the probability of a dry day
493 given that the previous day was wet. However, all the other BC methods provide satisfying
494 Pdw values, close to those of SAFRAN.

495 Interestingly, the Pwd maps (Fig. 15) are not completely the ”symmetric” of the Pdw maps.
496 Here, the conditional approach (Fig. 15(d)), ERA-I (Fig. 15(b)), as well as its direct shuffling

497 (Fig. 15(f)) underestimate the dry day probabilities (particularly strongly for the latter two
498 datasets). The independent BC (Fig. 15(c)) shows better Pwd values, although too high
499 in the north-east region. However, the Pwd results the closest to those of the reference
500 dataset are obtained from the EC-BC model (Fig. 15(e)), which shows quite similar values
501 and spatial structures.

502 6. Conclusions and Discussion

503 a. Conclusions

504 In this paper, we have compared several univariate, bivariate and multivariate bias cor-
505 rection (BC) methods designed for specific multivariate properties:

- 506 • One univariate “independent BC” based on the CDFt approach;
- 507 • The “Conditional approach” (Piani and Haerter 2012) (here, based on CDFt) devel-
508 oped specifically for producing a correct two-dimensional inter-variable structure;
- 509 • The ‘Schaake Shuffle’ method (Clark et al. 2004) applied directly to raw (i.e., uncor-
510 rected) ERA-I reanalyses precipitation and temperature time series;
- 511 • The “Empirical Copula - Bias Correction” (EC-BC) approach constituted with the
512 Schaake Shuffle method applied to previously 1d-bias corrected time series (here,
513 through the CDFt method) of precipitation and temperature.

514 The Schaake method is based on temporal shuffling of the elements in each time series such
515 that the temporal rank structure is reconstructed.

516 Globally, on those datasets and with this experimental setting, although it is quite useful
517 for correction of the marginal distributions, the one-dimension CDFt bias correction alone
518 is not good at reproducing any of the inter-variable, spatial or temporal properties of the
519 observed data. This is true also for the univariate EQM method (not shown). In contrast,
520 the application of the EC-BC techniques clearly improves those properties. The conditional
521 and the shuffling methods improve the inter-variable properties – often, even when applied
522 directly to ERA-I data. This is not the case for the spatial structure where the conditional
523 technique – which was initially designed only for inter-variable structures – is not suitable,
524 whereas the EC-BC approach is quite efficient in general. This inappropriateness of the
525 conditional method is also visible in the temporal properties where auto-correlations are
526 underestimated. Again, in this temporal context, the EC-BC technique is relatively satisfying
527 for both temperature and precipitation.

528 As global conclusions:

- 529 • The one-dimensional BC method CDFt is not able to produce correct multidimensional
530 properties (similar results were obtained with the EQM method, not shown);
- 531 • The conditional technique – at least as applied in this experimental setup – is only
532 good for inter-variable properties reproduction;
- 533 • The EC-BC approach is good for both, inter-variable, spatial and temporal correlations.
534 The preliminary of 1d-BC before the shuffling procedure is nevertheless an important
535 requisite for precipitation since the combination “1d BC/shuffling” generally provides
536 the most satisfying results;
- 537 • Due to its easiness of coding, its speed of application and the good quality of its results

538 for both inter-variable, spatial and temporal properties, the Schaake Shuffle method
539 applied after a 1d BC method (i.e., the EC-BC approach) is a very good candidate for
540 all needs in multivariate bias correction.

541 Although not tested, the application of these BC methods to correct GCM outputs
542 instead of reanalysis data is expected to slightly degrade the results but to produce equivalent
543 rankings: the simpler methods should perform worse when based on GCM data due to
544 the GCM weather sequence that generally needs additional corrections, while the EC-BC
545 approach should continue to work well. More precisely, the 1d methods (CDF-t and EQM)
546 will basically reproduce the inter-variable, spatial and temporal properties of the input data.
547 So if those properties are wrong from the GCMs, they will be wrong as well for the 1d
548 corrected data. The conditional approach, by construction, should work fine to reconstruct
549 an inter-variable dependence close to that of the observations, even when driven by GCM
550 outputs. However, the spatial and temporal properties of the data corrected in following
551 this approach should stay relatively close to those of the GCM data. Moreover, although
552 the use of the Schaake shuffle directly to the GCM simulations should improve those, it is
553 expected that EC-BC will provide the best results in terms of the three types of properties
554 studied in this paper. Hence, by construction of the EC-BC approach, results similar to
555 those presented in this article can be expected on different regions or with different reference
556 or model datasets.

558 The general idea of the EC-BC and shuffling methods presented here is to re-shuffle the
559 predictive multivariate spatio-temporal data according to some rank structure derived from
560 training data. In doing so the data in the evaluation set receives a dependence structure
561 close to the dependence structure of the training dataset. More concretely, lets assume we
562 have training and test datasets, each with a multivariate spatio-temporal structure. With
563 the Schaake shuffle method, simply by shuffling the test data set in time such that the ranks
564 of the data in time is identical to that of the training data, we restore at least partly the
565 inter-variable, spatial and temporal dependencies of the training data set. Since the uni-
566 variate BC as presented above is a monotonic transformation of the data and is applied to
567 each variable and point in space independently, it has no influence on the copula function.
568 Shuffling can be performed prior or after univariate BC.

569

570 Note also that, if the CDFt method has been employed as univariate BC within the EC-
571 BC approach, other techniques can be used. This shuffling post-processing can be performed
572 based on most of the standard 1d-BC techniques. This interesting feature makes the proce-
573 dure flexible and easily applicable. Note that the shuffling can even be applied to most (if
574 not all) of the 1d statistical downscaling (SD) approaches. This application of the Schaake
575 shuffle to 1d-SD outputs should improve their temporal, spatial and inter-variable properties
576 as much as it has been shown for the BC methods in the present article. Therefore, it would
577 be interesting to compare such a multivariate SD based on shuffling post-processing to sta-
578 tistical downscaling models taking explicitly into account the multi-dimensional structure of

579 the data to be downscaled (e.g., Yang et al. 2005; Flecher et al. 2010; Vrac et al. 2007, for
580 multi-site, multi-variable and temporal dependences modeling respectively).

581

582 Moreover, other re-ordering of data might be applied to restore and preserve some spe-
583 cific structures. The Schaake method shuffles elements *in time*. In other words, one value
584 associated to a given location (grid-cell or station) stays associated to this location but is
585 placed at another time. However, one may want to allow shuffling values *both in time and*
586 *in space*. This could improve the reproduction of the spatial dependences. To do so, it is
587 easy to extend the Schaake approach: instead of computing ranks and shuffling values within
588 vectors, this is made within 2-dimensional matrices. Hence, one value initially associated
589 to a given location at a given time may be placed at another time and another location
590 after this “full” shuffling. This technique has been tested and the results (not shown) both
591 in terms of inter-variable, spatial and temporal properties are very similar to those of the
592 Schaake shuffling presented all along the present study, except for precipitation where this
593 full Schaake shuffle applied directly to ERA-I is not as efficient as the “regular” Schaake
594 shuffling. Note that this full shuffling could also be performed for different physical variables
595 at once. If the variables have the same units (e.g., all variables are temperature values), this
596 can make sense. However, if the variables are different (e.g., precipitation and temperature),
597 the shuffling of values between the two variables can be strongly inappropriate and quite
598 difficult to interpret afterward.

599

600 Finally, there are essential assumptions to BC and EC-BC. Univariate BC estimates a
601 transfer function (TF) between model and observations from the training data, and applies

602 this TF to the evaluation (or projection) dataset. The main assumption is that the relation
603 between model and observations remains unchanged during the projection period. However,
604 if the distribution of the model data changes in the projection period so does the distribution
605 of the projected values. EC-BC (through the Schaake shuffle) represents a method to restore
606 the dependence structure within the projected values, which is inherited from the dependence
607 structure of the observations in the training dataset. The dependence structure of the model
608 data is completely ignored. This is absolutely reasonable in our context of downscaling, since
609 we know that the dependence structure in the large-scale model is erroneous. However, EC-
610 BC also ignores potential changes in the dependence structure suggested by the model data.
611 This is an important assumption: the (spatial, temporal and/or inter-variable) dependence
612 structures do not change between the training period and the projection period. Although
613 this conservative assumption is reasonable and simplifies the bias corrections, it may not
614 be valid in a climate change context where the multivariate properties to be corrected may
615 evolve as well. Hence, if changes in the dependence properties or its temporal evolutions are
616 of interest, the development of models allowing to make the dependence structures change
617 in time or in function of some atmospheric covariates would be of great interest for both the
618 climate and impacts communities.

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623 well as for the EC-BC approach (with CDFt) should soon be made available on the CRAN
624 website² or upon request to the authors.

²<http://cran.r-project.org/>

REFERENCES

- 627 Chen, C., J. Haerter, S. Hagemann, and C. Piani, 2011: On the contribution of statistical
628 bias correction to the uncertainty in the projected hydrological cycle. *Geophys. Res. Lett.*,
629 **38**, L20403, doi:10.1029/2011GL049318.
- 630 Christensen, J., F. Boberg, O. Christensen, and P. Lucas-Picher, 2008: On the need for
631 bias correction of regional climate change projections of temperature and precipitation.
632 *Geophysical Research Letters*, **35** (20).
- 633 Clark, M., S. Gangopadhyay, L. Hay, B. Rajagopalan, and R. Wilby, 2004: The Schaake
634 shuffle: A method for reconstructing space-time variability in forecasted precipitation and
635 temperature fields. *J. Hydrometeor.*, **5**, 243–262.
- 636 Colette, A., R. Vautard, and M. Vrac, 2012: Regional climate downscaling with prior statis-
637 tical correction of the global climate forcing. *Geophysical Research Letters*, **39**, L13707,
638 doi:10.1029/2012GL052258.
- 639 Dee, D. P., et al., 2011: The ERA-Interim reanalysis: configuration and performance of the
640 data assimilation system. *Q.J.R. Meteorol. Soc.*, **137**, 553–597. doi: 10.1002/qj.828.
- 641 Déqué, M., 2007: Frequency of precipitation and temperature extremes over France in an
642 anthropogenic scenario: Model results and statistical correction according to observed
643 values. *Global Planet. Change*, **57**, 16 – 26.

- 644 Efron, B. and R. J. Tibshirani, 1993: *An Introduction to the Bootstrap*. Chapman & Hall,
645 436 pp.
- 646 Flecher, C., P. Naveau, D. Allard, and N. Brisson, 2010: A stochastic daily weather generator
647 for skewed data. *Water Resour. Res.*, **46** (W07519, doi:10.1029/2009WR008098).
- 648 Gneiting, T. and A. E. Raftery, 2007: Strictly proper scoring rules, prediction, and estima-
649 tion. *J. Amer. Stat. Assoc.*, **102** (477), 359–378, doi:10.1198/016214506000001437.
- 650 Gudmundsson, L., J. B. Bremnes, J. E. Haugen, and T. Engen-Skaugen, 2012: Technical
651 Note: Downscaling RCM precipitation to the station scale using statistical transformations
652 – a comparison of methods. *Hydrology and Earth System Sciences*, **16** (9), 3383–3390, doi:
653 10.5194/hess-16-3383-2012.
- 654 Haddad, Z. and D. Rosenfeld, 1997: Optimality of empirical z-r relations. *Q. J. R. Meteorol.*
655 *Soc.*, **123**, 1283–1293.
- 656 Hauke, J. and T. Kossowski, 2011: Comparison of values of pearson’s and spearman’s cor-
657 relation coefficients on the same sets of data. *Quaestiones geographicae*, **30** (2), 87–93.
- 658 Johnson, C. and N. Bowler, 2009: On the reliability and calibration of ensemble forecasts.
659 *Mon. Wea. Rev.*, **137**, 1717–1720.
- 660 Lavaysse, C., M. Vrac, P. Drobinski, M. Lengaigne, and T. Vischel, 2012: Statistical down-
661 scaling of the French Mediterranean climate: assessment for present and projection in an
662 anthropogenic scenario. *Nat. Hazards Earth Syst. Sci.*, **12**, 651–670, doi:10.5194/nhess-
663 12-651-2012.

664 Maraun, D., 2013: Bias correction, quantile mapping, and downscaling: Revisiting the
665 inflation issue. *Journal of Climate*, **26**, 2137–2143. doi: <http://dx.doi.org/10.1175/JCLI->
666 [D-12-00821.1](http://dx.doi.org/10.1175/JCLI-D-12-00821.1).

667 Meehl, G., 2007: *Global Climate Projections. In: Climate Change 2007: The physical basis.*
668 *Contribution of Working Group 1 to the fourth Assessment report of the Intergovernmental*
669 *Panel on Climate Change.* Solomon et al., Cambridge University Press, Cambridge, UK.

670 Michelangeli, P., M. Vrac, and H. Loukos, 2009: Probabilistic downscaling approaches: ap-
671 plication to wind cumulative distribution functions. *Geophys. Res. Lett.*, **36**, L11708,
672 doi:10.1029/2009GL038401.

673 Möller, A., A. Lenkoski, and T. L. Thorarinsdottir, 2012: Multivariate probabilistic fore-
674 casting using ensemble Bayesian model averaging and copulas. *Q. J. R. Meteorol. Soc.*,
675 doi:10.1002/qj.2009.

676 Muerth, M., et al., 2013: On the need for bias correction in regional climate scenarios to
677 assess climate change impacts on river runoff. *Hydrol. Earth Syst. Sci.*, **17**, 1189–1204,
678 doi:10.5194/hess-17-1189-2013.

679 Nelsen, R. B., 2006: *An Introduction to Copulas, 2nd edition.* Springer, New-York, 269 pp.

680 Oettli, P., B. Sultan, C. Baron, and M. Vrac, 2011: Are regional climate models rele-
681 vant for crop yield prediction in West Africa? *Environ. Res. Lett.*, **6**, doi: 10.1088/1748-
682 9326/6/1/014008.

683 Panofsky, H. and G. Brier, 1958: Some applications of statistics to meteorology. Tech. rep.,
684 University Park, Penn. State Univ., 224 pp.

- 685 Piani, C., J. Haerter, and E. Coppola, 2010: Statistical bias correction for daily precipitation
686 in regional climate models over Europe. *Theoretical and Applied Climatology*, **99**, 187–192,
687 doi:10.1007/s00704-009-0134-9.
- 688 Piani, C. and J. O. Haerter, 2012: Two dimensional bias correction of temperature and
689 precipitation copulas in climate models. *Geophys. Res. Lett.*, doi:10.1029/2012GL053839.
- 690 Piani, C., G. Weedon, M. Best, S. Gomes, P. Viterbo, S. Hagemann, and J. Haerter,
691 2010b: Statistical bias correction of global simulated daily precipitation and tem-
692 perature for the application of hydrological models. *J. Hydrol.*, **395**, 199–215,
693 doi:10.1016/j.jhydrol.2010.10.024.
- 694 Pinson, P., 2012: Adaptive calibration of (u, v)-wind ensemble forecasts. *Q. J. R. Meteorol.*
695 *Soc.*, **138**, 1273–1284.
- 696 Quintana-Segui, P., et al., 2008: Analysis of near surface atmospheric variables: validation
697 of the SAFRAN analysis over france. *J. Appl. Meteorol. and Climatol.*, **47**, 92–107.
- 698 Robertson, D. E., D. L. Shrestha, and Q. J. Wang, 2013: Post-processing rainfall forecasts
699 from numerical weather prediction models for short-term streamflow forecasting. *Hydrology*
700 *and Earth System Sciences*, **17 (9)**, 3587–3603, doi:10.5194/hess-17-3587-2013.
- 701 Roulin, E. and S. Vannitsem, 2012: Postprocessing of ensemble precipitation predictions
702 with extended logistic regression based on hindcasts. *Mon. Wea. Rev.*, **140**, 874–888.
- 703 Schaake, J., et al., 2007: Precipitation and temperature ensemble forecasts from single-valued
704 forecasts. *Hydrology and Earth System Sciences Discussions*, **4**, 655–717.

705 Schefzik, R., T. L. Thorarinsdottir, and T. Gneiting, 2013a: Uncertainty quantification in
706 complex simulation models using ensemble copula coupling. *Statistical Science*, **28** (4),
707 616–640.

708 Schefzik, R., T. L. Thorarinsdottir, and T. Gneiting, 2013b: Uncertainty quantification in
709 complex simulation models using ensemble copula coupling. *Statistical Science*, **28**, 616–
710 640.

711 Schoelzel, C. and P. Friederichs, 2008: Multivariate non-normally distributed random vari-
712 ables in climate research - introduction to the copula approach. *Nonlin. Processes Geo-*
713 *phys.*, **15**, 761–772.

714 Schuhen, N., T. L. Thorarinsdottir, and T. Gneiting, 2012: Ensemble model output statistics
715 for wind vectors. *Mon. Wea. Rev.*, **140**, 3204–3219.

716 Simmons, A. J. and J. K. Gibson, 2000: The ERA-40 project plan. Era- 40 project rep.
717 series 1, ECMWF, Shinfield Park, Reading, United Kingdom.

718 Sklar, A., 1959: Fonctions de répartition à n dimensions et leurs marges. Tech. Rep. 229-231,
719 Publ. Inst. Statist. Univ. Paris 8.

720 Thorarinsdottir, T. L., T. Gneiting, and N. Gissibl, 2013a: Calibration diagnostics for point
721 process models via the probability integral transform. *SIAM/ASA Journal on Uncertainty*
722 *Quantification*, **1**, 150–158, doi:10.1137/130907550.

723 Thorarinsdottir, T. L., M. Scheuerer, and C. Heinz, 2013b: Assessing the calibration of
724 high-dimensional ensemble forecasts using rank histograms. *arXiv:1310.0236*.

725 Tisseuil, C., M. Vrac, G. Grenouillet, M. Gevrey, T. Oberdorff, A. Wade, and S. Lek,
726 2012: Strengthening the link between hydro-climatic downscaling and species distribu-
727 tion modelling: Climate change impacts on freshwater biodiversity. *Science of the Total*
728 *Environment (STOTEN)*, **424**, 193–201, doi: 10.1016/j.scitotenv.2012.02.035.

729 Vautard, R. and 25 authors, 2013: The simulation of European heat waves from an ensemble
730 of regional climate models within the EURO-CORDEX project. *Climate Dynamics*, DOI
731 10.1007/s00382-013-1714-z.

732 Verkade, J., J. Brown, A. Weerts, and A. H. Reggiani, P. Weerts, 2013: Post-
733 processing ECMWF precipitation and temperature ensemble reforecasts for operational
734 hydrologic forecasting at various spatial scales. *Journal of Hydrology*, **501**, 73–91,
735 doi:10.1016/j.jhydrol.2013.07.039.

736 Vigaud, N., M. Vrac, and Y. Caballero, 2013: Probabilistic downscaling of GCM scenar-
737 ios over southern India. *International Journal of Climatology*, **33 (5)**, 1248–1263, DOI:
738 10.1002/joc.3509.

739 Voisin, N., F. Pappenberger, D. P. Lettenmaier, R. Buizza, and J. C. Schaake, 2011: Appli-
740 cation of a medium-range global hydrologic probabilistic forecast scheme to the ohio river
741 basin. *Wea. Forecasting*, **26**, 425–446.

742 Voisin, N., J. C. Schaake, and D. P. Lettenmaier, 2010: Calibration and downscaling methods
743 for quantitative ensemble precipitation forecasts. *Wea. Forecasting*, **25**, 1603–1627.

744 Vrac, M., P. Drobinski, A. Merlo, M. Herrmann, C. Lavaysse, L. Li, and S. Somot, 2012:
745 Dynamical and statistical downscaling of the French Mediterranean climate: uncertainty

746 assessment. *Nat. Hazards Earth Syst. Sci.*, **12**, 2769–2784, doi:10.5194/nhess-12-2769-
747 2012.

748 Vrac, M., M. Stein, and K. Hayhoe, 2007: Statistical downscaling of precipitation through
749 nonhomogeneous stochastic weather typing. *Clim. Res.*, **34**, 169–184.

750 White, R. and R. Toumi, 2013: The limitations of bias correcting regional climate model
751 inputs. *Geophysical Research Letters*, **12 (40)**, 2907–2912, DOI: 10.1002/grl.50612.

752 Wilks, D. S., 2014: Multivariate ensemble-MOS using empirical copula. *Submitted to "Quart.*
753 *J. Roy. Meteor. Soc."*.

754 Wood, A., L. Leung, V. Sridhar, and D. Lettenmaier, 2004: Hydrologic implications of
755 dynamical and statistical approaches to downscaling climate model outputs. *Clim. Change*,
756 **62 (189–216)**.

757 Yang, C., R. E. Chandler, V. S. Isham, and H. S. Wheater, 2005: Spatialtemporal rain-
758 fall simulation using generalized linear models. *Water Resour. Res.*, **41 (W11415)**,
759 **doi:10.1029/2004WR003739**).

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777 $k()$ indicates the rank within the sample. 40

TABLE 1. Reference data of sample size 4 for the illustration of the ‘‘Schaake shuffle’’. $k()$ indicates the rank within the sample.

Training				Prediction				Schaake Shuffle			
$x_T^{(i)}$	$k(x_T^{(i)})$	$y_T^{(i)}$	$k(y_T^{(i)})$	$x_P^{(i)}$	$k(x_P^{(i)})$	$y_P^{(i)}$	$k(y_P^{(i)})$	$x_{PSS}^{(i)}$	$k(x_{PSS}^{(i)})$	$y_{PSS}^{(i)}$	$k(y_{PSS}^{(i)})$
0.3	1	1.1	1	0.7	3	1.3	2	0.2	1	1.1	1
0.5	2	1.7	3	0.5	2	1.8	4	0.5	2	1.4	3
0.9	4	1.2	2	0.2	1	1.1	1	0.9	4	1.3	2
0.8	3	1.9	4	0.9	4	1.4	3	0.7	3	1.8	4

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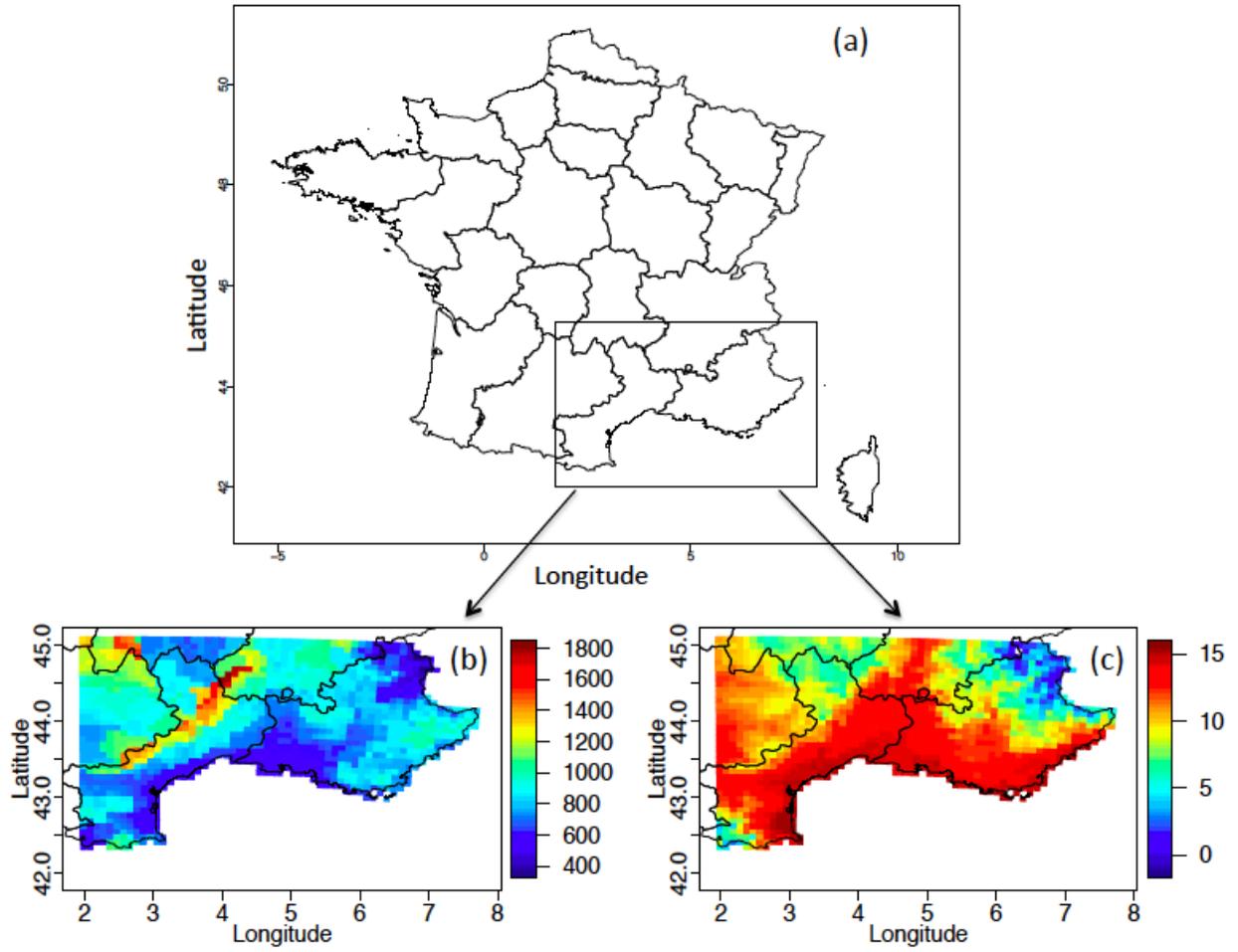


FIG. 1. (a) Map of France with the region of interest in a box, as well as (b) the mean cumulated annual precipitation and (c) the mean daily temperature.

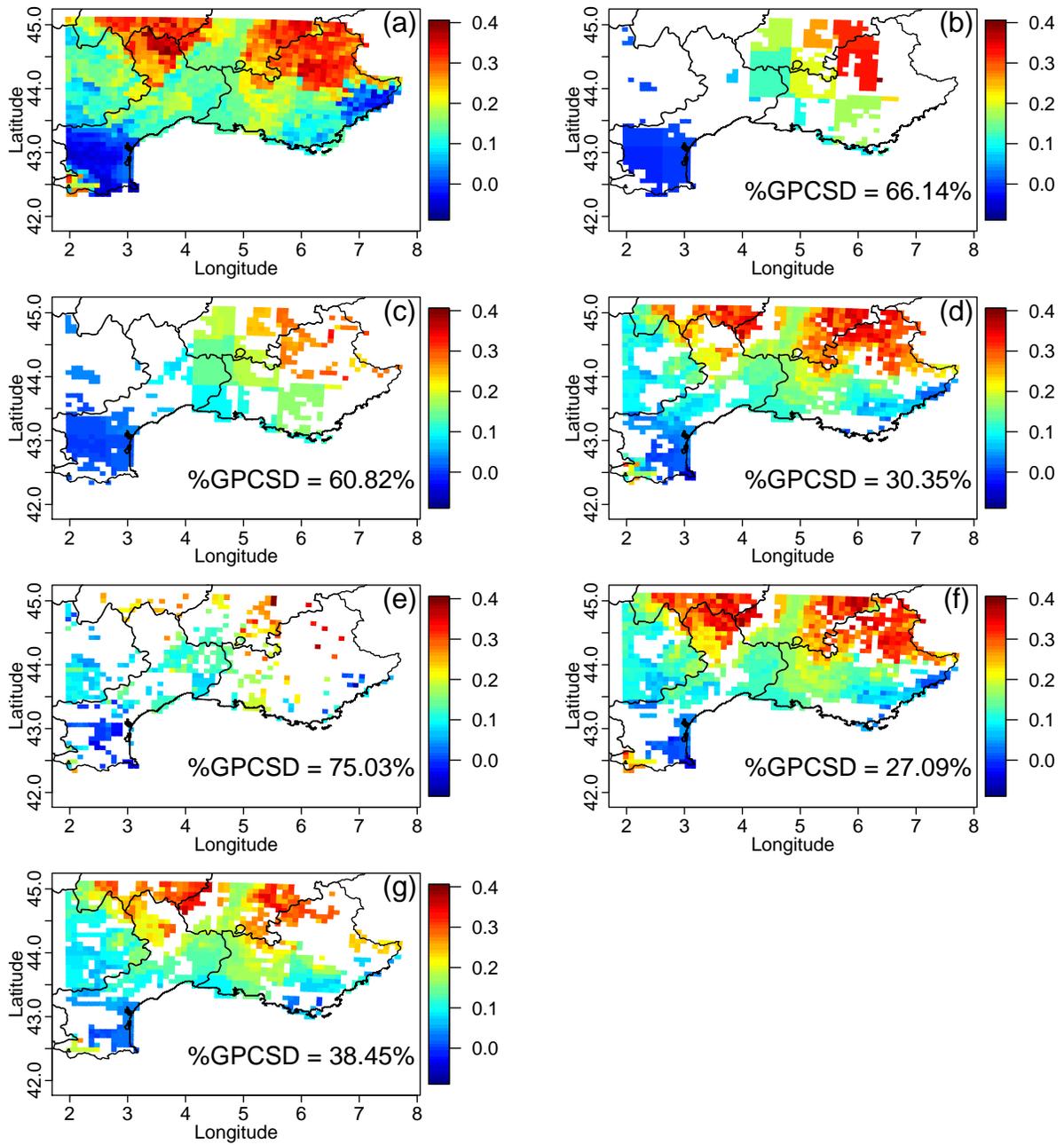


FIG. 2. Maps of inter-variable (PR, T) spearman correlations for the different approaches in winter: (a) SAFRAN; (b) ERA-I; (c) independent BC (through CDFt); (d) conditional BC of T2 given PR; (e) conditional BC of PR given T2; (f) EC-BC; (g) Schaake shuffle on ERA-I. The percentage of grid-points with correlation significantly different (%GPCSD) from that of SAFRAN is indicated on each panel.

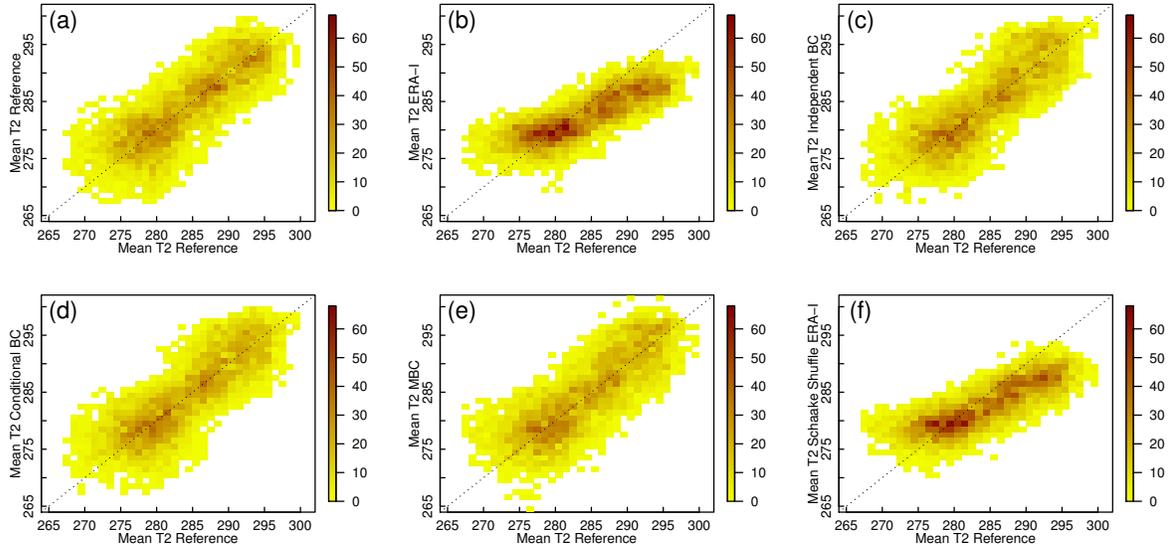


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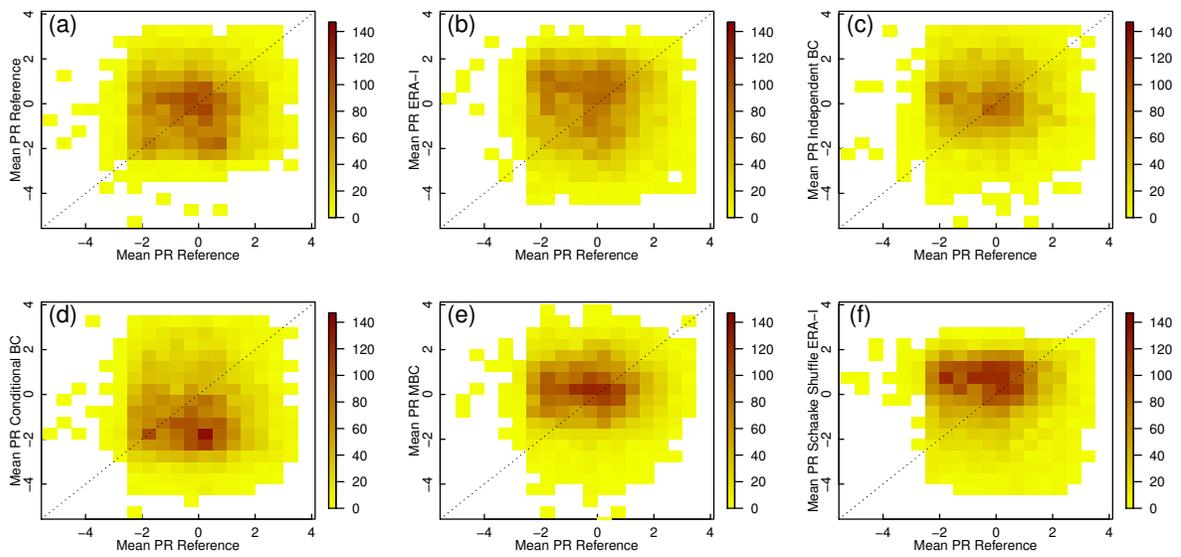


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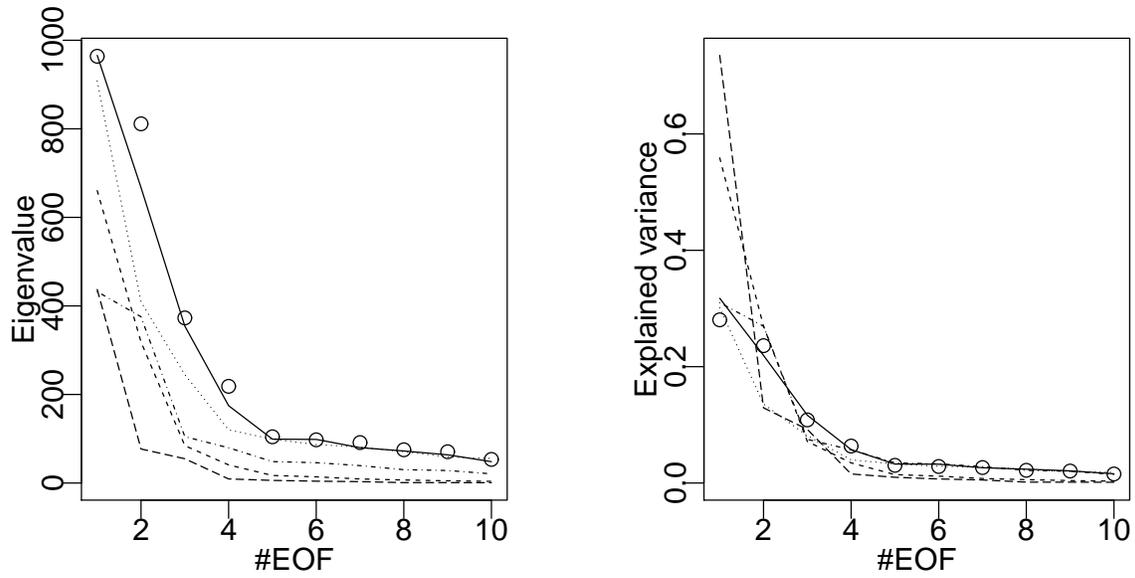


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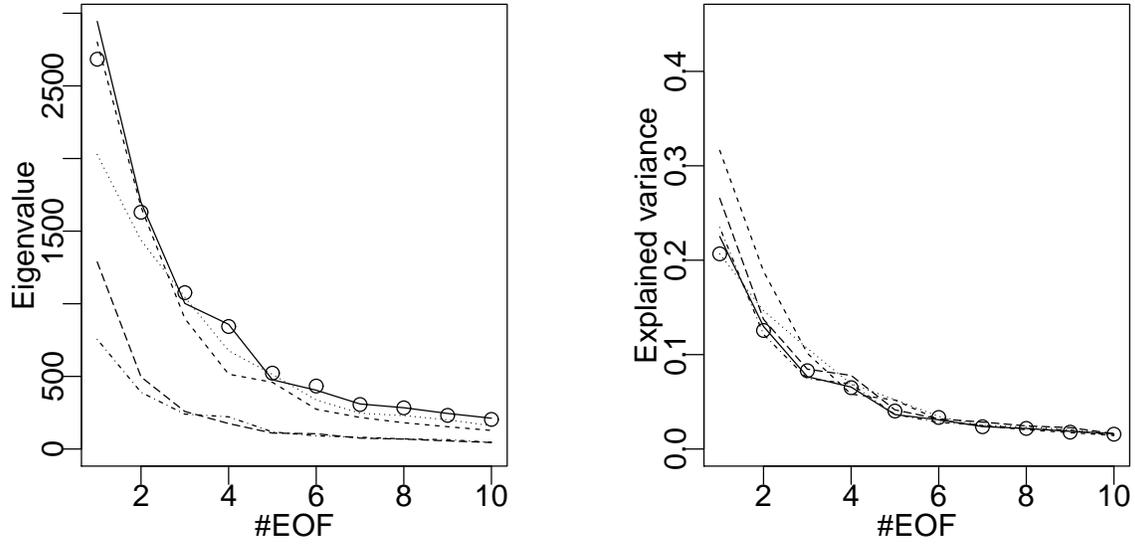


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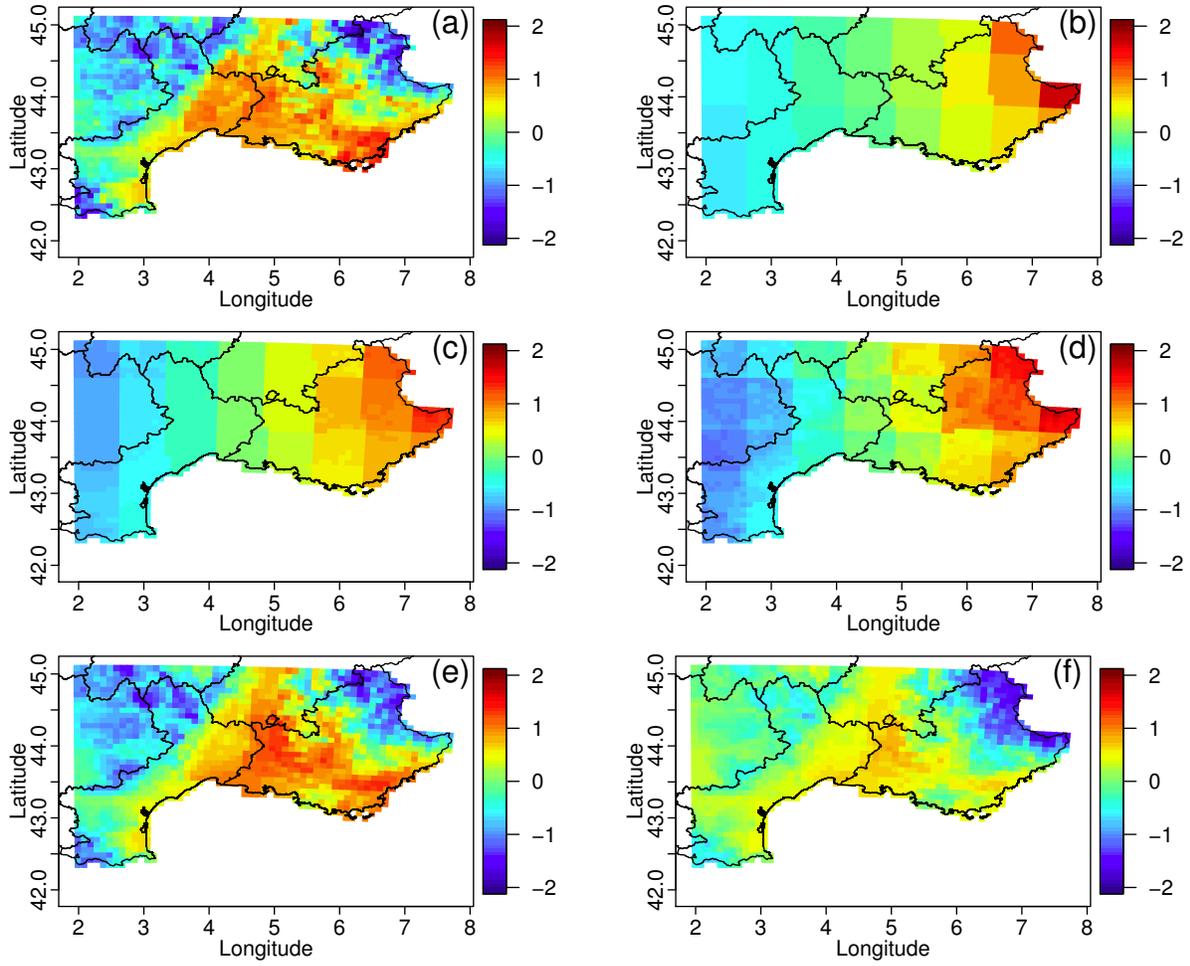


FIG. 7. First EOF of 2m temperature for (a) reference, (b) ERA-I, (c) independent bias correction, (d) conditional approach with T2 given PR, (e) EC-BC and (f) Schaake shuffle on ERA-I without BC.

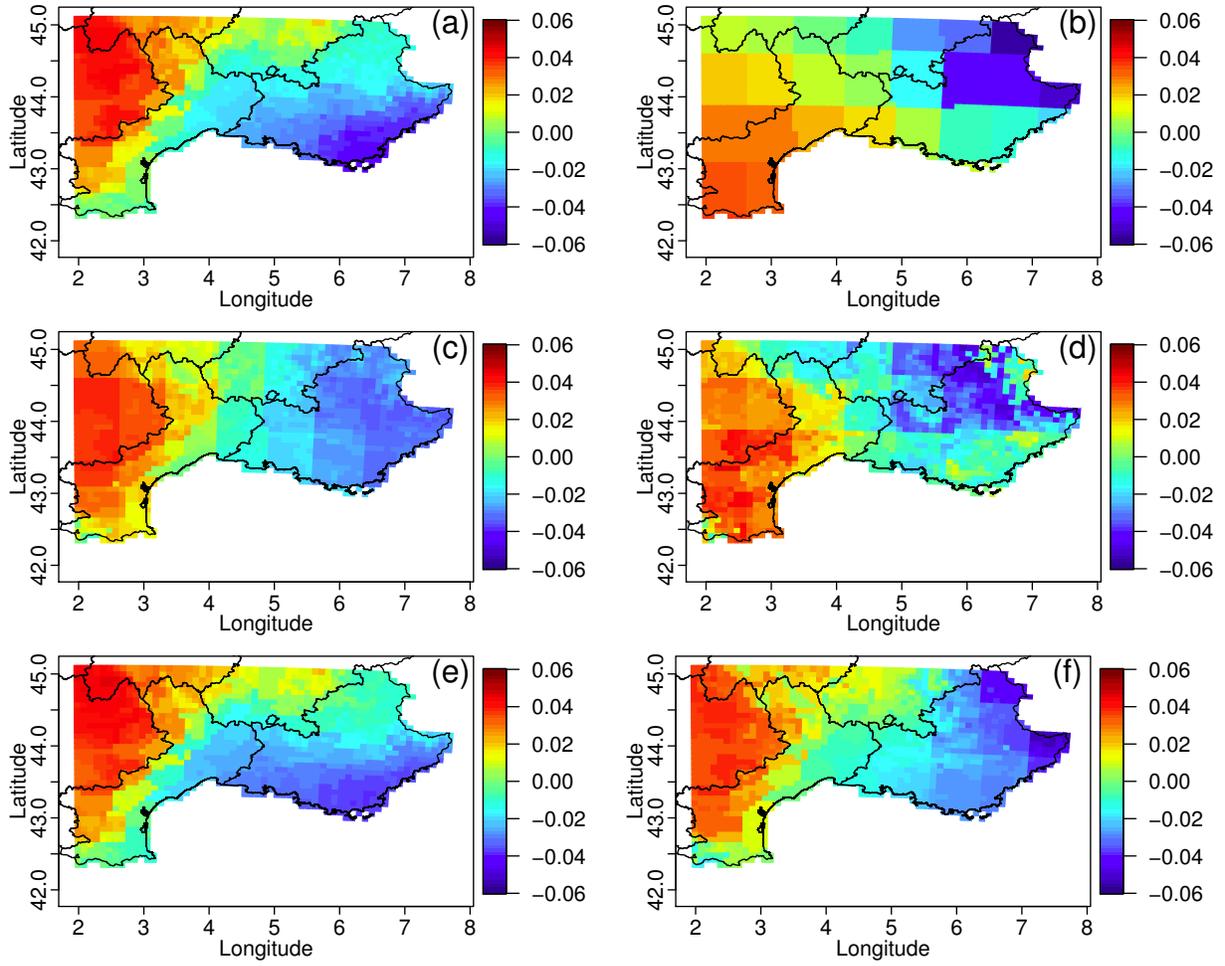


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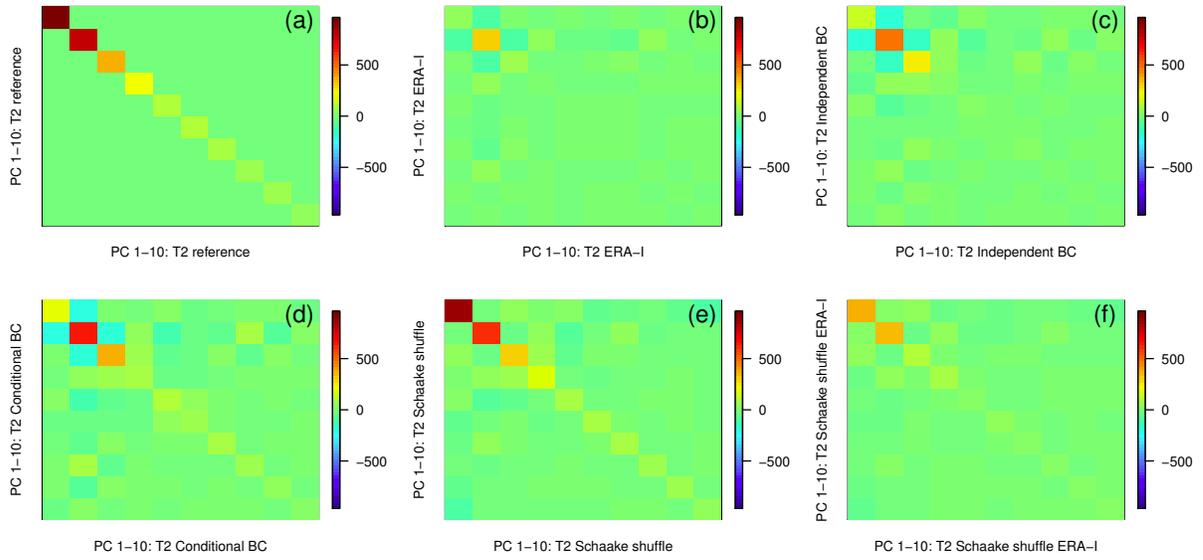


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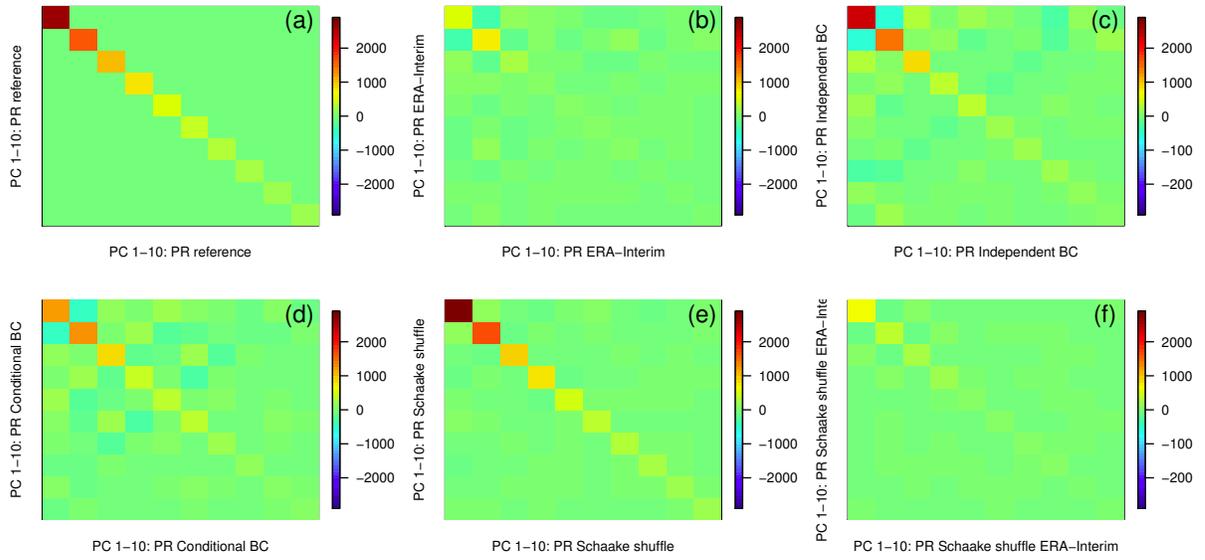


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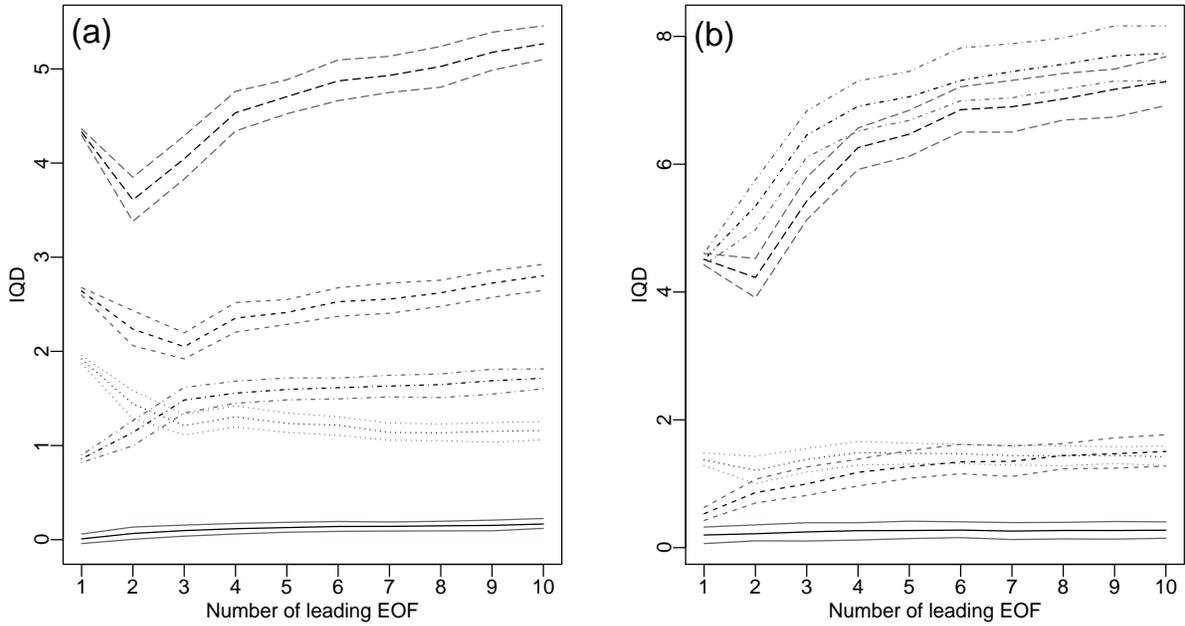


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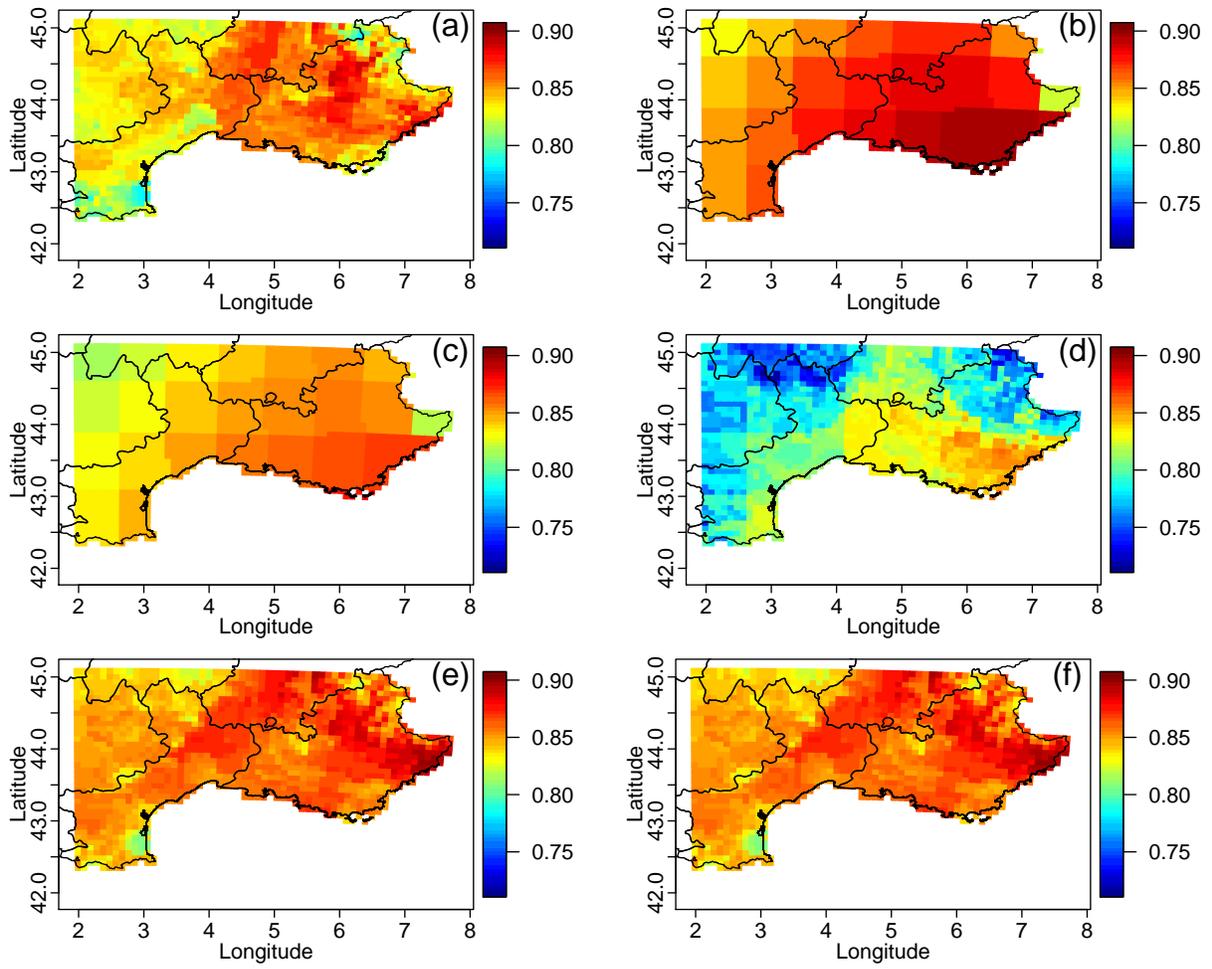


FIG. 12. Maps of 1-day lag temperature auto-correlations in winter for (a) reference, (b) ERA-I, (c) independent bias correction, (d) conditional approach with T2 given PR, (e) EC-BC and (f) Schaake shuffle on ERA-I without BC.

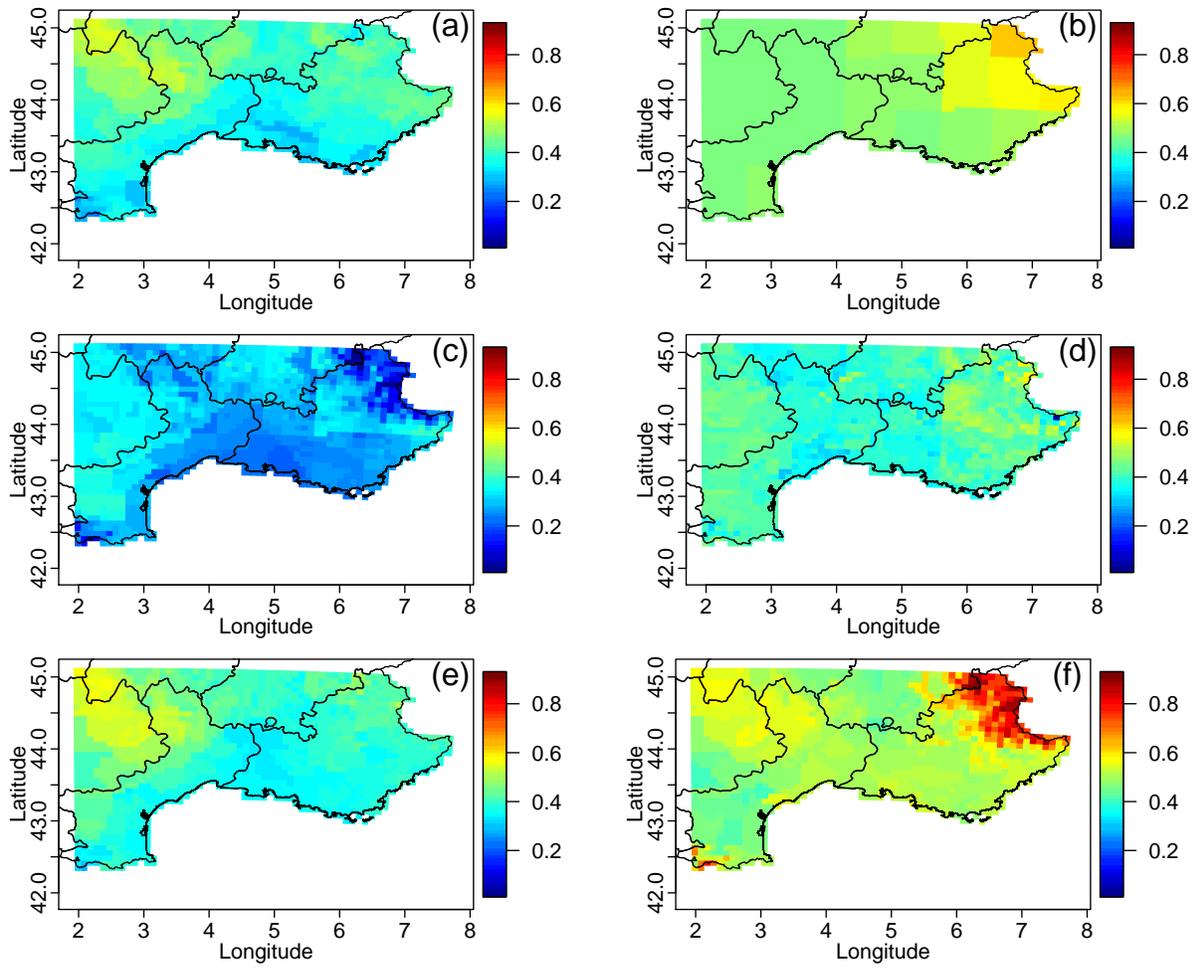


FIG. 13. Maps of 1-day lag precipitation auto-correlations in winter for (a) reference, (b) ERA-I, (c) independent bias correction, (d) conditional approach with PR given T2, (e) EC-BC and (f) Schaake shuffle on ERA-I without BC.

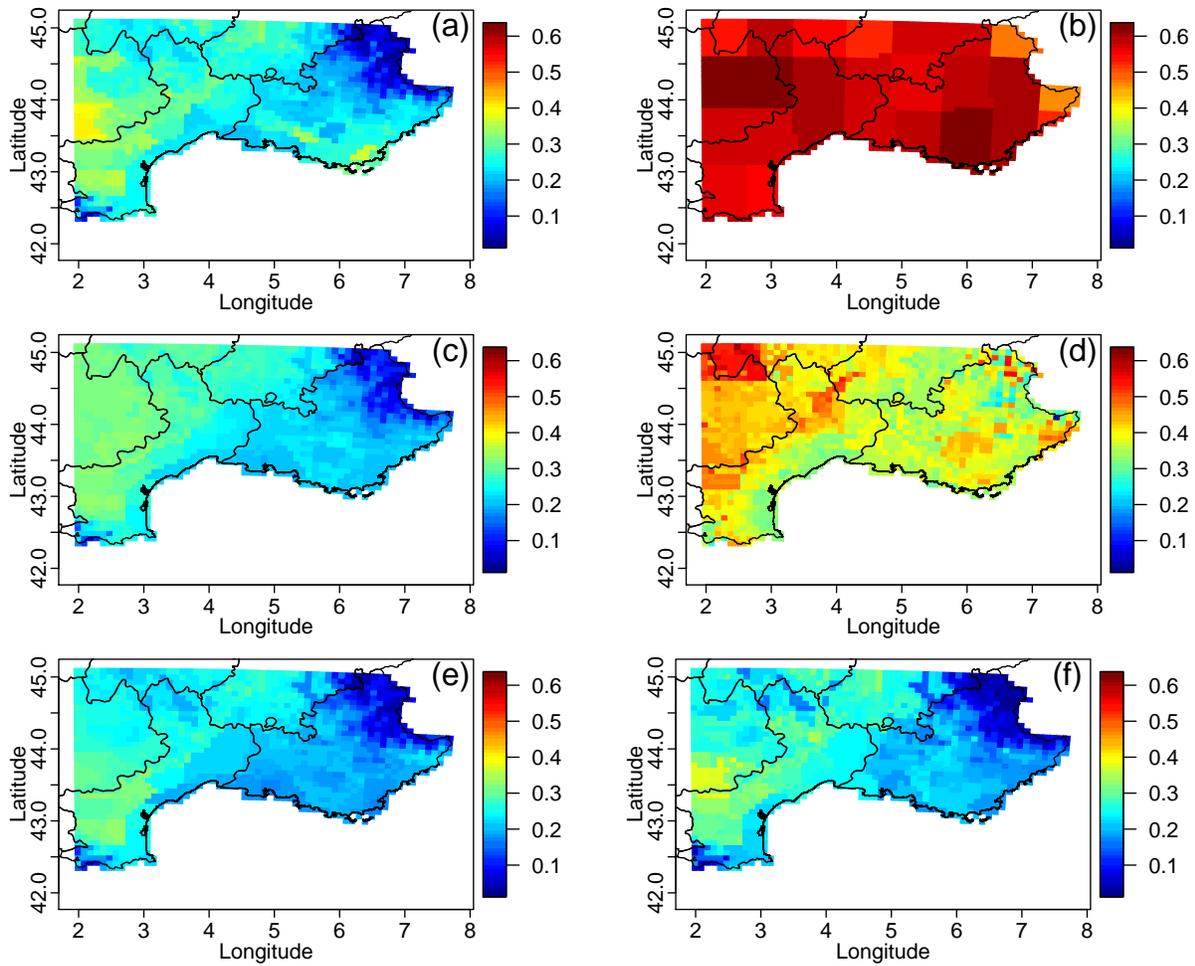


FIG. 14. Maps of daily probability of a dry rain given that the previous day was wet – i.e., $Proba(dry|wet)$ – in winter for (a) reference, (b) ERA-I, (c) independent bias correction, (d) conditional approach with PR given T2, (e) EC-BC and (f) Schaake shuffle on ERA-I without BC.

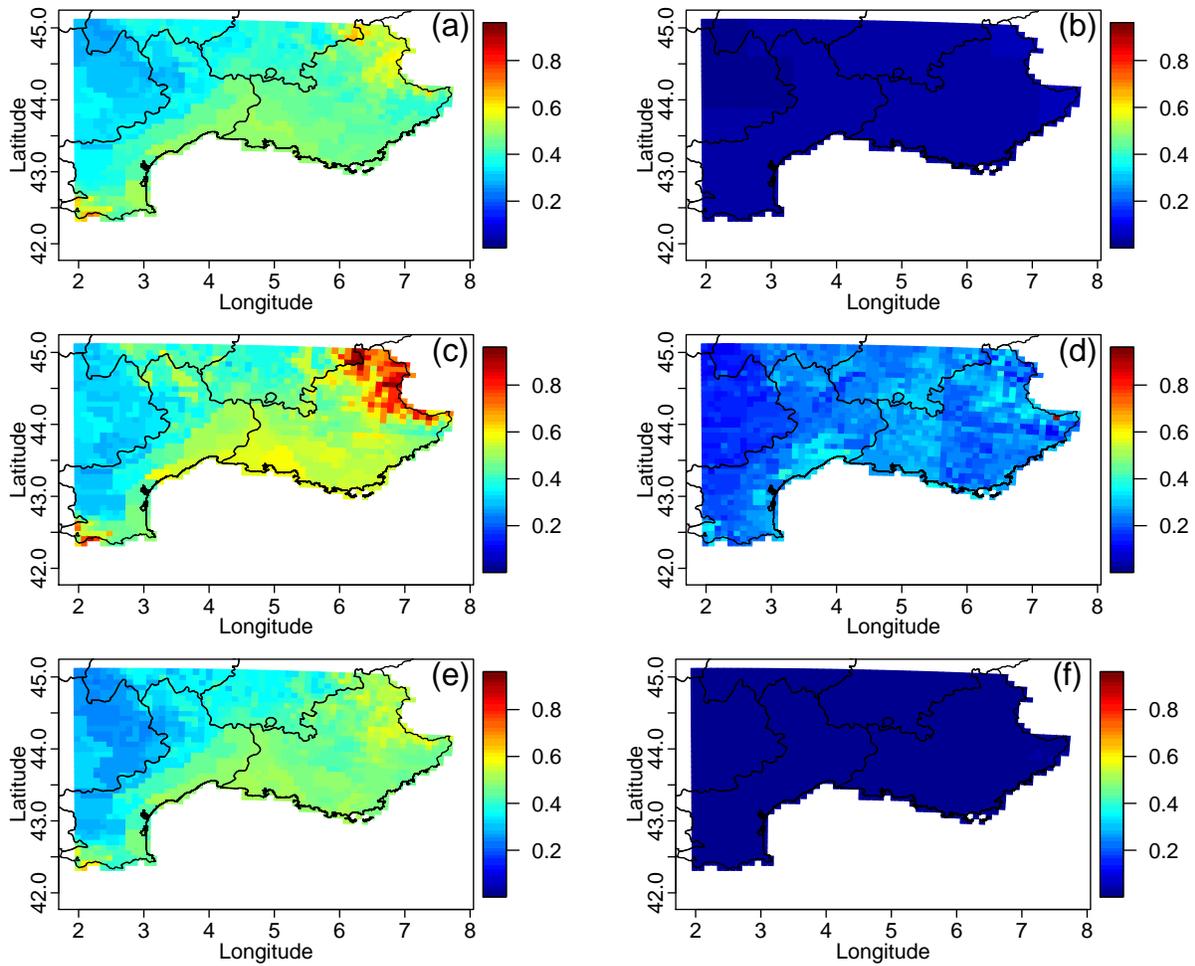


FIG. 15. Maps of daily probability of rain occurrence (i.e., wet day) given that the previous day was dry – i.e., $Proba(wet|dry)$ – in winter for (a) reference, (b) ERA-I, (c) independent bias correction, (d) conditional approach with PR given T2, (e) EC-BC and (f) Schaake shuffle on ERA-I without BC.